Rapid Node Cardinality Estimation in Heterogeneous Machine-to-Machine Networks

Sesha Vivek Y., P. Hari Prasad, Rajesh Kumar, Sachin Kadam, Gaurav S. Kasbekar

***Abstract*—Machine-to-Machine (M2M) networks are an emerging technology with applications in various fields including smart grids, healthcare, vehicular telematics, smart cities etc. Heterogeneous M2M networks contain different types of nodes, e.g., nodes that send emergency, periodic and normal type data. An important problem is to rapidly estimate the number of active nodes of each node type in every time frame in such a network. In this paper, we design an estimation scheme for estimating the active node cardinalities of each node type in a heterogeneous M2M network with three types of nodes. Our scheme consists of two phases– in phase 1, coarse estimates are computed and these estimates are used to compute the final estimates to the required accuracy level in phase 2. We analytically derive a condition that can be used to decide as to which of two possible approaches is to be used in phase 2. Using simulations, we show that our proposed scheme requires significantly fewer time slots to execute compared to separately executing a well-known estimation protocol designed for a homogeneous network in prior work thrice to estimate the cardinalities of the three node types, even though both these schemes obtain estimates with the same accuracy.**

I. Introduction

Machine-to-Machine (M2M) communications is emerging as a key technology for connecting together a very large number of autonomous devices that require minimal to zero human intervention in order to generate, process and transmit data [1]. M2M networks have extensive applications in various fields including smart grids, health care, vehicular telematics, smart cities, security and public safety, agriculture and industrial automation [2].

The design of efficient networking protocols to cater to the increasing number of M2M devices is turning out to be an important research field [2]. In particular, the design of medium access control (MAC) protocols for M2M networks is challenging because they have a number of unique characteristics, e.g., (i) network access needs to be provided to an extremely large number of M2M devices, (ii) most M2M devices are battery powered and have limited power availability, (iii) the quality of service (QoS) requirements in M2M applications differ from those in Human-to-Human (H2H) communications and are also different for different M2M devices [3]. A key component of a MAC protocol for M2M networks is an estimation protocol that rapidly estimates the number of active devices (i.e., the devices that currently have some data that needs to be sent to the base station) in every time frame [3]. These estimates can be used to find the optimal values of various parameters of the MAC protocol, e.g., contention probability, contention period, data transmission period etc, in each time frame [4]–[6]. For example, recall that for the Slotted ALOHA protocol, the optimal contention probability is the reciprocal of the number of active nodes [7].

There has been extensive research on the problem of node cardinality estimation in M2M networks and in Radio Frequency Identification (RFID) systems (see the following paragraphs for a review of these papers); however, with the exception of our prior work [8], all the papers in the existing research literature address the problem of node cardinality estimation in a *homogeneous* network, i.e., a network consisting of only one type of nodes. In contrast, in this paper, we address the problem of obtaining separate estimates of the number of active nodes of each type in a *heterogeneous* network, i.e., a network with multiple types of nodes. Note that executing a node cardinality estimation protocol for a homogeneous network multiple times to obtain the active node cardinalities of each type in a heterogeneous network is inefficient. In this paper, we consider an M2M network containing three different types of nodes, which we refer to as Type 1 (T_1), Type 2 (T_2), and Type 3 (T_3) nodes; *e.g.*, these may be emergency, periodic and normal data type nodes respectively. We design an estimation protocol to rapidly obtain separate estimates of the number of active nodes of each data type.

Owing to the importance of active node cardinality estimation as part of the design of a MAC protocol, a lot of research has been carried out in estimating the number of active devices in a homogeneous M2M network [4]–[6]. Also, in [4]–[6], using the estimates obtained, the contention probabilities that maximize the throughput of their respective MAC protocols for M2M networks are determined. The problem of node cardinality estimation in M2M networks is similar to that of tag cardinality estimation in the context of RFID technology. In particular, in the latter context, an RFID reader estimates the number of tags, similar to the former context, in which a base station estimates the number of active nodes in an M2M network. Schemes for estimating the number of tags in an RFID system have been proposed in [9]–[15]. However, all of the above node cardinality estimation schemes [4]–[6], [9]–[15] are designed for homogeneous networks.

To the best of our knowledge, in prior literature there is only one paper, viz., our prior work [8], which designs a node cardinality estimation scheme for heterogeneous networks. We now compare the estimation scheme proposed in this paper with that proposed in [8]. After carefully reviewing various estimation protocols, including Enhanced Zero-Based estimator [9], Lottery Frame (LoF) based estimator [11], Probabilistic Estimating Tree estimator [12], Zero-One estimator [13], and Arbitrarily Accurate Approximation estimator [14], the authors of [16] have shown that for an estimation protocol for a homogeneous network to be efficient, *i.e.*, for it to take the

R. Kumar, S. Kadam and G. Kasbekar are with Department of Electrical Engineering, Indian Institute of Technology (IIT) Bombay, Mumbai, India. Sesha Vivek Y. is with Goldman Sachs, Bengaluru, India and P. Hari Prasad is with Daikin Industries Limited, Osaka, Japan. Their email addresses are {rajeshk, sachink, gskasbekar}@ee.iitb.ac.in, seshavivek.yenduri@gs.com, and hari.prasad@daikin.co.jp respectively. Sesha Vivek Y. and P. Hari Prasad worked on this research while they were with IIT Bombay.

