Truthful Reverse Auctions for Relay Selection with High Data Rates and Base Station Utility in D2D Networks

Aditya MVS, Harsh Pancholi, Priyanka P., Gaurav S. Kasbekar

***Abstract*—Device-to-Device (D2D) communication allows a cellular user (relay node) to relay data between the base station (BS) and another cellular user (destination node) experiencing poor direct channel conditions from the BS. However, the battery energy of a relay node gets depleted due its relaying activities, and hence, to compensate for this loss, relays need to be provided incentives. In this paper, we propose reverse auction mechanisms to assign a relay node to each destination node, when there are multiple potential relay nodes and multiple destination nodes, in each of the following three scenarios: 1) when relay nodes are allocated a fixed transmission power, 2) when relay nodes are allocated the transmission powers required to achieve the data rates desired by destination nodes, and 3) when the transmission powers of relay nodes are selected so as to approximately maximize the BS's utility. Monetary payments (incentives) are provided to the selected relay nodes in the auctions proposed for each of the above three scenarios. Also, in our model, the cost incurred due to interference caused by relay nodes to uplink cellular user communication is taken into account. We prove that all the proposed reverse auctions can be truthfully implemented as well as satisfy the individual rationality property. Using numerical computations, we show that in the fixed transmission power scenario, our proposed auction significantly outperforms an auction based on the widely used Vickrey-Clarke-Groves (VCG) mechanism in terms of the data rates achieved by destination nodes as well as the utility of the BS. Our proposed auctions are applicable to a variety of relaying schemes such as Normal relaying, Decode-and-Forward relaying, Amplify-and-Forward relaying and Selection relaying.**

I. Introduction

The demand from mobile users is rapidly increasing due to the proliferation of new applications such as video streaming services, online gaming etc. Long-Term Evolution (LTE)-Advanced is being extensively deployed worldwide to meet the growing demand [10]. Some of the objectives of LTE-Advanced are to provide improved cell-edge capacity relative to LTE [24] and decreased consumption of energy. Issues such as low signal to noise ratio (mainly at the cell edges) and coverage holes due to shadowing have to be tackled for throughput enhancement and improving cell edge capacity. As the link capacity of current technology is already close to the Shannon bound [21], the deployment of additional network infrastructure such as low-power base stations and dedicated relay nodes is considered as a possible solution. However, this involves huge deployment costs and also the number of subscribers in the network may not increase at the same rate in a particular region, which would make this solution unappealing for network operators. One alternative to avoid this is to use the concept of *Device-to-Device (D2D) communication* to improve the performance of a network [2]. D2D communication enables a mobile device to directly communicate with its peers bypassing the base station (BS) [8]. In this paper, we study a scenario where the BS requests some of the existing cellular users to act as relays between the BS and other cellular users to improve the throughput of cell-edge users and users that experience poor signal to noise ratio from the BS due to shadowing, and to extend the network coverage, i.e., the BS employs relaying using D2D communication. This also replaces a single high-powered link with two low-powered links, which can increase the energy efficiency of the network.

D2D communications, an innovative technique for next generation cellular networks, makes the relaying concept simpler with no need of introducing extra relay nodes in the network [2]. Also, it was shown in [26] that the achieved channel capacity in cellular networks in which D2D communication is used for relaying is enhanced when compared to the case without such relaying. We consider a scenario where D2D communication occurs *underlay*, i.e., D2D communication takes place on the same set of channels as traditional cellular communication (communication between the BS and cellular users) [2]. Note that underlay D2D communication increases the interference caused to the traditional cellular communication users. However, it is shown in [30] that through proper sharing of resources between the tradional cellular communication users and D2D users and control of transmission power, underlay D2D communication increases the overall throughput of the network.

As relaying of data (to another user with poor channel conditions from the BS) consumes energy, cellular users may not be willing to relay, since they would want to conserve battery energy for personal use in future. Thus, *incentives* must be provided by the centralized entity (BS or eNodeB) to make potential relays cooperate for throughput enhancement. In addition, although a BS can increase the achieved data rate of its cellular user experiencing poor channel conditions from the BS by selecting a relay which is willing to forward data to it, this will also increase the interference caused by the relay to its traditional cellular communication user which is using the same channel. So the costs incurred to the BS are: the incentives provided to the relay and the interference caused by the relay to its traditional cellular communication user. Thus, a BS has to select relays which can increase the throughput of the users experiencing poor channel conditions from the BS at a minimum expense to the BS and minimum interference to its traditional cellular communication users.

Apart from normal relaying, in which first the BS sends the

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message to the relay, which is ignored by the destination node, and then the relay forwards the message to the destination node, different *cooperative relaying schemes* [16] such as amplify-and-forward, decode-and-forward and selection relaying can be used. In each of the latter three schemes, the BS (source) transmits the message in the first time slot. Both the destination node and the relay receive this transmission in the first time slot. The relay node processes the received message (depending on the relaying scheme used) and sends it to the destination node in the second time slot. The destination node combines the transmission by the source in the first slot and by the relay in the second slot to form the received message. Cooperative relaying schemes have the advantage that they exploit space diversity to improve the achieved data rate [16].

In this paper, we consider a BS which requires relays to communicate with some of its cellular users (henceforth referred to as *destination nodes*) when the BS cannot communicate with the latter directly at sufficiently high data rates due to network coverage problems. This may be due to the location of the destination nodes in the shadow region of the BS or at the cell-edge. We study a *reverse auction* conducted by the BS in which the BS requests some of its users (henceforth referred to as *relay nodes*) to act as relays to its destination nodes. The BS provides monetary incentives to the relay nodes for acting as relays, since relays incur a cost due to their battery drain. For an auction to be feasible, the incentive provided by the BS to a relay node must be at least the cost incurred by the node for acting as a relay or else no node will participate in the auction. However, a relay node's incurred cost is private information of the node and is not known to the BS. Thus a greedy relay node can falsely declare the cost it incurs. Hence, mechanisms are required for ensuring that relay nodes *truthfully* declare the costs they incur. In this paper, we present reverse auctions that induce relay nodes to truthfully declare their costs. Specifically, in these auctions, potential relay nodes submit their incurred costs as bids to the BS, which then assigns a node to each destination node to act as a relay based on the bids submitted. The BS assigns a channel to transmit on and transmission power, along with providing a payment to each selected relay node. We propose reverse auctions for three different scenarios: 1) Constant power case, where the BS assigns a fixed transmission power to all the relay nodes, 2) Constant data rate case, where each destination node requests the BS for a desired data rate and the relay node assigned to a destination node must transmit at a power such that the desired data rate is achieved, and 3) Approximately maximizing the BS's utility case, where the assigned transmission power and the payment made to each relay node are determined by the BS such that the BS's utility is approximately maximized. Our proposed auctions for each of these three scenarios are applicable to all the above mentioned relaying schemes, viz., normal relaying, amplify-and-forward, decode-and-forward and selection relaying. We prove that all our proposed reverse auction mechanisms (i) guarantee truthful declaration of their incurred costs by relay nodes (i.e., are *incentive compatible* [20]), and (ii) satisfy the *individual rationality* property [20], i.e., the utility of a selected relay node is guaranteed to be non-negative. In the approximately maximizing the BS's utility case (scenario 3), the widely used Vickrey-Clarke-Groves (VCG) mechanism [20], on which several truthful auctions designed in prior work are based [3], [4], [5], [28] (see Section II), *is not applicable*. Also, we show via numerical computations that *our proposed auction for the constant power case (scenario 1) outperforms the auction based on the VCG mechanism [20] in terms of achieved data rates of destination nodes as well as BS utility*.

The rest of this paper is organised as follows. A review of related research literature is provided in Section II. Section III describes our network model and game formulation and gives a brief description of various relaying schemes that a BS can employ for relaying information. In Section IV, we describe our proposed auctions and show that they can be truthfully implemented and satisfy individual rationality. In Section V, we compare our proposed auctions with auctions based on the widely used VCG mechanism. In Section VI, we evaluate the performance of the proposed auctions via numerical computations. We provide conclusions in Section VII.

II. Related Work

In this section, we provide a review of related research literature. Relay assisted communication is studied in [9], [19], [31]. Here, the BS encourages its users to act as relays by providing them with incentives. However, the communication between a relay and a destination node is through a Wi-Fi channel. Hence there is no need for interference management. This is in contrast to the model in our paper, in which the communication between relays and destination nodes occurs underlay using cellular bands, and hence interference management is required.

Relay selection schemes in cooperative networks are studied in [3], [28]. An optimal relay assignment scheme called HERA in cooperative networks, which considers the selfish behaviour of the network users (relays) is proposed in [28]. However, the interference caused by the relays to the BS or cellular users is not considered as the availability of orthogonal channels for relays is assumed. Also, the transmission power of relays is selected arbitrarily. An auction based relay assignment scheme which considers interference in the calculations of the achieved data rates in cooperative networks is proposed in [3]. A centralized single round double auction scheme to select relays is proposed where the traffic flow users (source-destination pairs) and relays both submit their bids in the form of data rates achieved with and without using relays. Based on these bids, a maximum matching algorithm to increase the total capacity of the network is used to assign the relays and the Vickrey-Clarke-Groves (VCG) mechanism is used to determine the payment to the relays. Later a multi round auction where the relays are assigned to their buyers (cellular users) sequentially in a distributed network is proposed. However, the multi-round auction does not satisfy the incentive compatibility property. In contrast, all the proposed auctions in this paper satisfy the incentive compatibility property. Auction based allocation schemes in D2D networks are studied in [4], [11]. Optimal auction based resource allocation in D2D enabled multi-tier cellular networks is studied in [11]. A higher-tier BS acts as auctioneer whereas the D2D users and lower-tier BSs bid for channels and transmission power levels. The allocation mechanism is based on allocating channels and transmission power levels such that the total data rate is maximized while minimising the interference. Our work differs from the above papers [3], [11], [28] in that we consider both the uncertainty of information at the BS about the battery energy costs incurred by the relay nodes and the decrease in utility of the BS due to the interference caused by D2D communications. In our model, we not only consider the effect of interference by including it in the calculation of achieved data rates, but an additional loss term is introduced in the BS's utility that increases with the transmission power of a relay node. This is done to limit a relay node's transmission power.

