Sponsored data: On the Effect of ISP Competition on Pricing Dynamics and Content Provider Market Structures

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***Abstract*—We analyze the effect of sponsored data when Internet service providers (ISPs) compete for subscribers and content providers (CPs) compete for a share of the bandwidth usage by customers. Our model is of a full information, leader-follower game. ISPs lead and set sponsorship prices. CPs then make the binary decision of sponsoring or not sponsoring their content on the ISPs. Lastly, based on both of these, users make a two-part decision—choose the ISP to subscribe to, and amount of data to consume from each CPs through the chosen ISP. User consumption is determined by a utility maximization framework, sponsorship decision is determined by a non-cooperative game between CPs, and ISPs set their prices to maximize their profit in response to prices set by competing ISP. We analyze the dynamics of the prices set by ISPs, the sponsorship decisions of CPs, the market structure therein, and surpluses of the ISPs, CPs, users.**

This is the first analysis of the effect sponsored data in the presence of ISP competition. We show that inter-ISP competition does not inhibit ISPs from extracting a significant fraction of CP surplus, leaving CPs no better off (and sometimes worse off) as compared to the scenario where data sponsoring is disallowed. Moreover, ISPs often have an incentive to significantly skew the CP marketplace in favor of the most profitable CP.

I. Introduction

Market segmentation and discriminatory pricing are well known techniques [1], [2] that ISPs can use to increase revenues. A combination of inter-ISP competition and market expectations have rendered such schemes to be not so prevalent on the user side. Regulatory issues have also prevented the use of many smart data pricing schemes. However, sponsored data or zero-rating is a price discrimination technique that is being introduced by ISPs in many markets as a consumer friendly innovation and is gaining increased adaptation. In this scheme, the content provider (CP) pays the ISP charges for its content that is consumed by the users while the users do not pay the ISP charges for the same.

Regulatory response to sponsored data, or zero-rating, has been varied. In many countries, it is deemed to violate net neutrality regulations and is hence banned, e.g., Canada, Brazil, India, Chile, Sweden, Hungary. In many other countries it is allowed alongside net neutrality regulations that disallow discriminatory QoS schemes, e.g., USA, UK, Netherlands, Germany [3]. In fact, BEREC[1] guidelines stipulate a case by case analysis when zero-rating is a purely pricing practice, and leaves it to the national regulatory authorities.

Wherever allowed, it is expected that zero-rating and sponsored data schemes will become more prevalent and many companies are making plans to enter this 23 billion dollar market[2]. FreeBee on Verizon, BingeOn on T-Mobile, and the several avatars on AT&T, namely Sponsored Data, Data Perks, and Thanks, attest to their popularity. Some time ago, AT&T had revamped its Data Perks program to offer free DirecTV and other video services[3], Verizon was offering AOL Gameday and Hearst magazines via its FreeBee program[4] and T-Mobile has been offering free music and now streaming videos on BingeOn. There are also third party providers for such services, e.g., Aquto[5].

Our interest in zero-rating was sparked by the massive debate that occurred in India when Facebook wanted to introduce the zero-rated FreeBasics program. Around the same time, a zero-rated platform called AirTel Zero was announced in India. The Telecom Regulatory authority of India (TRAI) has since disallowed differential pricing by ISPs to their subscribers. And by extension zero-rating and sponsored data are disallowed. However, the ISPs provide many services that are subsidized in various ways that have an effect not different from zero-rating.

In this paper we study the effect of such services on the content provider market and on the surpluses of various stakeholders.

A. Previous work

The economics of discrimination and its effect on market structures, on investment incentives, and on stakeholder surpluses have been widely studied. In the provisioning of Internet service, one version of discrimination is called QoS discrimination. This is effected either by providing fast lanes for preferred CPs or by giving them transmission priorities or a combination of the two. The effect of QoS discrimination is analyzed in, e.g., [4]–[6]. With QoS discrimination, the improved quality of experience drives users toward the

[1]https://berec.europa.eu/

[2]https://www.mobilemarketer.com/ex/mobilemarketer/cms/news/research/20919.html

[3]https://www.theverge.com/2018/2/20/17032550/att-prepaid-plans-sponsored-data

[4]https://www.theverge.com/2016/1/19/10789522/verizon-freebee-sponsored-data-net-neutrality