minimum possible number of time slots to estimate the node cardinality for a given set of specifications of accuracy, it is necessary that the protocol have two phases– a phase for obtaining a coarse estimate, followed by a phase that uses the coarse estimate to achieve an accuracy target. Also, the authors of [16] have devised an improved protocol, viz., the simple RFID counting (SRC_S) protocol, which has two phases, for tag cardinality estimation in homogeneous RFID networks. Our proposed scheme in this paper is designed by extending the SRC_S protocol to estimate the active node cardinalities of each node type in a heterogeneous M2M network with three types of nodes. The estimation protocol proposed in our prior work [8] was designed by extending the LoF protocol [11], which is a node cardinality estimation protocol for a homogeneous network, for node cardinality estimation in a heterogeneous network; the estimation scheme proposed in this paper outperforms that in [8] since the former is designed by extending the SRC_S protocol [16], which has been shown to outperform the LoF protocol [11] in [16] in terms of number of slots required, for node cardinality estimation in a heterogeneous network.

The rest of this paper is organized as follows. The network model and problem formulation are described in Section II-A. The SRC_S protocol [16] is reviewed in Section II-B. The estimation protocol designed for a heterogeneous M2M network in our prior work [8] is reviewed in Section II-C. The node cardinality estimation scheme for heterogeneous M2M networks proposed in this paper is described in Section III-A. In Section III-B, we compute the expected number of slots required by our scheme to execute. Our proposed scheme consists of two phases and one of two possible approaches is used in phase 2; the condition that is used to decide as to which approach is to be used in phase 2 is derived in Section III-C. We evaluate the performance of our proposed estimation scheme via simulations in Section IV. Finally, we provide conclusions and directions for further research in Section V.

II. Model, Problem Formulation and Background

A. The Node Cardinality Estimation Problem in a Heterogeneous M2M Network

Consider a heterogeneous M2M network consisting of a base station (BS) and three different types– say Type 1 (T_1), Type 2 (T_2), and Type 3 (T_3)– of nodes within its range as shown in Fig. 1. Time is divided into frames of equal durations, and in each frame only a subset of the nodes of each type have data to send to the BS. We call these nodes as *active* nodes. Let n_b be the number of active nodes of Type b, $b \in \{1,2,3\}$, in a given frame. Our objective is to rapidly estimate the number of active nodes, n_b, $b \in \{1,2,3\}$, of each type.

B. Review of SRC_S Protocol [16]

We now review the SRC_S protocol [16], which is a node cardinality estimation protocol for homogeneous networks, and which we extend for node cardinality estimation in heterogeneous networks.

The goal of a cardinality estimation protocol for a homogeneous network is to produce an estimate, say $\hat{n}$, for n (the actual number of active nodes), so that $P(|\hat{n}-n| \leq \varepsilon n) \geq 1-\delta$, where ε, the relative error, and δ, the required accuracy, are

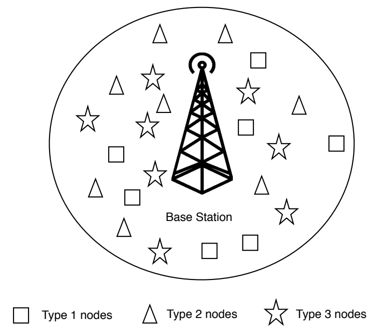


Figure 1: A base station with three different types of nodes within its range.

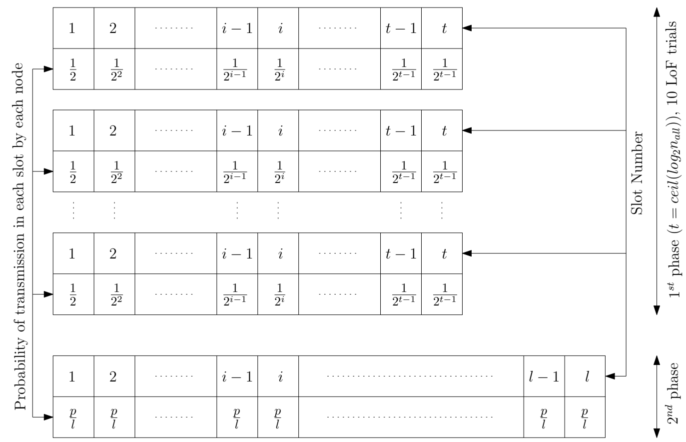


Figure 2: The figure shows the frame structure used in the SRC_S protocol.

user specified. SRC_S is a two phase protocol (see Fig. 2); at the end of the first phase (respectively, second phase), it finds a rough estimate $\tilde{n}$ (respectively, the final estimate $\hat{n}$) [16]. The first (respectively, second) phase of the protocol consists of a sequence of trials (respectively, a single trial), and each trial consists of a certain number of slots. The number of slots in a trial is called the length of the trial. At the beginning of each trial, the BS sends a command to the nodes. This causes the nodes to initialize their local state machines and potentially load new random numbers. Next, in each slot within that trial, a node transmits or does not transmit based on the command, its local state, and its random number. After a trial, a slot can be in one of the following three states: (i) *Empty*: There are no nodes that have transmitted in that slot, (ii) *Success*: Exactly one node has transmitted in that slot, (iii) *Collision*: More than one node have transmitted in that slot.