A relay assisted D2D communication scheme is studied in [5], in which the BS is the auctioneer and D2D user pairs are the bidders and the BS allocates relays, channels for transmission and their respective power levels to the D2D user pairs. A D2D pair is allotted a relay if the relay results in increase in its data rate. The allocation mechanism maximizes the total increase in valuations (which depends on the achieved data rates) of all D2D user pairs. The VCG mechanism is followed in determining the payments to the relays and also to ensure truthfulness. In this paper, we too consider an auction conducted by a BS to assign relay nodes to the destination nodes. However, in our work the relay nodes are selected to assist the communication between the BS and destination nodes instead of assisting the communication between a pair of D2D users. Reverse auctions are also studied in [4] where truthfulness is achieved by following the second price auction. An auction conducted by the primary user in a Cognitive Radio Network to select a relay node to transmit its data is proposed in [13]. The auction is modelled as an optimal stopping problem where the primary user receives bid information from relays one by one and designs an optimal stopping policy. At the stopping time, the primary user selects the relay node. It is proved that the proposed auction satisfies individual rationality and can be truthfully implemented. However, the authors did not consider the cost of interference due to deployment of the relay in the network and only considered a single relay assignment. In contrast, in the model in this paper, we consider the cost of interference and the assignment of multiple relay nodes. A double auction, based on finding a maximum matching in a bipartite graph, for optimal assignment of relays in a cellular network consisting of multiple cellular users and multiple relay nodes is proposed in [29]. A cellular user is assigned a relay node only when there is an increment in channel capacity by cooperation. Three assignment problems are examined: 1) Maximizing the total number of edges in a matching, 2) Maximizing the total channel capacity in the network, 3) Maximizing the social welfare in the network. A similar network setting is used in [17]. As before, a bipartite graph is constructed, but with a difference that the edge weight now represents the energy efficiency of the source-relay-destination link which is defined as the ratio of channel capacity of the link to the total power consumed by the source node and the relay node. A maximum matching is found to obtain an efficient relay assignment. However, the auctions proposed in [17], [29] do not satisfy the incentive compatibility condition. This is in contrast to our proposed auction, which is also based on maximum matching in a bipartite graph, but is proved to satisfy the incentive compatibility condition.

Also, all the truthful auction mechanisms above [3], [4], [5], [28] use some form of the VCG mechanism [20]. In contrast, this paper proposes novel reverse auction based schemes *which differ from the VCG mechanism-based scheme*, in order to incentivize the relay nodes. Also, in all the above works the transmission power of a relay is either arbitrarily selected (fixed) [3], [5], [13], [28], [29] or selected to satisfy a certain *SINR* threshold [17] or selected to maximize the utility function considered [11]. In contrast, in this paper we consider three scenarios: 1) the BS assigns a fixed transmission power P to all the relay nodes, 2) the BS assigns transmission powers to different relay nodes to achieve the desired data rates of destination nodes, and 3) the BS selects the transmission power of each relay node to approximately maximize the BS's utility. In scenario 3, the *VCG-mechanism is not applicable*. Also, our numerical results show that in scenario 1), *our proposed reverse auction scheme outperforms the VCG mechanism based scheme in that it assigns relay nodes to destination nodes such that the achieved data rates of destination nodes and BS utility are higher*.

III. Network Model and Problem Formulation

A. Network Model

We consider a cellular network with multiple cells. We assume that an interference avoidance [18] algorithm is used by the BSs, and that this algorithm assigns spectrum resources (channels) to different BSs in each time slot such that inter-cell interference is negligible. So henceforth, we focus on a single cell which contains multiple cellular users. Fig. 1 depicts our network model; in this figure, the cell under consideration contains cellular users shown by stars and circles. We assume that time is divided into slots and in each slot, there would be some users that would need to receive data from the BS; among them, there could be some users which request the BS for relay aided communication. Let $\mathbf{D} = \{1, \ldots, D\}$ denote the set of cellular users which request for relay services. Henceforth, we refer to the users in $\mathbf{D}$ as "destination nodes"; these are shown by stars in Fig. 1. In order to deliver data to the destination nodes, the BS would send a relay request to a set of cellular users (relay nodes) in the cell which are willing to act as relays provided that they are compensated for their services. Let the set of relay nodes to which the request is sent be represented by $\mathbf{R} = \{1, 2, ..., R\}$; these are shown by circles in Fig. 1.

Information about channel conditions (qualities) is known to the BS through Channel State Information (CSI) conveyed by the cellular users. This CSI contains the channel gains between the BS and relays, between the BS and destination nodes and between the relays and destination nodes. This information can be estimated using reference signals, which are sent at known transmit powers are whose received powers are measured at the receivers [6]. For $i \in \mathbf{R}$, $j \in \mathbf{D}$, let $G_{i,j} \in K$ be the gain

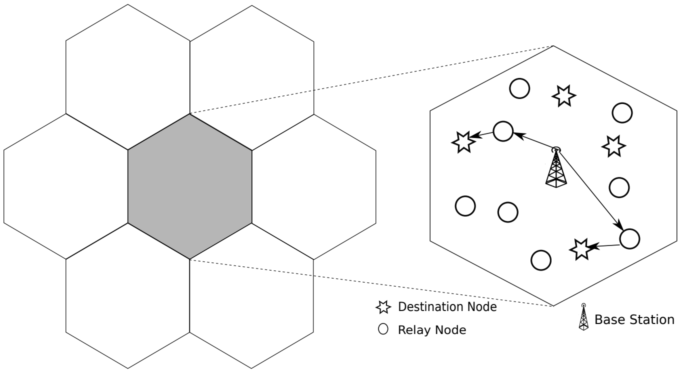

Fig. 1. The figure shows a cellular network with multiple cells. We assume that the BSs avoid inter-cell interference and each BS conducts an auction that assigns relay nodes present within its cell to its cellular users (destination nodes) that request relay services.

of the channel between relay node i and destination node j, where K represents the set of possible channel gain values, and let $G_{s,j} \in K$ be the channel gain between the BS and destination node j (s here represents the source which is the BS). Also, for $i \in \mathbf{R}$, let $G_{s,i} \in K$ be the gain of the channel between the BS and relay node i. We assume that all the above gains are known to the BS. Also, the gain, $G_{s,i}$, between the BS and relay node i and for each $j \in \mathbf{D}$, the gain, $G_{i,j}$, between relay node i and destination node j, are known to relay node i. Finally, the gain, $G_{i,j}$, between each relay node $i \in \mathbf{R}$ and destination node j and the gain, $G_{s,j}$, between the BS and destination node j are known to destination node j.

Now, battery power gets consumed when a cellular user acts as a relay and it is limited. Let B represent the set of all quantized battery power levels. Then, in a given time slot, a given relay node $i \in \mathbf{R}$ would be in some state $b_i \in B$. B also includes the dead state; node i cannot act as a relay if b_i is the dead state. Every relay node $i \in \mathbf{R}$ knows its own battery state b_i. However, b_i is private information of node i and is not known to the BS.

The relays assigned to destination nodes (using an auction) reuse the channels that are used by some cellular users for *uplink* (user to BS) communication and each relay node is allotted a unique channel. Also, before the auction to assign relays to destination nodes is conducted, the BS assigns a channel, which is also assigned to a cellular user for uplink communication, to each destination node $j \in \mathbf{D}$; a relay node which is assigned to a destination node uses this channel to communicate with its destination node. This pre-allocation of channels to destination nodes, for use by the relay nodes assigned to the destination nodes, is useful in estimating the interference at each destination node caused by the cellular user that transmits to the BS over the uplink using the same channel.

B. Game Formulation

1) Utility of a Relay Node: Consider a relay node $i \in \mathbf{R}$ which is assigned to destination node $j \in \mathbf{D}$. Let $\Gamma_{i,j}$ be the data rate achieved at destination node j with the help of relay node i. We assume that the payment made by the BS to the relay node is proportional to the achieved data rate $\Gamma_{i,j}$; thus, the payment made by the BS to the relay node would be $\beta\Gamma_{i,j}$, where β is the payment per unit data rate. The utility of the relay node is given by:

$$u_{i,j} = \beta\Gamma_{i,j} - E_{i,j} \tag{1}$$

where $E_{i,j}$ is the energy cost incurred by the relay node. The energy cost consists of two parts: 1) cost incurred while processing the received information from the BS and 2) cost incurred while transmitting the information to the destination node. Let $P_{c,i}$ denote the power required to process the received information and let $P_{i,j}$ be the power at which the relay node i transmits to destination node j. We assume that the total energy cost $E_{i,j}$ is a linear function of $P_{i,j} + P_{c,i}$ [1] and is given by:

$$E_{i,j} = \alpha_i(P_{i,j} + P_{c,i}) \tag{2}$$

where α_i is the cost per unit power, or, it can be said, the *valuation* relay node $i \in \mathbf{R}$ has for its power. α_i depends on b_i and is private information of node i. We assume that $P_{c,i}$ is proportional to the data rate, say $\Gamma_{s,i}$, of the information received by relay node i from the BS, i.e., $P_{c,i} = k\Gamma_{s,i}$ [12]; we also assume that the BS knows the constant k.

2) Utility of Base Station: Recall that we consider a cellular network in which D2D communication occurs underlay; in particular, we assume that each relay node i, which is assigned to a destination node, uses the same channel as some cellular user that communicates over the uplink with the BS.

The utility of the BS is given by:

$$U = \sum_{i,j} U_{i,j}, \tag{3}$$

where the summation is over all relay nodes $i \in \mathbf{R}$ and destination nodes $j \in \mathbf{D}$ such that j is assigned node i as relay. The contribution, $U_{i,j}$, to U from the pair (i, j) is a function of the revenue the BS gets from j, the payment made to the relay node i assigned to destination node j and the interference caused by relay node i at the BS since it uses the same channel as an uplink cellular user. Note that each destination node that receives relay service makes a payment to the BS as compensation. Let a be the revenue per unit transmission rate obtained by the BS from a destination node. We assume the cost of interference caused to the BS by assigning node i to destination node j as relay to be linearly dependent on $P_{i,j}$. Let $c_iP_{i,j}$ be this cost. Then $U_{i,j}$ is given by:

$$U_{i,j} = a\Gamma_{i,j} - \beta\Gamma_{i,j} - c_iP_{i,j}, \tag{4}$$

where $\Gamma_{i,j}$ is the data rate achieved at destination node j when it is assigned relay node i.