[5]http://www.aquto.com/

preferred CPs. Our interest in this paper is in price discrimination effected through a sponsorship program or a zero-rating platform. In this scheme, the content of the sponsoring or zero-rated CPs is free to the user while the user pays for the content from the non sponsoring CPs. Here, cheaper prices drive users towards sponsored or zero-rated content. (In the rest of the paper we will use the terms sponsored data and zero-rating interchangeably.) Examples of work that address price discrimination are [7]–[12]. In [7], [8], one ISP and one CP interact in a Stackelberg game. Two CPs and one ISP are considered in [9]–[14]. These papers differ in the interaction between the agents, the consumption model for the users, and the manner in which sponsorship is effected. However, the key conclusion in all of them is qualitatively similar—as the revenue rate of the CPs increase, the ISP can achieve higher profits than in the case where sponsorship is not allowed. Further, CPs with lower revenue rate possibly lose more on their surplus either due to sponsorship costs, or due to competition with free content. This can make them become less profitable in the short term and potentially nonviable in the long term.

All of the preceding work considered only one ISP and this begs the natural question: Would ISP competition reduce the ability of the ISPs to extract the CP surplus? Specifically, would the ISPs be as powerful as the models that use only one ISP indicate. Surprisingly, the answer is in the affirmative, albeit with some qualifications.

We mention here that the only prior work that we are aware of that considers zero-rating with ISP-competition is [15]. This is a purely numerical study, where the strategic interaction between the competing ISPs and the resulting equilibria are not considered.

B. Preview

In the next section we set up the notation and the model for the leader-follower game involving two ISPs as leaders, two CPs following the ISPs, and a continuum of users following the CPs. We begin by describing the user behavior for a given set of ISP prices and CP sponsorships. This is then used to determine the market share of the ISPs. We then describe how the CPs make the sponsorship decision for a given set of prices from the ISPs. Finally, the determination of the prices by the ISPs is also detailed.

In Section III, we derive the best response strategies of one ISP for a given set of prices and the sponsorship configuration of the CPs on the competing ISP. The key contribution in this section is that

- there is a threshold on CP profitability beyond which the ISP will price its data sponsorship service such that at least one of the CPs will sponsor its data (Theorem 1), and
- at least one of the CPs has less surplus than it would have had if the ISP did not operate a data sponsorship program (Lemma 7).

In Section IV, we use the results of the previous section to have the ISPs sequentially determine their best response prices in response to the sponsorship configuration on the competing ISP, in a tâtonnement-like iterative process. The following are the key contributions in this section.

- For a wide range of parameter sets, we find that numerically, the iterative process converges to an equilibrium rather quickly. Further, at equilibrium the CPs choose the same configuration on both the ISPs.
- In some cases, the equilibrium configuration is the same as that in the case where each ISP acts as a monopoly. As a result, the ISPs are unaffected by inter-ISP competition, and both ISPs are able to extract a fraction of the CP-side surplus. At least one CP, and sometimes both CPs, end up worse off in the process, compared to the scenario where data sponsorship is not permitted (Theorems 3 and 4).
- In some cases, inter-ISP competition results in a *prisoner's dilemma*, causing both ISPs to induce a sub-optimal sponsorship configuration. However, this does not necessarily result in a benefit for CPs. At least one CP, and sometimes both CPs, still end up worse off, compared to the scenario where data sponsorship is not permitted.

In Sections V and VI, we consider natural generalizations of our model, and demonstrate that these generalizations do not qualitatively change our conclusions above. Specifically, in Section V, we consider asymmetric ISPs, differentiated by the stickiness of their customer base. In Section VI, we consider the extension to three CPs (and two ISPs).

We conclude with a discussion and some policy prescriptions in Section VII. The key input to policy planners from the preceding is that although ISP and CP competition can provide price stability, data sponsorship practices enable ISPs to extract a substantial portion of CP surplus—importantly, this ability is not diminished by inter-ISP competition. The resulting asymmetry of benefits drives smaller CPs towards significantly lower profitability and possibly exiting the market. Thus, while data sponsorship may provide improved surplus to users in the short run, it can also, in the long run, diminish competition in the CP marketplace. (Note however, that our model does not capture such long term effects.)