The first phase of the SRC_S protocol consists of a sequence of independent trials of the LoF protocol [11]; let $t_{n,max}$ be the number of trials of the LoF protocol conducted. Fig. 3 shows a single trial of the LoF protocol. Let n_{all} be the total number of nodes manufactured and $t = \lceil \log_2 n_{all} \rceil$. In each trial of the LoF protocol, every active node randomly chooses a slot according to the following distribution: for $i \in \{1,2,\ldots,t-1\}$, the i^{th} slot is chosen with probability $(1/2^i)$ and the t^{th} slot is chosen with probability $(1/2^{t-1})$. Each active node transmits in its chosen slot. For each $t_n \in \{1,2,\ldots,t_{n,max}\}$, let $j(t_n)$ be the smallest number $j \in \{1,2,\ldots,t\}$, such that the j^{th} slot is empty in the t_n^{th} trial. The estimate of the number of active nodes found in the t_n^{th} trial is $\tilde{n}(t_n) = 1.2897 \times 2^{j(t_n)-1}$. At the end of all $t_{n,max}$ trials, the average of their outputs is the estimated number of active nodes, *i.e.*, $\tilde{n} = \text{mean}\{\tilde{n}(1), \tilde{n}(2), \ldots, \tilde{n}(t_{n,max})\}$. Now, the number of trials, $t_{n,max}$, is determined based on the required accuracy δ. For example, for $\delta = 0.2$, $t_{n,max} = 10$ is used.

Slot Number

1	2	....	$i-1$	i	....	$t-1$	t
$\frac{1}{2}$	$\frac{1}{2^2}$	....	$\frac{1}{2^{i-1}}$	$\frac{1}{2^i}$	....	$\frac{1}{2^{t-1}}$	$\frac{1}{2^{t-1}}$

Probability of transmission in each slot by each node

Figure 3: The figure shows a trial of the LoF protocol.

Phase 2 of the SRC_S protocol consists of a single trial of l slots; each active node independently participates (respectively, does not participate) in the trial with probability p (respectively, $1-p$). Also, each node that participates transmits in a slot selected uniformly at random from the l slots (see Fig. 2). The parameter value $p = \min(1, 1.6l/\tilde{n})$ is used. The parameter l is a function of the relative error ε and it is found from a numerical lookup table, which is constructed by executing the SRC_S protocol for different values of n, and finding the value of l required to achieve a given value of ε [16]. Note that the expected fraction of empty slots, out of the l slots, is $(1-p/l)^n$. The protocol counts the number of empty slots, say z, out of the l slots. The final estimate generated by the protocol is $\hat{n} = \ln(z/l)/\ln(1-p/l)$.

C. Review of Node Cardinality Estimation Protocol for Heterogeneous M2M Networks Proposed in [8]

We now briefly review the node cardinality estimation scheme proposed in our prior work [8] since we use it as part of the estimation scheme proposed in this paper. The scheme proposed in [8] extends the LoF protocol [11] for obtaining separate estimates of the active node cardinalities of each node type in a heterogeneous M2M network with three types of nodes (see Section II-A). Also, the scheme proposed in [8] consists of 3 stages (see Fig. 4) and we henceforth refer to it as the "3-stage technique".

As in Section II-B, let t be the number of slots required for the execution of a single trial of the LoF protocol. Stage 1 of the 3-stage technique consists of t blocks, B_i, $i \in \{1, \ldots, t\}$ (see Fig. 4). Each block, B_i, is divided into two slots $S_{i,1}$ and $S_{i,2}$. Each active node of each of the three types chooses a block number at random according to the distribution used in LoF (see Section II-B), *i.e.*, for $i \in \{1, 2, \ldots, t-1\}$, block B_i is chosen with probability $(1/2^i)$ and block B_t is chosen with probability $(1/2^{t-1})$. T_1 active nodes whose chosen block is B_i transmit symbol α in both slots, $S_{i,1}$ and $S_{i,2}$, of block B_i. T_2 (respectively, T_3) active nodes whose chosen block is B_i transmit symbol β only in slot $S_{i,1}$ (respectively, $S_{i,2}$). Stage 1 concludes with this. Now, it has been shown in [8] that if a collision occurs in at most one slot of a given block B_i, then the set of types of nodes that transmitted in block B_i can be unambiguously inferred by the BS. However, for some blocks B_i of stage 1, collisions in both slots of the block may occur; in this case, the BS has ambiguity about the types of nodes that transmitted in those particular blocks. To resolve the ambiguity, after the end of this stage, the BS transmits the list of all block numbers for which collisions in both their slots occured via a broadcast packet (BP) (see Fig. 4). In stage 2, there are K slots, where K is the number of blocks in stage 1 in which a collision occurred in both of its slots. In the i^{th} slot of stage 2, T_1 nodes that transmitted in the i^{th} block of stage 1 in which collisions occurred in both the slots, transmit symbol α. T_2 and T_3 nodes do not transmit in stage 2. Now, it is easy to see that at the end of stage 2, the BS unambiguously knows the set of block numbers of stage 1 in which T_1 nodes transmitted. However, if in stage 2, there are collisions in some of the slots, ambiguity remains with the BS on whether T_2 and T_3 nodes transmitted in the corresponding blocks of stage 1 or not. To resolve this ambiguity, after the end of stage 2, the BS transmit a BP containing the list of blocks of stage 1 for which collisions occurred in the corresponding slots of stage 2. Suppose there are R blocks in this list. In stage 3, $2R$ slots are used. For $i \in \{1, \ldots, R\}$, T_2 (respectively, T_3) active nodes corresponding to the i^{th} block in the above list transmit symbol β in the $(2i-1)^{th}$ (respectively, $(2i)^{th}$) slot of stage 3. It is easy to see that at the end of stage 3, the BS unambiguously knows the sets, say $\mathscr{I}_1$, $\mathscr{I}_2$ and $\mathscr{I}_3$, of block numbers of stage 1 in which T_1, T_2 and T_3 nodes respectively transmitted. For $k \in \{1, 2, 3\}$, let j_k be the smallest number j such that no T_k node transmitted in the j^{th} block of stage 1. Then the estimate of the number of active nodes of T_k is $1.2897 \times 2^{j_k - 1}$ (see Section II-B).