3) Objective: Our objective in this paper is to design an auction that can be conducted by the BS to assign to each destination node, a unique relay node. The two desirable properties of any auction are i) it must satisfy the property of individual rationality (IR) [20], and ii) it must be truthfully implementable [20]. An auction satisfies IR if no relay gets a

[1] All our results readily generalize to the case when $E_{i,j} = \alpha_i(P_{i,j} + P_{c,i}) + P_0$ where P_0 is a constant.

negative utility under any outcome of the auction [20]. Also an auction is truthfully implementable if revealing its true valuation α_i is the dominant strategy for each relay node $i \in \mathbf{R}$ [20]. In Section IV, we describe our proposed reverse auctions, which satisfy the above two properties, designed for three scenarios: (A) Constant power case, where the BS assigns a fixed transmission power P to each relay node, (B) Constant data rate case where the BS assigns a transmission power to each relay node i to achieve the desired data rate, say Γ_j, at the destination node j to which it is assigned, and (C) the case where the BS selects the relay nodes's transmission powers to approximately maximize its own total utility (U in (3)). We prove that in each case the auction satisfies IR and can be truthfully implemented. We design auctions for four different relaying schemes– normal relaying, amplify-and-forward, decode-and-forward and selection relaying [16].

Remark 1: All our results readily generalize to the case where the interference cost function $c_i P_{i,j}$ in (4) is replaced with any other function of $P_{i,j}$ which is differentiable, strictly increasing and convex.

C. Relaying Schemes

In this subsection, we briefly describe some basic cooperative communication protocols, any one of which may be employed by relay nodes assigned to destination nodes, using our proposed reverse auctions, for forwarding data. Consider a relay node i which is assigned to destination node j. In all the following relaying schemes, we divide each time slot into two equal parts, which we denote by mini-slot 1 and mini-slot 2. In mini-slot 1, the BS transmits the message and this transmission is received by both the relay node i and destination node j. In mini-slot 2, the relay node i retransmits the message it received in mini-slot 1 (possibly after processing it), whereas the BS does not transmit any message. Depending upon the relaying scheme employed, this retransmitted signal can simply be an exact copy of the signal that relay node i received in mini-slot 1 or its decoded version. The next few paragraphs give a brief overview of the operation of various relaying schemes and the data rates achieved at destination node j through them.

1) Normal Relaying Scheme: In the normal relaying scheme, the BS transmits its message in mini-slot 1, which is received by the relay node, but ignored by the destination node; in mini-slot 2 the relay node forwards the received message to the destination. This kind of relaying operation can only extend the range of the communication or save transmission power but does not achieve any diversity gain. The data-rate capacity of this relaying scheme is determined by the weaker of the two links– the link from the BS to the relay node and that from the relay node to the destination node. The data rate achieved at the destination node j is given by:

$$\Gamma_{i,j} = \min\left\{\frac{W}{2}\log_2(1+SINR_{s,i}), \frac{W}{2}\log_2(1+SINR_{i,j})\right\}. \tag{5}$$

where s denotes the BS, P_s is the power at which the BS transmits, W is bandwidth of the channel, $SINR_{s,i} = \frac{P_s G_{s,i}}{I_{s,i}+N_{s,i}}$ is the signal to interference ($I_{s,i}$) plus noise ($N_{s,i}$) ratio of the link between the BS and relay node i, $SINR_{i,j} = \frac{P_{i,j} G_{i,j}}{I_{i,j}+N_{i,j}}$ is the signal to interference ($I_{i,j}$) plus noise ($N_{i,j}$) ratio of the link between the relay node i and destination node j. Note that $\frac{W}{2}\log_2(1+SINR_{s,i})$ (respectively, $\frac{W}{2}\log_2(1+SINR_{i,j})$) is the Shannon capacity of the channel between the BS and relay node i (respectively, relay node i and destination node j); the factor $\frac{1}{2}$ appears in each capacity expression since communication occurs on each of the above two channels for $\frac{1}{2}$ of the duration of a time-slot. In this work, we assume that the channel gain between the BS and the relay node is sufficiently high so that the BS can adjust its transmission power P_s to make the capacities of both links equal. So now, the data-rate capacity equals:

$$\Gamma_{i,j} = \frac{W}{2}\log_2(1+SINR_{i,j}). \tag{6}$$

2) Amplify-and-Forward Relaying Scheme: The amplify-and-forward (AF) scheme is a simple relaying scheme in which, in mini-slot 1, the BS transmits the message to the relay and the destination node; also, the relay node amplifies the received signal and in mini-slot 2, it forwards the amplified version of the signal to the destination node [16]. Apart from its simplicity and low cost, its advantage is that the relay node does not need to decode and re-encode the received signal. However, a major limitation of this scheme is that the noise in the signal received at the relay node also gets amplified. The data-rate capacity of the AF cooperative relaying protocol is given by [16]:

$$\Gamma_{i,j} = \frac{W}{2}\log_2\left(1+SINR_{s,j}+\frac{SINR_{s,i}SINR_{i,j}}{1+SINR_{s,i}+SINR_{i,j}}\right), \tag{7}$$

where $SINR_{s,j} = \frac{P_s G_{s,j}}{I_{s,j}+N_{s,j}}$ is the signal to interference ($I_{s,j}$) plus noise ($N_{s,j}$) ratio of the link between the BS and destination node j, $SINR_{s,i}$ and $SINR_{i,j}$ are as defined above for the normal relaying scheme.

3) Decode-and-Forward Relaying Scheme: In the decode-and-forward (DF) relaying scheme, in mini-slot 1, the BS transmits the message to the relay and the destination node; the relay node decodes the received signal from the BS and re-encodes it before forwarding it to the destination node in mini-slot 2 [16]. As a result of decoding and encoding the received signal, the relay node incurs an additional processing cost. The data-rate capacity of this cooperative relay protocol is given by [16]:

$$\begin{aligned}\Gamma_{i,j} = \min\Big\{&\frac{W}{2}\log_2(1+SINR_{s,i}),\\ &\frac{W}{2}\log_2(1+SINR_{s,j}+SINR_{i,j})\Big\},\end{aligned} \tag{8}$$

where the $SINR$ terms are as defined above for the AF case.

4) Selection Relaying Scheme: Unlike fixed relaying schemes like AF and DF, cooperative communication is employed only if the channel conditions satisfy certain conditions in the selection relaying protocol. The BS transmits the message to the relay node and the destination node in mini-slot 1 as in the AF and DF cooperative schemes. But the relay node forwards this signal only if the SINR from the BS to the relay node is above a certain threshold ζ. If this threshold constraint on the $SINR$ is satisfied, then the relay node forwards the signal using the DF protocol, otherwise the BS again transmits the same signal to the destination node in mini-slot 2 [16]. So the data-rate capacity of the selection relaying cooperative communication protocol is given by [16]:

$$\Gamma_{i,j} = \begin{cases} \frac{W}{2}\log_2(1+2SINR_{s,j}), & \text{if } SINR_{s,i} < \zeta, \\ \frac{W}{2}\log_2(1+SINR_{s,j}+SINR_{i,j}), & \text{otherwise.} \end{cases} \tag{9}$$

If $SINR_{s,i} < \zeta$, then relay node i is not assigned to destination node j.

IV. Proposed Reverse Auctions

First, we briefly explain some terminology and notations from graph theory that are used in this section. A graph $G = (V, E)$, with node set V and edge set E, is a *bipartite graph* if V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in E is between a node in V_1 and a node in V_2 [27]. We represent a bipartite graph as $G = (V_1, V_2, E)$. A *matching* $m \subset E$ in a bipartite graph is a collection of edges such that no two edges have a common endpoint [27]. A matching m is *maximal* if $m \cup e$ is not a matching for any edge $e \in E \setminus m$ [27].

In this section, we present our proposed auctions that allocate to each destination node $j \in \mathbf{D}$, a relay node $i \in \mathbf{R}$. In Sections IV-A, IV-B and IV-C, we present our proposed auctions for the constant power case, constant data rate case and the case where the transmit power is selected to approximately maximize the BS's utility respectively.

A. Constant Power Case

In general, in a cellular network, the BS can either allocate different transmit power levels to different cellular users (e.g., taking into account the current channel gains) or assign a fixed transmit power to all cellular users [23]. Although the former scheme, a variable transmit power scheme, allows a more flexible allocation, the fixed power allocation scheme is easier to implement due to its simplicity and also the loss in performance is negligible compared to the former for dense deployments of BSs [14], [25]. So in this subsection, we consider the case where the BS assigns a fixed transmission power P to all the relay nodes. By (1) and (2), the utility of a relay node i if it is assigned to destination node j is:

$$u_{i,j} = \beta_i \Gamma_{i,j} - \alpha_i(P + P_{c,i}). \tag{10}$$

A relay node gets 0 utility if it is not assigned to any destination node. We now propose an auction which is based on matching in bipartite graphs. First, each relay i declares its valuation, α_i, to the BS. Then we construct a complete bipartite graph [2] $G = (\mathbf{R}, \mathbf{D}, E)$, where $\mathbf{R}$ (respectively, $\mathbf{D}$) is the set of all relay nodes (respectively, destination nodes). The weight of the edge between relay node $i \in \mathbf{R}$ and destination node $j \in \mathbf{D}$ is defined to be $\frac{\alpha_i(P+P_{c,i})}{\Gamma_{i,j}}$. Let (i, j) denote the edge between relay node $i \in \mathbf{R}$ and destination node $j \in \mathbf{D}$. Also, let $\mathbf{M}$ denote the set of all possible maximal matchings in the above graph. For every maximal matching $m \in \mathbf{M}$, we define a corresponding weight w_m, which is equal to the sum of weights of all the edges in m. Let R_m denote the set of all relay nodes which are in the neighbourhood [3] of $\mathbf{D}$ under the matching m. The proposed algorithm is based on finding a maximal matching with the minimum weight. If we denote $w_{min} = \min_{m\in\mathbf{M}} w_m$ and $m_{min} = \operatorname{argmin}_{m\in\mathbf{M}} w_m$, we select the relay nodes $R_{m_{min}}$ as the auction winners, each of which is assigned to its neighbour in the set $\mathbf{D}$ under the matching m_{min}. We denote for every relay node i, $w_{m_{min}^{-i}} = \min_{m\in\mathbf{M}, i\notin R_m} w_m$ and $\mathbf{M}_i$ as the set of all maximal matchings such that for every $m \in \mathbf{M}_i$ we have $i \in R_m$ and $w_m \le w_{m_{min}^{-i}}$. If a relay node $i \in R_{m_{min}}$, then for every $m \in \mathbf{M}_i$, we define:

$$p_{i,m} = \left(w_{m_{min}^{-i}} - w_m + \frac{\alpha_i(P + P_{c,i})}{\Gamma_{i,j}} \right) \Gamma_{i,j}, \tag{11}$$

where $j \in \mathbf{D}$ is the adjacent vertex of node i in matching m. The payment given to relay node i is $p_i = \max_{m\in\mathbf{M}_i} p_{i,m}$. The sequence of steps that implements the above auction are described in Fig. 2.