II. Model and preliminaries

We consider two competing ISPs and two competing CPs. Each ISP operates a zero-rating platform, and CPs have the option of sponsoring their content by joining the zero-rating platform of one or both ISPs. ISP j ($j \in \{1, 2\}$) charges p_j dollars per unit of data to its subscribers and a sponsoring charge of q_j dollars per unit of data on CPs that zero-rate their content.[6] CPs derive their revenue via advertisements; CP i ($i \in \{1, 2\}$) makes a revenue of a_i dollars per unit of data consumed by users. Users subscribe to exactly one of the two ISPs and consume content of the CPs through that ISP. Further, the volume of user consumption is determined by the ISP charges and the utility obtained.

We capture the strategic interaction between the users, CPs, and ISPs via a three-tier leader follower model, as shown in Figure 1.

[6]Such usage-based pricing is prevalent in the mobile Internet space [16].

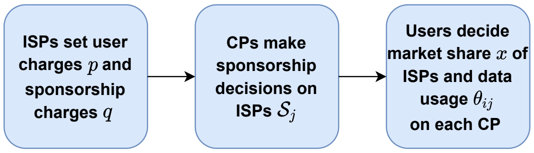

Fig. 1: Three-tier leader-follower interaction between ISPs, CPs, and users

1) ISPs 'lead' by setting sponsorship charges. For simplicity, we assume that user charges are equal, i.e., $p_1 = p_2 = p$, and are exogenously determined. [7]
2) CPs respond to sponsorship charges by making the binary decision of whether or not to sponsor their content on each ISP.
3) Finally, the user base responds to the actions of the CPs by determining the fraction of subscribers of each ISP. Moreover, subscribers of each ISP determine their consumption of each CP's content.

In the following, we describe in detail our behavior model of the user base, followed by our models for the behavior of the CPs and the ISPs. Proofs of the results stated in this section can be found in Appendix A.

A. User behavior

We begin by describing the consumption profile of users of ISP j ($j \in \{1, 2\}$), and subsequently describe how the market split across ISPs is determined.

Behavior of users of ISP j: Let $\mathcal{N} = \{1, 2\}$ denote the set of CPs. The set of sponsoring CPs on ISP j is denoted by $\mathcal{S}_j$ and $\mathcal{O}_j = \mathcal{N} \setminus \mathcal{S}_j$ denotes the set of non-sponsoring CPs on ISP j. We denote the configurations $\mathcal{S}_j = \emptyset$, $\mathcal{S}_j = \{1\}$, $\mathcal{S}_j = \{2\}$, and $\mathcal{S}_j = \{1, 2\}$ by NN, SN, NS and SS respectively.

We assume that users derive a utility of $\psi_i(\theta)$ from consuming θ bytes of data from CP i within a billing cycle. Here, $\psi_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuously differentiable, concave and strictly increasing function. We further assume that each user has a 'capacity to consume' c bytes, which is the maximum amount of data (across both CPs) a user can consume in a billing cycle. Let $\theta_{i,j}$ denote the consumption of CP i content by users of ISP j. Thus, we take $\theta_j = (\theta_{i,j}, i \in \mathcal{N})$ to be the unique solution (z_1^*, z_2^*) of the following optimization.

$$\begin{array}{cl} \max\limits_{z=(z_1,z_2)} & \sum_{i \in \mathcal{N}} \psi_i(z_i) - p \sum_{i \in \mathcal{O}_j} z_i \\ \text{s.t.} & \sum_{i \in \mathcal{N}} z_i \leq c, \quad z \geq 0 \end{array} \tag{1}$$

Here, the optimization variable z_i represents the data consumption of a user on CP i. The first term in the objective function above is the utility derived from content consumption, and the second term is the price paid by the user to ISP j for the consumption of non-sponsored content. Since p is assumed to be determined exogenously, it follows that the solution of the above optimization depends only on the sponsorship configuration $M_j \in \{\text{NN,SN,NS,SS}\}$ on ISP j. We sometimes write the solution of (1) as $\theta^{M_j} = (\theta_i^{M_j}, i \in \mathcal{N})$ to emphasize this dependence. We denote the optimal value of (1) by u^{M_j}.