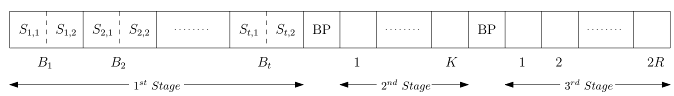


Figure 4: The figure shows the frame structure used in the 3-stage technique proposed in [8].

Now, the node cardinality estimate of each T_k obtained using the protocol proposed in [8] equals, and hence is as accurate as, the estimate that would have been obtained if the LoF protocol were separately executed thrice to estimate the number of active nodes of each type. However, under mild conditions, the amount of time needed by the estimation protocol proposed in [8] is much lower than the amount of time that would have been needed if the LoF protocol were separately executed thrice.

III. Proposed Node Cardinality Estimation Scheme for Heterogeneous M2M Networks

A. Proposed Node Cardinality Estimation Scheme

We now describe the proposed scheme, which is an extension of the SRC_S protocol for estimating the number of active nodes of each type in the model with a BS and three different types of nodes in its range described in Section II-A. The proposed scheme consists of two phases– they correspond to the two phases of the SRC_S protocol as explained below.

Recall from Section II-B that the first phase of the SRC_S protocol is a series of independent trials of the LoF protocol. Since it is shown in [8] that, under mild conditions, the 3-stage technique (described in Section II-C) takes less time compared to three separate executions of the LoF protocol for estimating the active node cardinalities of the three types of nodes, we use a series of independent executions of the 3-stage technique in the first phase of the proposed scheme.

At the end of the first phase of the proposed scheme, we obtain rough estimates, say $\tilde{n}_1$, $\tilde{n}_2$, and $\tilde{n}_3$, of the numbers of active nodes of T_1, T_2, and T_3 respectively. Note that these estimates are the same as those that would have been obtained if the first phase of the SRC$_S$ protocol were separately executed thrice for the three node types. Next, recall from Section II-B that the second phase of the SRC$_S$ protocol has a single trial. The number of slots, l, in the trial depends on the relative error ε we want (see Section II-B). We take the value of ε to be the same for all the three node types; hence, the length, l, of the trial would be the same for all the three node types. Let $p_b = \min\left(1, \frac{1.6l}{\tilde{n}_b}\right)$ for $b \in \{1,2,3\}$ (see Section II-B).

Now, one possible approach to execute the second phase of the proposed scheme is to separately execute phase 2 of the SRC$_S$ protocol for each of the three node types. Note that this requires a total of $3l$ time slots to execute. An alternative approach to execute the second phase of the proposed scheme is to use the 3-stage technique described in Section II-C with the change that in stage 1, l blocks are used and for $b \in \{1,2,3\}$, each node of Type b independently transmits with probability p_b in a block chosen uniformly at random from the l blocks and does not transmit with probability $1-p_b$. It is easy to see that at the end of stage 3, the BS unambiguously knows the sets, say $\mathscr{I}_1$, $\mathscr{I}_2$ and $\mathscr{I}_3$, of block numbers of stage 1 in which T_1, T_2 and T_3 nodes respectively transmitted. From the sets $\mathscr{I}_1$, $\mathscr{I}_2$ and $\mathscr{I}_3$, for each $b \in \{1,2,3\}$, z_b, which is the number of slots that would have been empty if phase 2 of the SRC$_S$ protocol were executed for T_b nodes, can be deduced. For each $b \in \{1,2,3\}$, the final estimate of the number of active nodes of Type b is calculated at the end of the second phase as $\hat{n}_b = ln(z_b/l)/ln(1-p_b/l)$ (see Section II-B). Note that irrespective of which of the above two approaches is used, *the final node cardinality estimate, $\hat{n}_b$, of each Type $b \in \{1,2,3\}$, obtained using the proposed scheme equals, and hence is as accurate as, the estimate that would have been obtained if the SRC$_S$ protocol were separately executed thrice to estimate the number of active nodes of each type.*

In order to minimize the execution time of the second phase of the proposed scheme, we have obtained a condition which, if satisfied, we use the 3-stage technique, else we separately execute phase 2 of the *SRC$_S$* protocol thrice for the three node types. This condition is derived in Section III-C.

B. Expected Number of Slots Required in Second Phase of the Proposed Scheme

Clearly, if three separate executions of the second phase of the *SRC$_S$* protocol are used in the second phase of the proposed scheme, then $3l$ slots are required. Now we compute the expected number of slots required in the second phase assuming that the 3-stage technique is used. This will be used in Section III-C to derive the condition using which we decide as to which approach to use in the second phase.