1: Construct a complete weighted bipartite graph $G = (\mathbf{R}, \mathbf{D}, E)$.
2: Define the weight of edge (i, j) to be $\frac{\alpha_i(P+P_{c,i})}{\Gamma_{i,j}}$.
3: Select a maximal matching m_{min} such that $m_{min} = \operatorname{argmin}_{m\in\mathbf{M}} w_m$.
4: If $(i, j) \in m_{min}$, where $(i, j) \in \mathbf{R} \times \mathbf{D}$, assign relay node i to destination node j.
5: If relay node i is assigned to destination node j, then it is paid $p_i = \max_{m\in\mathbf{M}_i} p_{i,m}$, else $p_i = 0$ and relay node i is not required to transmit any data.

Fig. 2. Auction for constant power case.

Theorem 1: The auction in Fig. 2 satisfies individual rationality and can be truthfully implemented.

Proof: Let us consider relay node $i \in \mathbf{R}$. We denote $w^i_{min} = \min_{m\in\mathbf{M}, i\in R_m} w_m$. Let us assume that node i is not selected as a relay when it reveals its valuation α_i truthfully. This implies that $w^i_{min} \ge w_{min}$. Assume that instead it declares α'_i. This leads to a change in the values of w_m, $m \in \mathbf{M}$. As a result, let w'_m denote the new weight of the maximal matching $m \in \mathbf{M}$. If $\alpha'_i > \alpha_i$, then $w_{min} = w'_{min} \le w^i_{min} < w'^i_{min}$. So node i is still not selected as a relay. If $\alpha'_i \le \alpha_i$, then node i is selected if $w'^i_{min} \le w_{min}$. Let $M'_i = \{m \in \mathbf{M} : i \in R_m, w'_m \le w'_{m_{min}^{-i}}\}$ (Note that in this case, $w'_{m_{min}^{-i}} = w_{min}$). The payment to node i is $p'_i = \max_{m\in\mathbf{M}'_i} p'_{i,m}$. But for every $m \in \mathbf{M}'_i$, we have:

$$\begin{aligned} p'_{i,m} &= \left(w'_{m_{min}^{-i}} - w'_m + \frac{\alpha'_i(P + P_{c,i})}{\Gamma_{i,j}} \right) \Gamma_{i,j} \\ &= \left(w_{min} - w'_m + \frac{\alpha'_i(P + P_{c,i})}{\Gamma_{i,j}} \right) \Gamma_{i,j}, \end{aligned}$$

[2] A bipartite graph $G = (V_1, V_2, E)$ is said to be complete if there is an edge between every $v_1 \in V_1$ and every $v_2 \in V_2$.

[3] The neighbourhood of a vertex v under the matching m is the set of all vertices which are connected by an edge in m with the vertex v. The neighbourhood of a set of vertices C under the matching m is the set $\{v : v$ is in the neighbourhood of a vertex $c \in C$ under $m\}$.

where $(i,j) \in m$. But we have $w_m - \frac{\alpha_i(P+P_{c,i})}{\Gamma_{i,j}} = w'_m - \frac{\alpha'_i(P+P_{c,i})}{\Gamma_{i,j}}$. Substituting this in the above equality, we get

$$\begin{aligned} p'_{i,m} &= (w_{min} - w_m)\Gamma_{i,j} + \alpha_i(P + P_{c,i}) \\ &\leq \alpha_i(P + P_{c,i}). \end{aligned}$$

The above inequality holds because $w_{min} \leq w_m$. Since $p'_i = \max_{m \in \mathbf{M}'_i} p_{i,m}$ and by (10), it follows that the utility of node i is ≤ 0 when it falsely declares its valuation to be α'_i. Now, let us consider the case where relay node i is selected and is assigned to destination node j when it declares its valuation α_i truthfully. Suppose node i declares α'_i instead and is still selected as a relay. Then $p'_{i,m}$ for each m is equal to $(w_{m^{-i}_{min}} - w'_m + \frac{\alpha'_i(P+P_{c,i})}{\Gamma_{i,j}})\Gamma_{i,j}$. But as stated above, we have $w_m - \frac{\alpha_i(P+P_{c,i})}{\Gamma_{i,j}} = w'_m - \frac{\alpha'_i(P+P_{c,i})}{\Gamma_{i,j}}$. So $p'_{i,m} = p_{i,m}$. By separately considering the cases $\alpha'_i < \alpha_i$ and $\alpha'_i > \alpha_i$, it can be checked that this implies that a node i which is selected as a relay when it reveals its true valuation will not get any additional benefit by manipulating its valuation. Also, if node i is selected as a relay when it reveals its true valuation, then the payment made to it is $p_i = \max_{m \in \mathbf{M}_i} p_{i,m}$. Since $p_{i,m} = \left(w_{m^{-i}_{min}} - w_m + \frac{\alpha_i(P+P_{c,i})}{\Gamma_{i,j}}\right)\Gamma_{i,j}$ and $w_m \leq w_{m^{-i}_{min}}$ for $m \in \mathbf{M}_i$, we have $p_{i,m} \geq \alpha_i(P + P_{c,i})$. So by (10), the utility of node i is ≥ 0. This proves the individual rationality property. ∎

Remark 2: Note that in the above auction, an expression for the data rate $\Gamma_{i,j}$ is not mentioned. The BS can choose the type of relaying scheme it wants to implement and a data rate expression is chosen accordingly. For example, if the BS chooses the decode-and-forward relaying scheme, then the data rate expression in (8) is used to calculate the achieved data rate at the destination node for each of the relay nodes in $\mathbf{R}$. It can be checked that Theorem 1 and its proof hold regardless of which of the four relaying schemes described in Section III-C is used.

We now prove that the auction in Fig. 2 can be implemented in polynomial time. We write the computational complexity of this auction in terms of the computational complexity of the *Hungarian algorithm* [15], which can be used to find the minimum weighted maximal matching in a bipartite graph $G = (\mathbf{R}, \mathbf{D}, E)$. The Hungarian algorithm has a time complexity of $O((R+D)^2 \log(R+D) + (R+D)RD)$ [7], where $R = |\mathbf{R}|$ and $D = |\mathbf{D}|$. Let $\mathcal{H}$ denote this time complexity.

Proposition 1: The time complexity of the auction in Fig. 2 is $O(D^2\mathcal{H})$.

The proof of the above Proposition is provided in Appendix A.

B. Constant Data Rate Case

In the constant data rate case, each destination node j requests data transmission to it at a certain fixed desired rate, say Γ_j. For example, this occurs when a destination node is streaming an audio or video file or is in an audio or video conference call. The relay which is assigned to this destination node must select its transmission power such that it achieves the desired data rate. The auction for this case is similar to the auction that is proposed for the constant power case, with the difference being that instead of assigning a constant power P for each of the relays, the BS now assigns a power $P_{i,j}$ to relay i assigned to destination node j such that $\Gamma_{i,j} = \Gamma_j$. If the required power $P_{i,j} > P_m$, where P_m is the maximum transmission power of a relay node, then we do not assign relay node i to destination node j. Closed form expressions for the power $P_{i,j}$ for each of the four relaying schemes described in Section III-C are provided in Appendix B. Similar to the constant power case, after each relay declares its valuation, α_i, to the BS, we construct a complete bipartite graph $(\mathbf{R}, \mathbf{D}, E)$; the weight of the edge between relay node i and destination node j is $\frac{\alpha_i(P_{i,j}+P_{c,i})}{\Gamma_j}$ if $P_{i,j} \leq P_m$ and ∞ if $P_{i,j} > P_m$. The payment to relay node i if it is assigned to destination node j is given by $p_i = \max_{m \in \mathbf{M}_i} p_{i,m}$, where:

$$p_{i,m} = \left(w_{m^{-i}_{min}} - w_m + \frac{\alpha_i(P_{i,j} + P_{c,i})}{\Gamma_j}\right)\Gamma_j \qquad (12)$$

The sequence of steps that implements the proposed auction is given in Fig. 3.

1: Construct a complete weighted bipartite graph $G = (\mathbf{R}, \mathbf{D}, E)$.
2: Define the weight of edge (i,j) to be $\frac{\alpha_i(P_{i,j}+P_{c,i})}{\Gamma_j}$ if $P_{i,j} \leq P_m$ and ∞ if $P_{i,j} > P_m$.
3: Select a maximal matching m_{min} such that $m_{min} = \operatorname{argmin}_{m \in \mathbf{M}} w_m$.
4: If $(i,j) \in m_{min}$ where $(i,j) \in \mathbf{R} \times \mathbf{D}$, assign relay node i to destination node j.
5: If relay node i is assigned to destination node j, then it is paid $p_i = \max_{m \in \mathbf{M}_i} p_{i,m}$, else $p_i = 0$ and relay node i is not required to transmit any data.