Through most of the paper, we make the assumption that the two CPs are *substitutable*, i.e., $\psi_1(\cdot) = \psi_2(\cdot) = \psi(\cdot)$; this simplifies notation and also enables us to highlight the impact of zero-rating in *skewing* the user consumption profile.[8] However, several of our results (including those stated in Section III, along with Theorems 2 and 3 in Section IV) generalize for $\psi_1(\cdot) \neq \psi_2(\cdot)$; see the discussion on non-substitutable CPs in Section VII. Under the CP-substitutability assumption, it is easy to see that the surpluses of users of ISP j under different sponsorship configurations are sorted as follows.

Lemma 1. $u^{SS} \geq u^{SN} = u^{NS} \geq u^{NN}$.

Finally, we note the following consequence of the above consumption model.

Lemma 2. *For any sponsorship configuration M, and $i \in \mathcal{N}$, $\theta_2^{SN} = \theta_1^{NS} \leq \theta_i^M$.*

Having described the content consumption profile of users of each ISP, we now describe how the user base gets divided across the ISPs. Toward that, we assume that the users can change their ISP subscription at any time.

Market split across ISPs: We model the distribution of users between ISPs using the *Hotelling model* [17]. Let $x = x_{M_1}^{M_2}$ denote the fraction of the user base subscribed to ISP 1. Under the Hotelling model, $x_{M_1}^{M_2}$ is the solution of the equation

$$u^{M_1} - t x_{M_1}^{M_2} = u^{M_2} - t(1 - x_{M_1}^{M_2}), \tag{2}$$

where $t > 0$ is a parameter of the model. This equation may interpreted as follows: We imagine the users as being distributed uniformly over the unit interval $[0, 1]$. ISP 1 is located at the left end-point of this interval, and ISP 2 is located at the right end-point. A user at position $x \in [0, 1]$ incurs a (virtual) transportation cost of tx to connect to ISP 1, and a (virtual) transportation cost $t(1 - x)$ to connect to ISP 2. Since each (non-atomic) user connects to the ISP that provides the higher payoff (surplus minus transportation cost), the market split is determined by (2). Note that the transportation cost captures the inherent *stickiness* of users to a certain ISP; users located in the left (respectively, right) half of the interval have an inherent preference for ISP 1 (respectively, ISP 2).[9] Moreover, a higher value of t implies increased user stickiness. To ensure a meaningful solution to (2), we assume that $t > u^{SS} - u^{NN}$. It then follows that the market share of ISP 1 is given by

$$x_{M_1}^{M_2} = \frac{u^{M_1} - u^{M_2} + t}{2t}. \tag{3}$$

Note that the Hotelling model has been extensively used in many similar situations, including in the modeling of ISPs,

[7] Indeed, in many markets, user expectations and inter-ISP competition have driven user-side pricing to be flat across providers.

[8] We do however explicitly capture asymmetry in the CP revenue rates a_j. Indeed, different CPs that offer comparable services may differ in their ability generate ad revenue.

[9] In practice, user stickiness may result from many considerations like inertia, high lead time to switch ISPs, and familiarity with the features and services offered by one's present ISP.

e.g., [5]. Further, a generalization is considered in Section V where we will assume that the stickiness of the users is not symmetric, i.e., the t is different for different ISPs. We conclude by collecting some immediate consequences of the Hotelling model.

Lemma 3. *For any given sponsorship configuration M_2 on ISP2, the market market share of ISP1 under different sponsorship configurations are related as follows:*

$$x_{NN}^{M_2} \leq x_{SN}^{M_2} = x_{NS}^{M_2} \leq x_{SS}^{M_2}.$$

This lemma is an immediate consequence of Lemma 1 and (3).

Lemma 4. *As $t \to \infty$, for any given sponsorship configurations M_1 and M_2 on ISPs 1 and 2, $x_{M_1}^{M_2} \to 0.5$.*

The above lemma, which is a direct consequence of (3), states that as user stickiness grows, the market shares of the ISPs become insensitive to their sponsorship configurations and approach a symmetric market split. In other words, as $t \to \infty$, the churn of users between ISPs diminishes, and each ISP can be thought of as a *monopoly*.