The number of slots required in the first stage is $2l$. Using the notation introduced in Section II-C, let K (respectively, $2R$) be the number of slots required in stage 2 (respectively, stage 3). Let $S^r_{i,1}$ (respectively, $S^r_{i,2}$), $i \in \{1,2,\ldots,l\}$ represent the result (collision, success or empty slot) of the first (respectively, second) slot of block B_i of stage 1. Also, let I_υ denote the indicator random variable corresponding to event υ, i.e., I_υ is 1 if υ occurs, else it is 0.

From Sections II-C and III-A, it is easy to see that $K = \Sigma_{i=1}^{l} I_{\{S^r_{i,1}=C, S^r_{i,2}=C\}}$, where C denotes collision. So,

$$E(K) = \Sigma_{i=1}^{l} P(S^r_{i,1}=C, S^r_{i,2}=C). \quad (1)$$

The conditions under which collisions occur in both the slots of block B_i are as follows:

1) At least two nodes of T_1 transmit in block B_i.
2) Exactly one node of T_1 and at least one node each of T_2 and T_3 transmit in block B_i.
3) At least two nodes each of T_2, T_3 and none of T_1 transmit in block B_i.

Let $Q_1(i)$, $Q_2(i)$, and $Q_3(i)$ denote the probabilities of the events in 1), 2), and 3) respectively. Since the probability of selecting a block B_i by the nodes of a given type $j \in \{1,2,3\}$ is the same for all the blocks B_i irrespective of i, we can write: $Q_j(i) = Q_j,\ j \in \{1,2,3\},\ i \in \{1,\ldots,l\}$. Hence,

$$P(S^r_{i,1}=C, S^r_{i,2}=C) = Q_1 + Q_2 + Q_3. \quad (2)$$

Also, $Q_1 = 1-u(n_1)-v(n_1)$, $Q_2 = v(n_1)(1-u(n_2))(1-u(n_3))$ and $Q_3 = u(n_1)(1-u(n_2)-v(n_2))(1-u(n_3)-v(n_3))$. Here, $u(n_b)$ is the probability that none of the nodes of Type b select a given block and $v(n_b)$ is the probability that exactly one node of Type b selects a given block. So for $b \in \{1,2,3\}$, $u(n_b) = (1-\frac{p_b}{l})^{n_b}$ and $v(n_b) = n_b\frac{p_b}{l}(1-\frac{p_b}{l})^{n_b-1}$, where $p_b = \min\left(1, \frac{1.6l}{n_b}\right)$ (see Section III-A).

By (1) and (2):

$$E(K) = l(Q_1+Q_2+Q_3). \quad (3)$$

Also:

$$E(R) = lQ_1, \quad (4)$$

since in stage 3, only those nodes of T_2 and T_3 transmit for which collisions occurred in both the slots of the corresponding blocks of stage 1 due to two or more T_1 nodes transmitting (see Sections II-C and III-A). The total expected number of slots required by the 3-stage technique is $2l+2+E(K)+2E(R)$. (Note that two slots are required for BPs (see Fig. 4)).

C. Condition Used to Select Approach to be Used in Phase 2

From the description of the 3-stage technique in Sections II-C and III-A, it can be seen that in stage 1, if a T_1 node chooses block B_i, it transmits in both the slots $S_{i,1}$ and $S_{i,2}$, whereas if nodes of T_2 or T_3 select block B_i, they transmit only in one of $S_{i,1}$ or $S_{i,2}$. So, the number of collisions due to T_1 nodes is high compared to those due to T_2 or T_3. Also, clearly the numbers of slots required in stage 2 and stage 3 increase with the number of collisions in stage 1. Therefore, the numbers of slots required in stage 2 and stage 3 increase rapidly when the number of T_1 nodes is increased. Hence, we develop a condition on n_1: if it is less than a certain value, we use the 3-stage technique in phase 2 of the proposed scheme, else we use three separate trials of the second phase of the *SRC$_S$* protocol. It is possible to check whether the condition holds because we already have a rough estimate of n_1, i.e.,

$\tilde{n}_1$, from phase 1 using which it can be checked whether the condition holds.

Now, to derive the condition, note that the use of the 3-stage technique is profitable only if the number of slots required when this technique is used is less than $3l$; also, note that the number of slots required increases with increase in $\tilde{n}_2$ and $\tilde{n}_3$. So, we keep $\tilde{n}_2$ and $\tilde{n}_3$ very large, i.e., we let them approach infinity, and we derive a condition on $\tilde{n}_1$ for which the expected number of slots required when the 3-stage technique is used is less than $3l$. This ensures that when this condition is satisfied, the expected number of slots required by the 3-stage technique is $\leq 3l$ regardless of the values of n_2 and n_3. Now, recall from Section III-B that the expected number of slots required by the 3-stage technique is $2l+2+E(K)+2E(R)$. So the required condition is: $2l+2+E(K)+2E(R) \leq 3l$, *i.e.*,

$$E(K)+2E(R) \leq l-2. \tag{5}$$

Since $\tilde{n}_2$ and $\tilde{n}_3$ are assumed to be very large, they are $>> 1.6l$. Therefore $p_2 = \min\left(1, \frac{1.6l}{\tilde{n}_2}\right) = \left(\frac{1.6l}{\tilde{n}_2}\right)$ and similarly $p_3 = \left(\frac{1.6l}{\tilde{n}_3}\right)$ (see Section III-A). Let the functions $u(\cdot)$ and $v(\cdot)$ be as defined in Section III-B. For very large values of $\tilde{n}_2$:

$$\lim_{\tilde{n}_2\to\infty} u(\tilde{n}_2) = \lim_{\tilde{n}_2\to\infty} \left(1-\frac{1.6}{\tilde{n}_2}\right)^{\tilde{n}_2} = e^{-1.6}. \tag{6}$$

$$\lim_{\tilde{n}_2\to\infty} v(\tilde{n}_2) = \lim_{\tilde{n}_2\to\infty} 1.6\left(1-\frac{1.6}{\tilde{n}_2}\right)^{\tilde{n}_2} = 1.6e^{-1.6}. \tag{7}$$