Fig. 3. Auction for constant data rate case.

Theorem 2: The auction in Fig. 3 satisfies individual rationality and can be truthfully implemented.

The proof is similar to that of Theorem 1 and is omitted for brevity. Also, it can be checked that Theorem 2 holds regardless of which of the four relaying schemes described in Section III-C is used.

Proposition 2: The time complexity of the auction in Fig. 3 is $O(D^2\mathcal{H})$.

The proof is similar to that of Proposition 1 and is omitted for brevity.

C. Selection of Power to Approximately Maximize BS Utility

In this subsection, we design an auction in which the BS requests each relay node to transmit at a power that will approximately maximize the BS's utility. Let $P_{i,j}$ denote the power at which the BS requires relay node i to transmit to destination node j [4]. The BS makes a payment of $\beta_{i,j}\Gamma_{i,j}$ if relay node i is assigned to destination node j. So by (1) and (2) the utility of relay node i is given by:

$$u_{i,j} = \beta_{i,j}\Gamma_{i,j} - \alpha_i(P_{i,j} + P_{c,i}), \qquad (13)$$

[4] $P_{i,j} = 0$ if relay node i is not assigned to destination node j.

and by (4) the contribution to the utility of the BS (U in (3)) from pair (i, j) when relay node i is assigned to destination node j is given by:

$$U_{i,j} = (a - \beta_{i,j})\Gamma_{i,j} - c_i P_{i,j}. \quad (14)$$

For the individual rationality condition to be satisfied, $u_{i,j} \geq 0$ $\forall i, j$; so by (13), $\beta_{i,j}\Gamma_{i,j} \geq \alpha_i(P_{i,j} + P_{c,i})$. However, by (14) the BS gets maximum utility when $\beta_{i,j}\Gamma_{i,j} = \alpha_i(P_{i,j} + P_{c,i})$. So the maximum contribution to the utility of the BS from pair (i, j) when relay node i is assigned to destination node j and relay node i transmits at power $P_{i,j}$ is:

$$U_{i,j} = a\Gamma_{i,j} - \alpha_i(P_{i,j} + P_{c,i}) - c_i P_{i,j}. \quad (15)$$

Since the only variable in the above expression is $P_{i,j}$, we find the power that maximizes $U_{i,j}$. Suppose $P^*_{i,j}$ [5] maximizes $U_{i,j}$ in (15). $P^*_{i,j}$ depends on the type of relaying scheme employed by the BS. Closed form expressions for the power $P^*_{i,j}$ for each of the four relaying schemes described in Section III-C are provided in Appendix B. The maximum contribution to the utility of the BS from pair (i, j) when relay node i is assigned to destination node j is:

$$U^*_{i,j} = a\Gamma^*_{i,j} - \alpha_i(P^*_{i,j} + P_{c,i}) - c_i P^*_{i,j} \quad (16)$$

where $\Gamma^*_{i,j}$ is the data rate achieved at destination node j when relay node i is transmitting at power $P^*_{i,j}$.

For our proposed auction, first, each relay i declares its valuation, α_i, to the BS. Then we construct a complete bipartite graph $G = (\mathbf{R}, \mathbf{D}, E)$. For each pair of nodes $(i, j) \in \mathbf{R} \times \mathbf{D}$, nodes i and j are connected by an edge whose weight is $U^*_{i,j}$. We denote the set of all possible maximal matchings as $\mathbf{M}$ and an individual matching by m. For every matching $m \subset E$, we define w_m as the sum of weights of all the edges $(i, j) \in m$. For every matching m, we define a set R_m which consists of all relay nodes that are in the neighbourhood of $\mathbf{D}$ under the matching m. If we denote $w_{max} = \max_{m \in \mathbf{M}} w_m$ and $m_{max} = \operatorname{argmax}_{m \in \mathbf{M}} w_m$, then we select the relay nodes in $R_{m_{max}}$ as the winners of the auction. Each relay node in $R_{m_{max}}$ is assigned to its neighbour in $\mathbf{D}$ under the maximal matching m_{max}. For every relay node i we denote $w^{-i}_{max} = \max_{m \in \mathbf{M}, i \notin R_m} w_m$. A relay node i which is assigned to destination node j is paid:

$$p_{i,j} = (w_{max} - w^{-i}_{max} + \alpha_i(P^*_{i,j} + P_{c,i})). \quad (17)$$

The sequence of steps that implements our proposed auction is given in Fig. 4.

Theorem 3: The auction in Fig. 4 satisfies individual rationality and can be truthfully implemented.

Proof: Consider node i. Let $u_{i,k}$ denote the utility of relay node i when it declares its valuation truthfully and is assigned to destination node k. k can be a pseudo user if relay node i is not assigned to any destination node when it reveals its true valuation. Then $u_{i,k}$ is simply zero. Let us assume that relay node i manipulates its valuation and declares α'_i instead. This will change the weights of all matchings in the set $\mathbf{M}^c_i = \{m \in \mathbf{M} : (i, l) \in m \text{ for some } l \in \mathbf{D}\}$. Let the new

[5] In case $P^*_{i,j} \leq 0$, then we choose $P_{i,j} = 0$, else $P_{i,j} = P^*_{i,j}$.

1: Construct a complete weighted bipartite graph $G = (\mathbf{R}, \mathbf{D}, E)$.
2: Define the weight of edge (i, j) to be $U^*_{i,j}$.
3: Select a maximal matching m_{max} such that $m_{max} = \operatorname{argmax}_{m \in \mathbf{M}} w_m$, where $\mathbf{M}$ is the set of all maximal matchings and w_m represents the sum of weights of all edges in the maximal matching m.
4: If $(i, j) \in m_{max}$ where $(i, j) \in \mathbf{R} \times \mathbf{D}$, assign relay node i to the destination node j.
5: If relay node i is assigned to destination node j, then it is paid $p_{i,j} = (w_{max} - w^{-i}_{max} + \alpha_i(P^*_{i,j} + P_{c,i}))$ and transmits at power $P^*_{i,j}$, else it is paid 0 and relay node i is not required to transmit any data.

Fig. 4. Auction to approximately maximize the BS's utility.

weight of the matching $\bar{m} \in \mathbf{M}^c_i$ be $w'_{\bar{m}}$. If some matching $m \in \mathbf{M}^c_i$ is the matching with maximum weight w'_m, then relay node i is assigned to a destination node. Otherwise relay node i is not assigned to any destination node in which case its utility is 0. Assume that some $m \in \mathbf{M}^c_i$ is the matching with the maximum weight w'_m and that relay node i is assigned to destination node j in matching m. Suppose when relay node i declares α'_i, it is assigned transmit power $P'_{i,j}$, and $\Gamma'_{i,j}$ is the data rate achieved at destination node j when relay node i transmits at power $P'_{i,j}$; also, let us denote the new weight of the edge (i, j) as $U'_{i,j}$. We then have the following equality: $w'_m - U'_{i,j} = w_m - U^*_{i,j}$. Now by (1), (2) and (17), the utility of relay node i is given by:

$$\begin{aligned}
u'_{i,j} =& w'_m - w^{-i}_{max} + \alpha'_i(P'_{i,j} + P_{c,i}) - \alpha_i(P'_{i,j} + P_{c,i}) \\
=& w_m - U^*_{i,j} + U'_{i,j} - w^{-i}_{max} + \alpha'_i(P'_{i,j} + P_{c,i}) \\
&- \alpha_i(P'_{i,j} + P_{c,i}) \quad (\text{since } w'_m - U'_{i,j} = w_m - U^*_{i,j}) \\
=& w_m - w^{-i}_{max} + a\Gamma'_{i,j} - \alpha'_i(P'_{i,j} + P_{c,i}) - c_i P'_{i,j} \\
&- a\Gamma^*_{i,j} + \alpha_i(P^*_{i,j} + P_{c,i}) + c_i P^*_{i,j} \\
&+ \alpha'_i(P'_{i,j} + P_{c,i}) - \alpha_i(P'_{i,j} + P_{c,i}) \qquad (\text{by } (16)) \\
=& w_m - w^{-i}_{max} + a\Gamma'_{i,j} - \alpha_i(P'_{i,j} + P_{c,i}) - c_i P'_{i,j} \\
&- (a\Gamma^*_{i,j} - \alpha_i(P^*_{i,j} + P_{c,i}) - c_i P^*_{i,j}) \\
\leq& w_m - w^{-i}_{max}
\end{aligned}$$

The inequality holds because when relay node i declares its valuation truthfully, $P^*_{i,j}$ maximizes $U_{i,j}$. If node i is not selected when it declares its valuation truthfully, then we have $w_m - w^{-i}_{max} = w_m - w_{max} \leq 0$ and if node i is selected when it declares its valuation truthfully, then $w_m - w^{-i}_{max} \leq w_{max} - w^{-i}_{max} = u_{i,k}$. This proves that the above auction can be truthfully implemented. Also, since from (17), the utility of relay node i assigned to a destination node j is $w_{max} - w^{-i}_{max} \geq 0$, the proposed auction satisfies the individual rationality property. ■

Proposition 3: The time complexity of the auction in Fig. 4 is $O(D\mathcal{H})$.

The proof of the above Proposition is provided in Appendix A.