B. CP behavior

In this subsection, we describe our model of CP behavior. Recall that in our leader-follower model, CPs lead the users and follow the ISPs, i.e., they decide whether on not to sponsor their content on ISPs 1 and 2 based on sponsorship charges announced by the ISPs, knowing *ex-ante* that the user base will respond to their actions based on the model presented in Section II-A. Since each CP seeks to maximize its own profit, it is natural to capture the outcome of their interaction as a Nash equilibrium.

Note that in general, each CP may choose to either sponsor or not sponsor its content on each ISP. This means that there are four possible actions per CP, and sixteen possible sponsorship configurations in all. To avoid the resulting analytical (and notational) complexity, we make the following simplifying assumption.

Assumption 1. *The CPs can only reconsider their sponsorship decision on a single ISP at a time.*

Assumption 1 is natural if there is a contractually binding period associated with the decision to sponsor one's content on an ISP, say ISP 1, with the opportunity to form (or renew) a sponsorship contract with ISP 1 arising periodically and out of sync with similar opportunities to sponsor on ISP 2.

Under Assumption 1, it is meaningful to ask the question: *Given a sponsorship configuration $M_2 \in \{NN,SN,NS,SS\}$ on ISP 2, when is $M_1 \in \{NN,SN,NS,SS\}$ a Nash equilibrium sponsorship configuration on ISP 1?* In the remainder of this section, we address this question.

Consider an arbitrary sponsorship M_2 configuration on ISP 2. If CP 1 chooses to sponsor its content on ISP 1, its surplus is given by

$$x(a_1 - q_1)\theta_{1,1} + (1-x)\big[(a_1 - q_2)\theta_{1,2}\mathbb{1}_{\{M_2 \in \{SS,SN\}\}} + a_1\theta_{1,2}\mathbb{1}_{\{M_2 \in \{NS,NN\}\}}\big].$$

The first term above captures the surpluses from ISP 1 (revenue from ISP 1 users minus the sponsorship charge paid to ISP 1). Note that this term contains the market share of ISP 1 as a factor; also recall that we take the 'volume' of the user base to be unity. The second term captures the surplus of CP 1 from ISP 2. Similarly, if CP 2 chooses not to sponsor its content on ISP 1, its surplus is given by

$$xa_1\theta_{1,1} + (1-x)\big[(a_1 - q_2)\theta_{1,2}\mathbb{1}_{\{M_2 \in \{SS,SN\}\}} + a_1\theta_{1,2}\mathbb{1}_{\{M_2 \in \{NS,NN\}\}}\big].$$

It is important to note that in the above equations, x, $\theta_{1,1}$ and $\theta_{1,2}$ depend on the actions of both CPs. The conditions for the different sponsorship configurations on ISP 1 to be a Nash equilibrium are derived in Appendix A.

C. ISP behavior

We now describe our model for ISP behavior. ISPs derive their revenue from two sources: from users (subscribers) for the consumption of non-sponsored content, and from CPs for the consumption of sponsored content. Thus, the surplus of ISP 1 is given by $x\left[\sum_{i \in \mathcal{S}_1} q_i\theta_{i,1} + \sum_{i \in \mathcal{O}_1} p\theta_{i,1}\right]$, whereas that of ISP 2 is given by $(1-x)\left[\sum_{i \in \mathcal{S}_2} q_i\theta_{i,2} + \sum_{i \in \mathcal{O}_2} p\theta_{i,2}\right]$.

The ISPs, being leaders of the three-tier leader-follower interaction, set sponsorship prices as to induce the most profitable Nash equilibrium among CPs on their zero-rating platform. Specifically, we assume that given a sponsorship configuration on, say ISP 2, when ISP 1 sets the sponsorship price on its zero-rating platform, the most profitable (for ISP 1) Nash equilibrium between the CPs on its platform emerges.[10] Note that in our model, the impact of the action of any ISP depends on the prevailing sponsorship configuration on the other. In other words, the interaction between the ISPs has *memory*.