Similarly for very large values of $\tilde{n}_3$: $\lim_{\tilde{n}_3\to\infty} u(\tilde{n}_3) = e^{-1.6}$ and $\lim_{\tilde{n}_3\to\infty} v(\tilde{n}_3) = 1.6e^{-1.6}$. Now, we consider the cases (I) $\tilde{n}_1 < 1.6l$ and (II) $\tilde{n}_1 \geq 1.6l$ separately, and in each case, find the values of $\tilde{n}_1$ for which the condition in (5) holds.

1) Case I: $\tilde{n}_1 < 1.6l$: This implies $p_1 = \min\left(1, \frac{1.6l}{\tilde{n}_1}\right) = 1$. By (3) and (4), we get:

$$E(K) = l\Big[1-\left(1-\frac{1}{l}\right)^{\tilde{n}_1} - \frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1-1} + \frac{\tilde{n}_1}{l}\Big(1 - \frac{1}{l}\Big)^{\tilde{n}_1-1}\left(1-e^{-1.6}\right)^2 + \left(1-\frac{1}{l}\right)^{\tilde{n}_1}\left(1-2.6e^{-1.6}\right)^2\Big] \tag{8}$$

$$E(R) = l\Big[1-\left(1-\frac{1}{l}\right)^{\tilde{n}_1} - \frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1-1}\Big] \tag{9}$$

Hence:

$$E(K)+2E(R) = l\Big[3\Big(1-\left(1-\frac{1}{l}\right)^{\tilde{n}_1} - \frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1-1}\Big) + 0.6370\frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1-1} + 0.2257\left(1-\frac{1}{l}\right)^{\tilde{n}_1}\Big] \tag{10}$$

Now, substituting from (10) into (5) and simplifying, the condition in (5) becomes:

$$2.7743\left(1-\frac{1}{l}\right)^{\tilde{n}_1} + 2.3630\frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1-1} \geq 2+\frac{2}{l} \tag{11}$$

Proposition 1: Assume that $l \geq 100$. Inequality (11) holds when $\tilde{n}_1 \leq 0.864l$. Also, it does not hold when $0.883l \leq \tilde{n}_1 < 1.6l$.

The proof of Proposition 1 is relegated to the Appendix. Assuming that $l \geq 100$ (which would most likely be the case in practice), Proposition 1 shows that whenever $\tilde{n}_1 \leq 0.864l$ (respectively, $0.883l \leq \tilde{n}_1 < 1.6l$), $E(K)+2E(R) \leq l-2$ (respectively, $E(K)+2E(R) > l-2$) and hence the 3-stage technique takes less (respectively, more) time on average than doing separate trials for each type of node in phase 2 of the proposed scheme.

2) Case II: $\tilde{n}_1 \geq 1.6l$: This implies $p_1 = 1.6l/\tilde{n}_1$. Using (3) and (4) and simplifying, we get:

$$\begin{aligned} E(K)+2E(R) &= l\Big[3-2.7743\left(1-\tfrac{1.6}{\tilde{n}_1}\right)^{\tilde{n}_1} \\ &\quad -3.7808\left(1-\tfrac{1.6}{\tilde{n}_1}\right)^{\tilde{n}_1-1}\Big] \\ &= h(l,\tilde{n}_1) \quad \text{(say)}. \end{aligned} \tag{12}$$

Proposition 2: $h(l,\tilde{n}_1) \geq 1.4221l$ for $l \geq 4$.

The proof of Proposition 2 is relegated to the Appendix. Assuming that $l \geq 4$ (which would most likely be the case in practice), Proposition 2 shows that whenever $\tilde{n}_1 \geq 1.6l$, $E(K)+2E(R) \geq 1.4221l > l-2$, and hence the condition in (5) is not met and doing separate trials for each type of nodes takes less time on average than using the 3-stage technique in phase 2 of the proposed scheme.

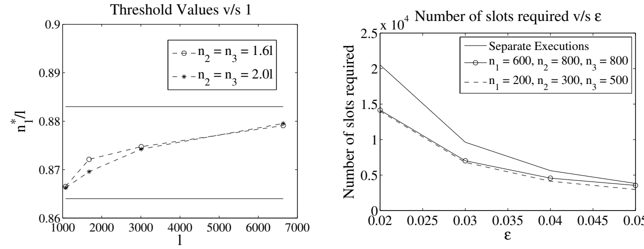
In summary, the analysis of Cases I and II shows that when $\tilde{n}_1 \leq 0.864l$ (respectively, $\tilde{n}_1 \geq 0.883l$), the 3-stage technique takes less (respectively, more) time on average than doing separate trials for each type of node in phase 2 of the proposed scheme. It is unclear from the analysis as to which technique takes less time when $\tilde{n}_1 \in (0.864l, 0.883l)$. This question is addressed via simulations in Section IV.