V. Reverse Auctions based on the VCG Mechanism

The Vickrey-Clarke-Groves (VCG) mechanism [20] is the most widely used strategy-proof method for allocation of resources and deciding on the payments to be made in standard economic models where users are rational. In this section we will give a brief description of the VCG mechanism and compare the proposed auction schemes with the VCG mechanism. Let N be the set of players (agents) and $|N| = n$. Each player $i \in N$ has private information, say α_i, called its *type*. All players's types define a type vector $\alpha = (\alpha_1, \ldots, \alpha_n)$. A mechanism [20] defines a set of strategies A_i, for each player $i \in N$, from which player i selects a strategy a_i. By the *direct revelation principle* [20], we can assume that each player declares its type as its strategy. Thus the resulting strategy vector is $a = (\alpha_1, \ldots, \alpha_n)$. A mechanism computes allocation [6] o and payment vector $p = (p_1, \ldots, p_n)$ as a function of strategy vector a. p_i is the payment given to agent i. For each possible allocation o, agent i's preferences are given by a valuation function $v_i(\alpha_i, o)$. If the utility of agent i is denoted by $u_i(\alpha_i, a)$, an assumption required for the VCG mechanism to apply is that agents are rational and have quasi-linear utility functions of the form:

$$u_i(\alpha_i, a) = v_i(\alpha_i, o) + p_i. \tag{18}$$

Under the VCG scheme, the allocation o^* that satisfies the following condition is selected [20]:

$$\sum_{i=1}^{n} v_i(\alpha_i, o^*) \geq \sum_{i=1}^{n} v_i(\alpha_i, o) \ \ \forall o, \tag{19}$$

and the payment p_i is given by [20]:

$$p_i = \sum_{j \neq i} v_j(\alpha_j, o^*) - \sum_{j \neq i} v_j(\alpha_j, o^*_{-i}), \tag{20}$$

where o^*_{-i} is the allocation that would have been selected under the VCG scheme if agent i did not participate in the mechanism.

We now consider all the three scenarios discussed in the previous section and apply the VCG mechanism. The modelled game with R relay nodes denoted by the set $\mathbf{R} = \{1, ..., R\}$ and D destination nodes denoted by the set $\mathbf{D} = \{1, \ldots, D\}$ can be described as a mechanism as follows: Each relay node $i \in \mathbf{R}$ in the network environment is an agent and has private information α_i (its type). In our model, the payment to relay node i is (see (1) and (2)):

$$p_i = \beta \sum_{j=1}^{D} \Gamma_{i,j} y_{i,j}$$

and the valuation of relay node i is:

$$v_i(\alpha_i, o) = -\alpha_i \sum_{j=1}^{D} (P_{i,j} + P_{c,i}) y_{i,j}, \text{where} \tag{21}$$

$$y_{i,j} = \begin{cases} 1, & \text{if } i \text{ is assigned to } j \text{ under the allocation o,} \\ 0, & \text{else.} \end{cases} \tag{22}$$

Also, $\sum_{j=1}^{D} y_{i,j} \leq 1$ for all $i \in \mathbf{R}$ and $\sum_{i=1}^{R} y_{i,j} = 1$ for all $j \in \mathbf{D}$. The inequality says that a relay may be assigned to one destination node or none and the equality says that every destination node is assigned exactly one relay. Note that the set of variables $\{y_{i,j} : i \in \mathbf{R}, j \in \mathbf{D}\}$ constitute the allocation o. The VCG mechanism is truthfully implementable and satisfies the property of individual rationality [20].

We now compare the proposed auctions with those based on the VCG mechanism.

(i) Constant power case: In this scenario, in the proposed auction, relay nodes are assigned to destination nodes such that the following expression is minimized (see Fig. 2):

$$\sum_{j=1}^{D} \sum_{i=1}^{R} \frac{\alpha_i (P + P_{c,i})}{\Gamma_{i,j}} y_{i,j} \tag{23}$$

But by (19) and (21), under the VCG mechanism, relay nodes are assigned to destination nodes such that the following expression is minimized:

$$\sum_{j=1}^{D} \sum_{i=1}^{R} \alpha_i (P + P_{c,i}) y_{i,j} \tag{24}$$

where $\sum_{j=1}^{D} y_{i,j} \leq 1$ for all $i \in \mathbf{R}$ and $\sum_{i=1}^{R} y_{i,j} = 1$ for all $j \in \mathbf{D}$. From the above, it can be seen that the assignment of relay nodes to destination nodes under the proposed auction may differ from that under the VCG mechanism. Also, from (24), it can be seen that the VCG mechanism selects the D nodes in $\mathbf{R}$ with the D smallest values of the quantity $\alpha_i(P + P_{c,i})$ as relays. Since the VCG mechanism assigns relay nodes to destination nodes with the sole purpose of minimising the expression in (24), its outcome may be any arbitrary assignment of the D relay nodes in $\mathbf{R}$ with the D smallest values of $\alpha_i(P + P_{c,i})$ to the nodes in $\mathbf{D}$; note that every such assignment minimizes the quantity in (24). Also, under the VCG mechanism, the payment to every selected relay node is (from (20)) $\alpha_l(P + P_{c,l})$, where $\alpha_l(P + P_{c,l})$ is the $(D+1)$'st lowest value from the set $\{\alpha_m(P + P_{c,m}) : m \in \mathbf{R}\}$. The rest of the nodes get a payment of 0. Note that every relay node that is selected under the VCG mechanism is paid the same amount $\alpha_l(P + P_{c,l})$. Also, note that under the VCG mechanism, a selected relay node is not assigned to any specific destination node. This is in contrast to the proposed auction, where the auction assigns every selected relay node to a specific destination node since the expression in (23) is minimized. Although both the proposed auction and the VCG mechanism satisfy individual rationality and can be truthfully implemented, we will show via numerical computations in Section VI that our proposed auction outperforms the VCG mechanism in terms of the data rates achieved by the destination nodes. Intuitively, this is because in contrast to the proposed auction, the VCG mechanism ignores the data rates $\Gamma_{i,j}$ (see (23) and (24)).

(ii) Constant data rate case: In this case, the proposed auction assigns relay nodes to destination nodes such that the following expression is minimized (see Fig. 3):

$$\sum_{j=1}^{D} \sum_{i=1}^{R} \frac{\alpha_i (P_{i,j} + P_{c,i})}{\Gamma_j} y_{i,j}. \tag{25}$$

[6] For example, in the context of an auction mechanism, an allocation may be a vector $Y = (Y_1, \ldots, Y_n)$, where Y_i is 1 if the good is allocated to bidder i and 0 else.

But by (19) and (21), under the VCG mechanism relay nodes are assigned to destination nodes such that the following expression is minimized:

$$\sum_{j=1}^{D}\sum_{i=1}^{R}\alpha_i(P_{i,j}+P_{c,i})y_{i,j}, \tag{26}$$

where $\sum_{j=1}^{D} y_{i,j} \leq 1$ for all $i \in \mathbf{R}$ and $\sum_{i=1}^{R} y_{i,j} = 1$ for all $j \in \mathbf{D}$. It can be easily verified that the assignment of relay nodes to destination nodes under the proposed auction may differ from that under the VCG mechanism.

(iii) Selecting power to approximately maximize the utility of the BS: Since the BS selects the transmission power so as to approximately maximize its utility, the VCG mechanism is not applicable since it does not specify how the transmission power should be set so as to approximately maximize the BS's utility.

VI. Numerical Results

In this section, we present a numerical evaluation of the performance of the proposed auctions. A hexagonal cell of radius 300 meters is considered with the relay nodes placed using a uniform random distribution in the cell. For modelling the channel, we considered distance dependent path loss along with lognormal shadow fading. Also, the channel is assumed to undergo Rayleigh fading. The battery state b_i of each relay node $i \in \mathbf{R}$ is assumed to be distributed uniformly at random in the set $\{0.1, 0.2, \ldots, 1\}$. The value of α_i is taken to be the reciprocal of the battery state value b_i. It is also assumed that each relay node has the same transmission bandwidth W. We take the number of destination nodes that request for relay services in a given time slot to be 10. We evaluated the performance of the proposed auctions in terms of the following metrics: data rates achieved by the destination nodes and utility of the BS. We repeated each experiment 100 times; each time, independently, the channel gains and battery states were randomly chosen according to their distributions, and the average values of the metrics over all the runs are depicted in the following plots. The simulation parameters are given in Table I.

TABLE I
Simulation Parameters

Parameter	Value
Cell type	Hexagonal
Cell radius	300 m
Propagation Model	Path loss with lognormal shadow fading and Rayleigh fading
a	2.5 units per Mbps
c_i	Uniformly distributed in $\{1, 2, \ldots, 10\}$
Battery state	Uni. dist. in $\{0.1, 0.2, \ldots, 1\}$
Bandwidth of node (W)	10^6 Hz
Noise power	-120dB
Standard deviation for shadow fading	8
Path loss Exponent	3.3
No. of destination nodes	10
P_{max}	1 W
P_s	10 W

A. Constant Power Case

Recall from Section V that the VCG mechanism, in addition to our proposed auction, can also be truthfully implemented. So we compared the utility of the BS and the achieved data rates of the destination nodes in the two auctions for various relaying schemes while increasing the number of available relay nodes from 20 to 100. The transmission power of each relay node is fixed at 1 W. Fig. 5 shows the average data rate achieved per destination node versus the number of relay nodes for each of the four relaying schemes described in Section III-C. From the figure, it can be seen that for all four relaying schemes, *the proposed auction significantly outperforms the VCG mechanism based auction in terms of the achieved data rates*. Also, from Fig. 5, in case of the normal relaying scheme, it is observed that the achieved data rate increases with the number of available relay nodes under the proposed auction, in contrast to the VCG mechanism where the data rate remains roughly constant. This is because of the fact that the VCG mechanism chooses the winning relays according to only the nodes's battery states (α_i) and $P_{c,i}$ (see (24)) which are randomly assigned and are independent of the channel gains (and hence achieved data rates). In contrast, the proposed auction selects the winners based on the values of α_i, $P_{c,i}$ as well as the SINRs of the channels between the BS and relay nodes and between the relay nodes and destination nodes (see (23)). As the number of available relay nodes increases, the likelihood that a node has a low α_i and $P_{c,i}$ and high SINR increases and thus the data rates provided by the auction winners increase. We observe the same trend under the proposed auctions for the amplify-and-forward, decode-and-forward and selection relaying schemes. The decrease in average data rates versus the number of relay nodes for these three schemes under the VCG mechanism in Fig. 5 is because of the following reason: as the number of relay nodes increases, so does the number of relay nodes that have low valuation (α_i) and $P_{c,i}$, and low $SINR$ value [7]; hence, the likelihood that nodes with low $SINR$ values are selected as relays increases (see (24)). The same effect happens in the normal relaying scheme too, but the performance of the VCG mechanism is poor even for lower number of relay nodes.