Definition 1 (System equilibrium). *A tuple (q_1, M_1, q_2, M_2) is said to be a system equilibrium if, for $j \in \{1,2\}$,*[11]

1) *Given sponsorship configuration M_{-j} on ISP $-j$, M_j is the most profitable Nash equilibrium (among the CPs) for ISP j under action q_j.*
2) *Given sponsorship configuration M_{-j} on ISP $-j$, the surplus of ISP j is maximized under action q_j.*

Note that under a system equilibrium, neither ISP has the incentive to unilaterally deviate from its action. Moreover, neither CP has the unilateral incentive to deviate from its sponsorship decision on one ISP given the prevailing sponsorship configuration on the other ISP.[12]

This concludes the description of our system model. Key notation is summarized in Table I. In Section III, we explore

[10] Note that if the action of an ISP allows for multiple Nash equilibria between the CPs (as per Lemma 8), we assume the ISP is able to induce the most profitable equilibrium. This is a standard approach for handling non-unique follower equilibria in leader-follower interactions [18].

[11] For any ISP j, we use the label $-j$ to refer to the other ISP.

[12] This is a weak notion of equilibrium, in the sense that it does not guarantee that CPs do not have the incentive to reverse their sponsoring decisions on *both* ISPs. However, we prove in Section IV that the system equilibria we observe do indeed possess this guarantee; see Theorems 3 and 4.

Notation	Description
$\mathcal{N}$	Set of CPs
$\mathcal{S}_j$	Set of sponsoring CPs on ISP j
p_j	User charge per unit of data on ISP j
q_j	Sponsorship charge per unit of data on ISP j
$\psi_i(\theta)$	User utility function for θ amount of data consumed at CP i
$\theta_{i,j}$	Amount of CP i data consumed by a user of ISP j
c	Capacity of data consumption by a user
$x_{M_1}^{M_2}$	ISP 1 market share under sponsorship configuration M_j on ISP j
$\theta_i^{M_j}$	CP i data consumed by user of ISP j under sponsorship configuration M_j
t_j	User transportation cost per unit distance from ISP j
a_i	CP i revenue per unit data consumed
ρ	Ratio of a_2 by a_1

TABLE I: Summary of notation

the optimal response of ISP 1 given a prevailing sponsorship configuration on ISP 2. Then, in Section IV, we analyze system equilibria.

III. ISP's best response strategy

In this section, we assume a fixed sponsorship configuration M_2 on ISP 2, and analyze the optimal strategy for ISP 1. This optimal strategy involves setting the sponsorship charge q_1 so as to induce the most profitable Nash equilibrium between the CPs on its zero-rating platform. We also study the impact of ISP 1's optimal strategy on the surplus of both ISPs, both CPs, and the user base.

The analysis of this section sheds light on the behavior we might expect from an ISP in a competitive marketplace. Indeed, the case M_2 = NN can also be thought as capturing the scenario where one ISP (ISP 1) operates a zero-rating platform, whereas the other (ISP 2) does not. Such a situation can happen when the competing ISP is slow to act; e.g., Sprint announced its zero rating service much later than its competitors. Moreover, the analysis of this section captures one step in the alternating best response dynamics we consider in the following section, providing insights into the observed system equilibria.

For notational simplicity, throughout this section, we take M_2 = NN. Our results easily generalize to arbitrary M_2. As in [12], we find it instructive to analyze ISP 1's optimal strategy in the scaling regime of *growing CP revenue rates*. After all, it is when CP revenue rates are large that ISPs have the incentive to offer zero-rating opportunities, so as to extract some of the CP-side surplus. Specifically, we consider $(a_1, a_2) = (a, \rho a)$, where $a > 0$ is a scaling parameter and $\rho \in (0, 1)$ is fixed. While the parameter a captures the overall monetization ability of the CPs, the parameter ρ captures the asymmetry in monetization ability across CPs. When ρ is small, this corresponds to the scenario where CP 1 has a considerably greater ability to monetize its content than CP 2, although their services are comparable from a user standpoint. As we shall see in this section and the next, the outcomes corresponding to this case differ considerably from the outcomes when $\rho \approx 1$, i.e., when the CPs are comparable in their ability to monetize their content. The proofs of the results stated in this section can be found in the appendix.

The following theorem sheds light on the sponsorship configurations induced by ISP 1 in the regime of growing CP revenue rates.