IV. Simulations

In this section, we evaluate the performance of the proposed node cardinality estimation scheme via simulations. Throughout, we assume that the targeted accuracy is $\delta = 0.2$.

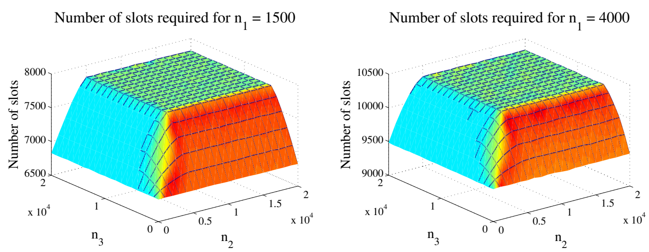
In Figs. 5a and 5b, the average number of slots required in the second phase of the proposed scheme when the 3-stage technique is used is plotted versus n_2 and n_3 for $n_1 = 1500$ and $n_1 = 4000$ respectively. It can be seen that in Fig. 5a, for all the values of n_2 and n_3 considered, the 3-stage technique takes less time than doing separate trials (which takes $3l = 9027$ slots). Also, in Fig. 5b, the 3-stage technique takes more time than doing separate trials. Since $1500 < 0.864 \times 3009$ and $4000 > 0.883 \times 3009$, these observations are consistent with the result derived in Section III-C that for $\tilde{n}_1 \leq 0.864l$ (respectively, $\tilde{n}_1 \geq 0.883l$), the 3-stage technique takes less (respectively, more) time than doing separate trials.

Fig. 6a (respectively, Fig. 6b) shows the number of slots required in the second phase of the proposed scheme when the 3-stage technique is used and when separate trials are used versus n_1 (respectively, n_2) for three different pairs of values of n_2 (respectively, n_1) and n_3. It can be seen that for each set of values of n_2 (respectively, n_1) and n_3, the number of slots required by the 3-stage technique increases in n_1 (respectively, n_2); this is because the number of collisions in stage 1 increases. Also, as n_2 (respectively, n_1) and n_3 increase,



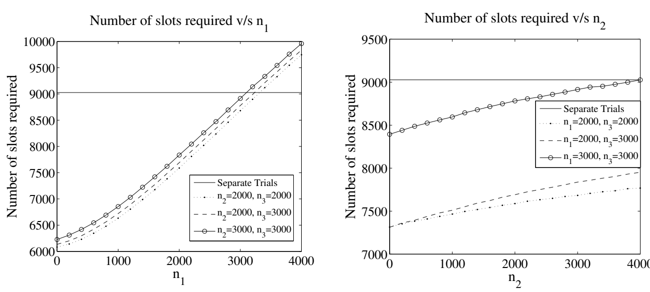
(a) (b)

Figure 7: Plot (a) shows n_1^*/l versus l and the two bounds 0.864 and 0.883 in Proposition 1. Plot (b) shows the average number of slots required by the proposed scheme and by the scheme in which three separate executions of the SRC_S scheme are used. In both plots, the following parameters are used: $\epsilon = [0.02, 0.03, 0.04, 0.05]$ and corresponding $l = [6638, 3009, 1674, 1075]$ [16].



(a) (b)

Figure 5: These plots show the average number of slots required in the second phase of the proposed scheme when the 3-stage technique is used for $n_1 = 1500$ and $n_1 = 4000$. The following parameters are used: $\epsilon = 0.03$ and $l = 3009$.



(a) (b)

Figure 6: These plots show the average number of slots required in the second phase of the proposed scheme when the 3-stage technique is used and when separate trials are used for different values of n_1, n_2 and n_3. The following parameters are used: $\epsilon = 0.03$ and $l = 3009$.

the number of slots required by the 3-stage technique increases; again, this is because the number of collisions increases.

Next, for given values of n_2 and n_3, let n_1^* be the value of n_1 for which the 3-stage technique and doing separate trials require equal numbers of slots to execute on average in the second phase of the proposed scheme; note that the value of n_1^* can be obtained by using a plot such as Fig. 6a and noting the value of n_1 at which the curve for the 3-stage technique intersects the horizontal line corresponding to doing separate trials. Fig. 7a shows a plot of n_1^*/l versus l for two different pairs of values of n_2 and n_3. It can be seen that $0.864 < n_1^*/l < 0.883$ for all values considered, which is consistent with the result derived in Section III-C that for $\tilde{n}_1 \leq 0.864l$ (respectively, $\tilde{n}_1 \geq 0.883l$), the 3-stage technique takes less (respectively, more) time than doing separate trials. Also, by using a plot such as Fig. 7a, we can find out n_1^*, using which we can in turn find out, for given values of $\tilde{n}_1$, $\tilde{n}_2$ and $\tilde{n}_3$, whether using the 3-stage technique or separate trials would take fewer slots in the second phase in practice– note that if $\tilde{n}_1 < n_1^*$ (respectively, $\tilde{n}_1 > n_1^*$), the 3-stage technique (respectively, separate trials) would take fewer slots.

Fig. 7b shows the total number of slots required (in both phases) by the scheme proposed in this paper and by the scheme in which three separate executions of the SRC_S scheme are used for estimating the node cardinalities of the three types of nodes versus the relative error ϵ for two different sets of values of n_1, n_2 and n_3. It can be seen that for all the values considered, the *total number of slots required by the proposed scheme is much lower than that required by the scheme that uses separate executions*; this shows the efficacy of the proposed scheme.