Fig. 6 plots the BS utilities under the proposed auction and VCG mechanism for each of the four relaying schemes versus the number of relay nodes. The trends in this figure are similar to those in Fig. 5; this is because, by (3) and (4), the utility of the BS is an increasing function of the achieved data rates of the destination nodes. In particular, for all four relaying schemes, *the proposed auction significantly outperforms the VCG mechanism based auction in terms of the BS's utility.*

[7]Note that since the locations of relay nodes are selected uniformly at random in a hexagonal cell, it is more likely that a relay node is located far from the BS (center of the hexagon) than close to it and hence it is likely to have a poor SINR.

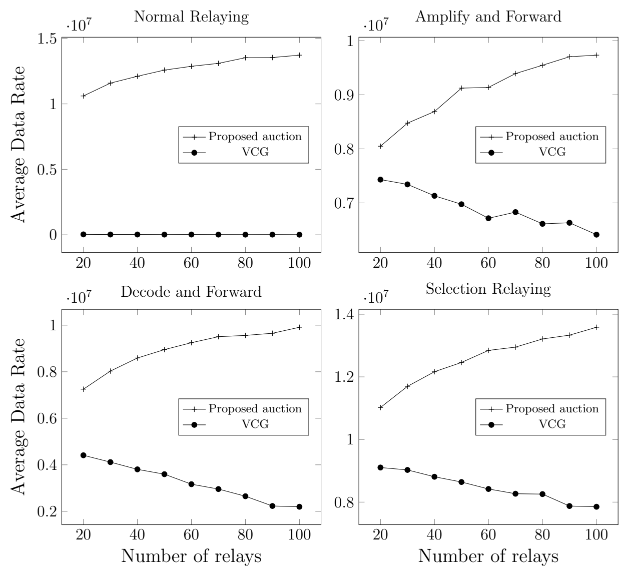

Fig. 5. The plots compare the average data rates achieved under the proposed auction with those under the VCG mechanism for various numbers of relay nodes and various relaying schemes for the constant power case.

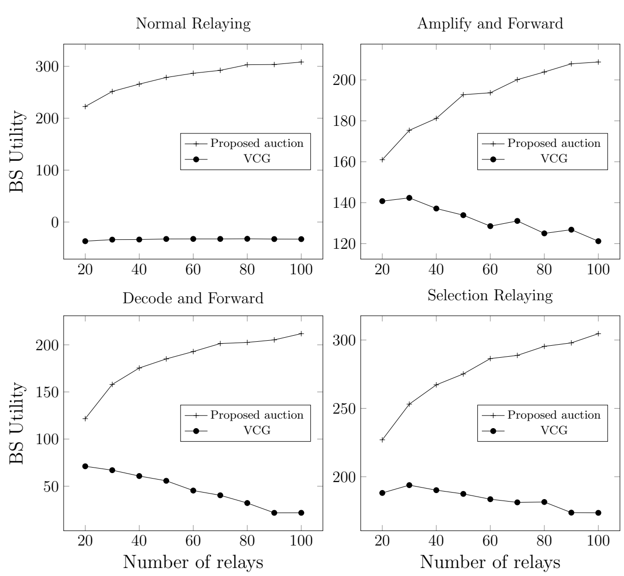

Fig. 6. The plots compare the BS utilities under the proposed auction and under the VCG mechanism for different numbers of relay nodes and various relaying schemes for the constant power case.

B. Constant Data Rate Case

Fig. 7 compares the performance of the proposed auction with that of the VCG mechanism based auction in terms of the BS utility for the constant data rate case. The two auctions perform similarly in terms of the BS utility. Intuitively, this is because of (3) and (4) and the fact that in the constant data rate case, the data rates at the destination nodes are the same (Γ_j) for both the proposed auction and the VCG mechanism based auction.

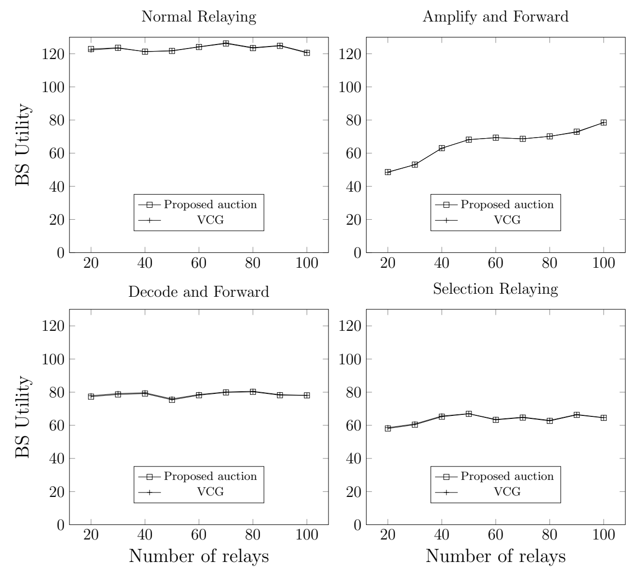

Fig. 7. The plots compare the BS utilities under the proposed auction and under the VCG mechanism for various numbers of relay nodes and for various relaying schemes for the constant data rate case.

C. Selection of Transmit Power to Approximately Maximize the BS's Utility

We compare the BS's utility under the proposed auction with that under the auction for the hypothetical case, where nodes are assumed to always truthfully reveal their valuations α_i. The latter auction makes the same assignment of relay nodes to destination nodes as in the proposed auction, but if relay node i is assigned to destination node j, then the former is paid $\alpha_i(P^*_{i,j} + P_{c,i})$, i.e., each relay node is paid only its incurred cost. This is in contrast to the proposed auction where each selected relay node i is paid an additional $w_{max} - w^{-i}_{max}$ (see (17)). The plots in Fig. 8 show that the BS's utility under the proposed auction is lower than that under the auction for the hypothetical case; this is because, when relay nodes may falsely declare their valuations (as in practice), they need to be paid more to incentivize them to truthfully declare their valuations. However, as the number of available relay nodes increases, the BS utilities under both the auctions increase, but the difference between the utilities changes very little. Thus the proposed auction performs well even in large networks.

VII. Conclusions

We considered a scenario in which some cellular users can relay data over D2D links to other cellular users with poor direct channel conditions from the BS. We proposed reverse auction mechanisms to assign a relay node to each destination node when there are multiple potential relay nodes and multiple destination nodes in each of the following three scenarios: 1) when relay nodes are allocated a fixed transmission power, 2) when relay nodes are allocated the transmission powers required to achieve the data rates desired by the destination nodes, and 3) when the transmission powers of relay nodes are selected so as to approximately maximize the BS's utility.

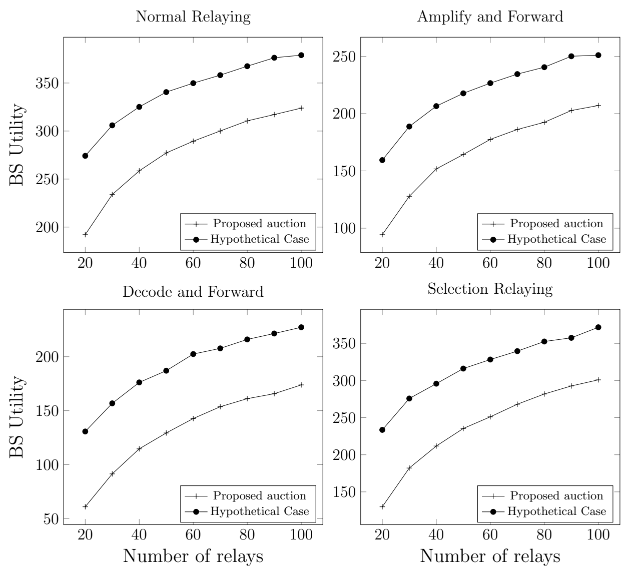

Fig. 8. In the BS utility maximisation case, for various relaying schemes and various numbers of available relay nodes, the above plots compare the BS utilities under the proposed auction with those under the auction for the hypothetical case, which is similar to the former with the difference that every relay node is assumed to truthfully reveal its valuation and is paid only its incurred cost.

We proved that all the proposed reverse auctions can be truthfully implemented as well as satisfy the individual rationality property. Using numerical computations, we showed that in the fixed transmission power scenario, our proposed auction significantly outperforms an auction based on the widely used VCG mechanism in terms of the data rates achieved by the destination nodes as well as the utility of the BS. Our proposed auctions are applicable to a variety of relaying schemes such as Normal relaying, Decode-and-Forward relaying, Amplify-and-Forward relaying and Selection relaying.

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Appendix

A. Proofs of Propositions 1 and 3

Proof of Proposition 1: Relays are assigned to destination nodes by finding the maximal matching of the bipartite graph $G = (\mathbf{R}, \mathbf{D}, E)$ with minimum weight (see step 3 in Fig. 2). This can be done using the Hungarian algorithm [15], which has a time complexity of $O(\mathcal{H})$.