Theorem 1. ***[ISP 1's profit maximizing strategy]*** *Let $(a_1, a_2) = (a, \rho a)$ where $a > 0$, and fixed $\rho \in (0, 1)$. There exists a threshold $a_s > 0$ such that*

1) *For $a \le a_s$, ISP 1 enforces an NN equilibrium.*
2) *For $a > a_s$, ISP 1 enforces an SN/SS equilibrium. Specifically, for $a > a_s$, there exists $\rho_s \in (0, 1)$ such that for $\rho \le \rho_s$, ISP 1 enforces an SN equilibrium, and otherwise, it enforces an SS equilibrium.*

Theorem 1 shows that when the revenue rates of both CPs are small, ISP 1 favors an NN configuration, since charging users is more profitable than charging the CPs. When the revenue rates cross a certain threshold, ISP 1 induces an SN/SS equilibrium depending on the values of a and ρ. Specifically, if ρ is small, i.e., CP 1 has a considerably higher revenue rate than CP 2, then ISP 1 favors an SN configuration. Indeed, in this case, it is in the interest of ISP 1 to skew user-side consumption in favor of CP 1, thanks to the greater potential for sponsorship revenue from CP 1 compared to CP 2. On the other hand, when $\rho \approx 1$, ISP 1 favors an SS configuration (for large enough a).

Next, we note that the threshold on CP revenue rates for sponsorship to be profitable for ISP 1 shrinks as user stickiness decreases. This is because when user stickiness is small (i.e., t is small), ISP 1 sees a sharp growth in its subscriber base once sponsorship kicks in.

Lemma 5. *Let $(a_1, a_2) = (a, \rho a)$ where $a > 0$, and fixed $\rho \in (0, 1)$. The sponsorship threshold a_S defined in Theorem 1 is an increasing function of t.*

Our next result highlights the benefit to ISP 1 from zero-rating.

Lemma 6. ***[ISP 1 surplus]*** *Let $(a_1, a_2) = (a, \rho a)$ where $a > 0$, and fixed $\rho \in (0, 1)$. Under the optimal strategy for ISP1 (given by Theorem 1), the profit $r_I(a)$ of ISP 1 varies with a as follows.*

1) *$r_I(a)$ is constant over $a \le a_s$.*
2) *For $a > a_s$, $r_I(a)$ is a strictly increasing, superlinear function of a, i.e., there exist constants $\nu > 0$ and κ such that $r_I(a) \ge \nu a + \kappa$ for $a > a_s$.*

Note that for $a > a_s$, ISP 1 profit grows at least linearly in a, implying that ISP 1 is able to extract a fraction of the CP revenues by optimally setting the sponsorship charge on its zero-rating platform.

Next, we turn to CP-side surplus. As the following lemma shows, the zero-rating platform leaves at least one CP worse off.

Lemma 7. ***[CP surplus]*** *Under the optimal strategy for ISP1 (given by Theorem 1), the following statements hold.*

1) *When ISP 1 induces an SN equilibrium, CP 1 makes the same profit as it would make under an NN configuration (or equivalently, without the zero rating platform on ISP 1). On the other hand, CP 2 makes a profit less than or equal to that it would make under an NN configuration.*

2) *Under an SS equilibrium, at least one of the CPs makes a profit less than or equal to that it would make under an NN configuration.*

Finally, we note that since zero-rating increases user surplus (see Lemma 1), it is clear that the surplus of subscribers of ISP 1, and also the aggregate surplus of the user base, is increased for $a > a_S$.

To summarize, the results in this section show that so long as the CP revenue rates are large enough, ISP 1 can set the sponsorship charges on its zero-rating platform so as to extract a considerable fraction of CP-side surplus, leaving one or both the CPs worse off. Moreover, ISP 1 also benefits from the growth of its subscriber base that results from the increased utility afforded to its users from sponsorship.

While the present section only considers the strategic behavior of a single ISP, in the following section, we seek to capture the strategic interaction between the ISPs.

IV. Equilibria of best response dynamics

The goal of this section is to study the strategic interaction between the ISPs, each ISP seeking to maximize its own profit. Since a characterization of the system equilibria associated with the three-tier interaction between the ISPs, the CPs, and the users is not analytically feasible (except in two limiting regimes; see Theorems 3 and 4), we explore the system equilibria obtained by simulating alternating best response dynamics between the ISPs, i.e., the ISPs alternatively play their optimal response to the prevailing sponsorship configuration on the other ISP. (The results of the previous section shed light on this optimal response.) These dynamics capture a myopic interaction between competing ISPs. Note that an equilibrium of these dynamics, i.e., a configuration where neither ISP adapts its action, is also a system equilibrium as defined in Section II-C. In this section, we analyze the properties of these equilibria (when they exist), highlighting the resulting sponsorship configurations, and also the surplus of the various parties.