V. Conclusions and Future Work

We designed an estimation scheme for rapidly obtaining separate estimates of the number of active nodes of each type in a heterogeneous M2M network with three types of nodes. Our scheme consists of two phases; we analytically derived a condition that can be used to decide as to which of two possible approaches is to be used in the second phase. Using simulations, we showed that our proposed scheme requires significantly fewer time slots to execute compared to separately executing the underlying estimation protocol, SRC_S [16], for homogeneous networks thrice, even though both these schemes obtain estimates with the same accuracy. In this paper, we considered a heterogeneous M2M network with three types of nodes. A direction for future research is to generalize our scheme to estimate the node cardinalities of each node type in a heterogeneous M2M network with T types of nodes, where $T \geq 2$ is an arbitrary integer.

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Appendix

Proof of Proposition 1: Consider

$$\begin{aligned}
&2.7743\left(1-\tfrac{1}{l}\right)^{\tilde{n}_1}+2.3630\tfrac{\tilde{n}_1}{l}\left(1-\tfrac{1}{l}\right)^{\tilde{n}_1-1}\\
&=2.7743\left(1-\tfrac{1}{l}\right)^{\tilde{n}_1}+2.3630\frac{\frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1}}{1-\frac{1}{l}}\\
&\geq 2.7743\left(1-\tfrac{1}{l}\right)^{\tilde{n}_1}+2.3630\tfrac{\tilde{n}_1}{l}\left(1-\tfrac{1}{l}\right)^{\tilde{n}_1} \qquad (13)
\end{aligned}$$

Let $\tilde{n}_1 = xl$. Then the quantity in (13) equals: $(1-\frac{1}{l})^{xl}(2.7743+2.3630x)$. Now, it can be easily shown that the function $g(l) = (1-\frac{1}{l})^l$ is increasing in l. Since $l \geq 100$, $g(l) \geq g(100) = 0.366$. Hence, the quantity in (13): $\geq (0.366)^x(2.7743+2.3630x) = f(x)$ (say).

It can be easily shown that: $f'(x) = -(0.366)^x(0.4255+2.3751x) < 0$, $\forall x > 0$. So $f(x)$ is a decreasing function for $x > 0$. Also, $f(0.864) = 2.021 \geq 2+2/l$ (since $l \geq 100$). Hence, for $x \leq 0.864$, $f(x) \geq 2.021$. It follows that the quantity in (13) is ≥ 2.021 for $x \leq 0.864$, or equivalently, $\tilde{n}_1 \leq 0.864l$. Hence, inequality (11) holds for $\tilde{n}_1 \leq 0.864l$ and $l \geq 100$.

Next, consider

$$\begin{aligned}
&2.7743\left(1-\frac{1}{l}\right)^{\tilde{n}_1}+2.3630\frac{\frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1}}{1-\frac{1}{l}}\\
&\leq 2.7743\left(1-\frac{1}{l}\right)^{\tilde{n}_1}+2.3630\frac{\frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1}}{0.99} \text{ (since } l \geq 100)\\
&= 2.7743\left(1-\frac{1}{l}\right)^{\tilde{n}_1}+2.3869\frac{\tilde{n}_1}{l}\left(1-\frac{1}{l}\right)^{\tilde{n}_1}\\
&= \left(1-\frac{1}{l}\right)^{xl}\left(2.7743+2.3869x\right) \qquad (14)
\end{aligned}$$

Now, $g(l) = (1-\frac{1}{l})^l < g(\infty) = e^{-1} = 0.3679$. Hence, the quantity in (14): $< (0.3679)^x(2.7743+2.3869x) = f_1(x)$ (say). Now, it is easy to show that $f_1'(x) = -(0.3679)^x(0.3874+2.3869x) < 0, \forall x > 0$. So $f_1(x)$ is decreasing function for $x > 0$. Also, $f_1(0.883) = 2.019 < 2.02 \leq 2+2/l$ (since $l \geq 100$). Hence, for $x \geq 0.883$, $f_1(x) < 2+2/l$. Hence, inequality (11) does not hold when $0.883l \leq \tilde{n}_1 < 1.6l$.

■

Proof of Proposition 2: It is easy to show that $(1-\frac{1.6}{\tilde{n}_1})^{\tilde{n}_1}$ is increasing in $\tilde{n}_1$ and its maximum value is $e^{-1.6}$ at $\tilde{n}_1$=∞. Also,

$$\begin{aligned}
(1-\frac{1.6}{\tilde{n}_1})^{\tilde{n}_1-1} &= \frac{(1-\frac{1.6}{\tilde{n}_1})^{\tilde{n}_1}}{1-\frac{1.6}{\tilde{n}_1}}\\
&\leq \frac{e^{-1.6}}{1-\frac{1.6}{\tilde{n}_1}}\\
&\leq \frac{e^{-1.6}}{1-\frac{1}{l}} \text{ (since } \tilde{n}_1 \geq 1.6l)\\
&\leq \frac{e^{-1.6}}{1-\frac{1}{4}} \text{ (since } l \geq 4).\\
&= 0.2692
\end{aligned}$$

Hence, $h(l,\tilde{n}_1) \geq l[3-2.7743\times e^{-1.6}-3.7808\times 0.2692] = 1.4221l$. ■