Next, we find the time complexity of computing the payment p_i made to a relay node i assigned to destination node j (see step 5 in Fig. 2). Let $\mathbf{M}_i^k$ denote the set of all maximal matchings m such that m contains the edge (i,k) and $w_m \le w_{m_{min}^{-i}}$. Then we have $\mathbf{M}_i = \cup_{k\in\mathbf{D}}\mathbf{M}_i^k$. Note that $p_i = \max_{m\in\mathbf{M}_i} p_{i,m} = \max_{k\in\mathbf{D}}\left(\max_{m\in\mathbf{M}_i^k} p_{i,m}\right)$. Let $p_i^k = \max_{m\in\mathbf{M}_i^k} p_{i,m}$. Then by (11), we can write:

$$p_i^k = w_{m_{min}^{-i}}\Gamma_{i,k} + \alpha_i(P + P_{c,i}) - \Gamma_{i,k}\min_{m\in\mathbf{M}_i^k} w_m. \quad (27)$$

Next, $\min_{m\in\mathbf{M}_i^k} w_m$ for a given destination node $k \in \mathbf{D}$ can be found as follows. Find the maximal matching of the complete bipartite graph $G^{-\{i,k\}} = \left(V\setminus\{i,k\}, E^{-(i,k)}\right)$, where $E^{-(i,k)} = \{e \in E : e \neq (i,l),(l,j) \forall l \in \mathbf{R}\cup\mathbf{D}\}$, with minimum weight. This can done using the Hungarian algorithm on the graph $G^{-\{i,k\}}$. Let $m_{min}^{G^{-\{i,k\}}}$ denote the maximal matching of $G^{-\{i,k\}}$ with the least weight and let $w_{min}^{G^{-\{i,k\}}}$ denote the weight of this matching. If $w_{min}^{G^{-\{i,k\}}} \le w_{m_{min}^{-i}} - \frac{\alpha_i P}{\Gamma_{i,k}}$, then let $\min_{m\in\mathbf{M}_i^k} w_m = w_{min}^{G^{-\{i,k\}}} + \frac{\alpha_i P}{\Gamma_{i,k}}$, else let $\min_{m\in\mathbf{M}_i^k} w_m = \infty$. Next, substituting the value of $\min_{m\in\mathbf{M}_i^k} w_m$ into (27), p_i^k can be found. Finally, we calculate the payment as $p_i = \max_{k\in\mathbf{D}} p_i^k$. Since for calculating the payment p_i to relay node i, we run the Hungarian algorithm on the bipartite graph $G^{-\{i,k\}}$ for every $k \in \mathbf{D}$, the time complexity of computing the payment p_i is $O(D\mathcal{H})$. Since the payment needs to be computed for each of the D relay nodes that are assigned to destination nodes, the overall time complexity is $O\left(D^2\mathcal{H}\right)$. The result follows. ■

Proof of Proposition 3: Relays are assigned to destination nodes by finding the maximum weighted maximal matching of the bipartite graph $G = (\mathbf{R},\mathbf{D},E)$ (see step 3 in Fig. 4). This can be done using the Hungarian algorithm [15] and the time complexity of this operation is $O(\mathcal{H})$.

Next, to compute the payment to relay node i assigned to destination j (see (17)), w_{max}^{-i} needs to be found. This can be computed by finding the maximum weighted maximal matching of the bipartite graph $G^{-i} = (V\setminus i, E^{-\{i\}})$, where $E^{-\{i\}} = \{e \in E : e \neq (i,k), \forall k \in \mathbf{D}\}$, using the Hungarian algorithm. So the time complexity of computing the payment made to each of the D relay nodes assigned to destination nodes is $O\left(D\mathcal{H}\right)$. The result follows. ■

B. Expressions for the Transmission Power of a Relay

Here, we provide expressions for the transmission power of a relay node under various relaying schemes for the constant data rate scenario and approximate BS utility maximization scenario. Let $\gamma_{i,j} = \frac{G_{i,j}}{N_{i,j}+I_{i,j}}$ and $\gamma_{s,j} = \frac{G_{s,j}}{N_{s,j}+I_{s,j}}$. Let P_m denote the maximum transmission power of a relay node. $P_{i,j}$ is the power at which relay node i is required to transmit under the constant data rate scenario if it is assigned to destination node j, which requests a data rate of Γ_j, and $P_{i,j}^*$ is the power at which relay node i is required to transmit if it is assigned to destination node j to approximately maximize the BS's utility.

1) Normal Relaying: For the constant data rate scenario, when relay node i is assigned to destination node j, which requests a data rate of Γ_j, by (6), the data rate Γ_j in terms of the transmit power $P_{i,j}$ is given by:

$$\Gamma_j = \frac{W}{2}\log_2(1 + P_{i,j}\gamma_{i,j}).$$

From the above equation, the transmission power $P_{i,j}$ required by relay node i is:

$$P_{i,j} = \frac{4^{\frac{\Gamma_j}{W}} - 1}{\gamma_{i,j}}.$$

Next, from (15), at the transmission power $P_{i,j}^*$ which approximately maximizes the BS's utility, we have:

$$\frac{\partial U_{i,j}}{\partial P_{i,j}} = a\frac{\partial \Gamma_{i,j}}{\partial P_{i,j}} - \alpha_i - c_i = 0. \quad (28)$$

Substituting (6) in the above equation and solving for $P_{i,j}^*$ [8], we get:

$$P_{i,j}^* = \frac{aW}{2\ln 2(\alpha_i + c_i)} - \frac{1}{\gamma_{i,j}}.$$

2) Amplify-and-Forward: For the constant data rate scenario, when relay node i is assigned to destination node j, we obtain the transmission power required by relay node i as in the normal relaying scheme. Putting $\Gamma_{i,j} = \Gamma_j$ in (7) and solving, we get:

$$P_{i,j} = \frac{\left(4^{\frac{\Gamma_j}{W}} - 1 - SINR_{s,j}\right)(1 + SINR_{s,i})}{\left(1 + SINR_{s,i} + SINR_{s,j} - 4^{\frac{\Gamma_j}{W}}\right)\gamma_{i,j}}.$$

In the approximately maximizing the BS utility scenario, we follow the same procedure as used for the normal relaying scheme. We find roots of (28) by substituting (7) for $\Gamma_{i,j}$ and as a result we get a quadratic equation. It can be easily seen that one root is always negative and hence we take the larger root of the quadratic equation, which is as follows, as the transmission power [9] of relay node i:

$$\begin{aligned}&(1 + SINR_{s,i} + SINR_{s,j})P_{i,j}^2\gamma_{i,j}^2\\&+(1 + SINR_{s,i})(2 + 2SINR_{s,j} + SINR_{s,i})P_{i,j}\gamma_{i,j}\\&+(1 + SINR_{s,j})(1 + SINR_{s,i})^2\\&= aW\frac{SINR_{s,i}(1 + SINR_{s,i})}{\ln 4(\alpha_i + c_i)}\gamma_{i,j}.\end{aligned}$$

3) Decode-and-Forward: From (8), there are two possible cases:

Case 1: If $\frac{W}{2}\log_2(1+SINR_{s,i}) \ge \frac{W}{2}\log_2(1+SINR_{s,j}+SINR_{i,j})\ \forall\ 0 < P_{i,j} \le P_m$, then we have:

$$\Gamma_{i,j} = \frac{W}{2}\log_2(1 + SINR_{s,j} + SINR_{i,j}). \quad (29)$$

In the constant data rate scenario, the transmission power at which relay node i must transmit to destination node j is given by:

$$P_{i,j} = \frac{4^{\frac{\Gamma_j}{W}} - 1 - SINR_{s,j}}{\gamma_{i,j}},$$

[8] When $P_{i,j}^* < 0$, we set $P_{i,j}^* = 0$ and when $P_{i,j}^* > P_m$, we set $P_{i,j}^* = P_m$. This process is followed for all the relaying schemes.

[9] The transmission power is 0 if both roots are negative.

In the approximately maximizing the BS utility scenario, the same procedure is followed as in the normal relaying case. We find the root of (28) by substituting (29) for $\Gamma_{i,j}$. The transmission power required by relay node i while transmitting to destination node j for approximately maximizing the BS's utility is:

$$P_{i,j}^* = \frac{aW}{\ln 4(\alpha_i + c_i)} - \frac{1 + SINR_{s,j}}{\gamma_{i,j}}$$

Case 2: In Case 2, $\exists P_0$ such that $\frac{W}{2}\log_2(1 + SINR_{s,i}) > \frac{W}{2}\log_2(1 + SINR_{s,j} + SINR_{i,j}) \;\forall\; 0 < P_{i,j} < P_o$ and $\frac{W}{2}\log_2(1 + SINR_{s,i}) \leq \frac{W}{2}\log_2(1 + SINR_{s,j} + SINR_{i,j})$ $\forall\; P_o \leq P_{i,j} \leq P_m$.

In the constant data rate scenario, in the case when $0 < P_{i,j} < P_o$, we have $\frac{W}{2}\log_2(1 + SINR_{s,i}) > \frac{W}{2}\log_2(1 + SINR_{s,j} + SINR_{i,j})$; so:

$$\Gamma_{i,j} = \frac{W}{2}\log_2(1 + SINR_{s,j} + SINR_{i,j})$$

The expression for $P_{i,j}$ in this case is the same as in Case 1. In the case when $P_o \leq P_{i,j} \leq P_m$, we have $\frac{W}{2}\log_2(1 + SINR_{s,i}) \leq \frac{W}{2}\log_2(1 + SINR_{s,j} + SINR_{i,j})$; so:

$$\Gamma_{i,j} = \frac{W}{2}\log_2(1 + SINR_{s,i}).$$

Since the data rate is independent of $P_{i,j}$, we set $P_{i,j} = P_o$ as this will minimize the interference cost to the BS (see (4)).

In the approximately maximizing the BS utility scenario, in Case 2, we find the maximum contribution to the utility of the BS across the two cases: $0 < P_{i,j}^* < P_o$ and $P_o \leq P_{i,j}^* < P_m$. When we assume that $0 < P_{i,j}^* < P_o$, we substitute $\Gamma_{i,j} = \frac{W}{2}\log_2(1 + SINR_{s,j} + SINR_{i,j})$ in (28) and obtain the transmission power $P_{i,j}^*$ that maximizes (15). If $P_{i,j}^* \geq P_o$, then we set $P_{i,j}^* = P_o$ and if $P_{i,j}^* < 0$, we set $P_{i,j}^* = 0$. We find the corresponding contribution to the utility of the BS (see (4)) which we denote by U_1^*. Next, we repeat the process assuming that $P_o \leq P_{i,j}^* < P_m$. In this case we substitute $\frac{W}{2}\log_2(1 + SINR_{s,i})$ as the expression for data rate in (28). It can be seen that $P_{i,j}^* = P_o$. We calculate the corresponding contribution to the BS utility (see (4)), which we denote by U_2^*. If $U_1^* > U_2^*$, then the expression for $P_{i,j}^*$ is similar to the one obtained in Case 1, else $P_{i,j}^* = P_o$.

4) Selection Relaying: From (9), the capacity of the selection relaying protocol if relay node i is assigned to destination node j is given by:

$$\Gamma_{i,j} = \frac{W}{2}\log_2(1 + SINR_{s,j} + SINR_{i,j}).$$

The expressions for transmission powers are the same as those given in Case 1 of the decode-and-forward relaying scheme. Note that if $SINR_{i,j} < \zeta$, then relay i is not assigned to destination node j.