Our numerical experiments yield two interesting observations:

- The alternating best response dynamics either converge quickly (in 5 to 8 rounds) or (in some cases) oscillate indefinitely.
- When the dynamics do converge, the equilibrium is *symmetric*, i.e., of the form (q, M, q, M).

This last observation leads us to analyze the implications of symmetric system equilibria:

Theorem 2. *Under any symmetric system equilibrium of the form (q, M, q, M), the following holds.*

1) *If $M \in \{SN, NS\}$, then the CP that sponsors on both ISPs makes the same profit as it would if zero-rating were not permitted. On the other hand, the CP that does not sponsor on both ISPs makes a profit less than or equal to that it would if zero-rating were not permitted.*
2) *If $M = SS$, at least one of the CPs makes a profit less than or equal to that it would make if zero-rating were not permitted.*

Theorem 2 highlights that under any symmetric system equilibrium, at least one of the CPs is worse off, compared to the case where zero-rating is not permitted. In the absence of inter-ISP competition, a similar observation was made in [12]; Theorem 2 highlights that *competition at the ISP level does not necessarily translate to improved surplus at the CP level*. The proof of Theorem 2 is presented in the appendix.

We now report the results of our numerical experiments. Throughout, we use $\psi(\theta) = \log(1 + \theta)$. We set the initial configuration on ISP 2 to be NN, and allow ISP 1 to play first (although we observe that the limiting behavior of the dynamics is robust to the initial condition).

A. Equilibrium sponsorship configurations

We first report the (experimentally observed) limiting sponsorship configurations from the best response dynamics over the $a_1 \times a_2$ space. Interestingly, in all cases, we observe that the equilibrium (when the dynamics converge) is symmetric across the ISPs, i.e., both ISPs arrive at the same sponsorship configuration. Moreover, these equilibrium configurations have the same structural dependence on a_1 and a_2 as we saw in the 'single-step best response' characterization in Section III; see Figure 2(a). When a_1 and a_2 are small, the equilibrium involves both ISPs in an NN configuration, as expected. Moreover, when $a_1 \gg a_2$ or $a_2 \gg a_1$, both ISPs arrive at an equilibrium wherein the more profitable CP sponsors. Finally, when a_1 and a_2 are comparable and large enough, both ISPs induce both CPs to sponsor. We also observe that there are certain intermediate regions in the $a_1 \times a_2$ space where the best response dynamics oscillate.

Next, we compare the limiting behavior of the best response dynamics for different values of the transportation cost parameter t; see Figures 2(a)–2(c). Recall that increasing t implies increasing user stickiness, and thus a diminishing dependence of one ISP's action on the other. Note that as t grows, the region of the $a_1 \times a_2$ where the ISPs induce one or both CPs to sponsor shrinks. Interestingly, this is the result of a *prisoner's dilemma* between the ISPs: When t is small, i.e., when inter-ISP user churn is significant, each ISP has the unilateral incentive to induce sponsorship even at small CP revenue rates, to benefit from the resulting increase in its subscriber base. However, once one ISP induces sponsorship, the other ISP is also incentivized to induce sponsorship to recover its lost market share. As a result, the ISPs arrive at an equilibrium that leaves them both worse off; this will also be apparent from the plots of ISP surplus reported later.

On the other hand, when t is large, then each ISP's market share is relatively insensitive to the other's actions, and so the ISPs induce sponsorship only when it is mutually beneficial for them to do so. This also explains why as t becomes large, the region of the $a_1 \times a_2$ space where the best response dynamics oscillate diminishes. Finally, we compare the limiting behavior of the best response dynamics for different values of the 'capacity to consume' c; see Figures 3(a)–3(c). Note that when c is small, there is only a modest growth in user-side consumption from zero-rating. As a result, the equilibrium is NN on both ISPs except when the CP revenue rates are

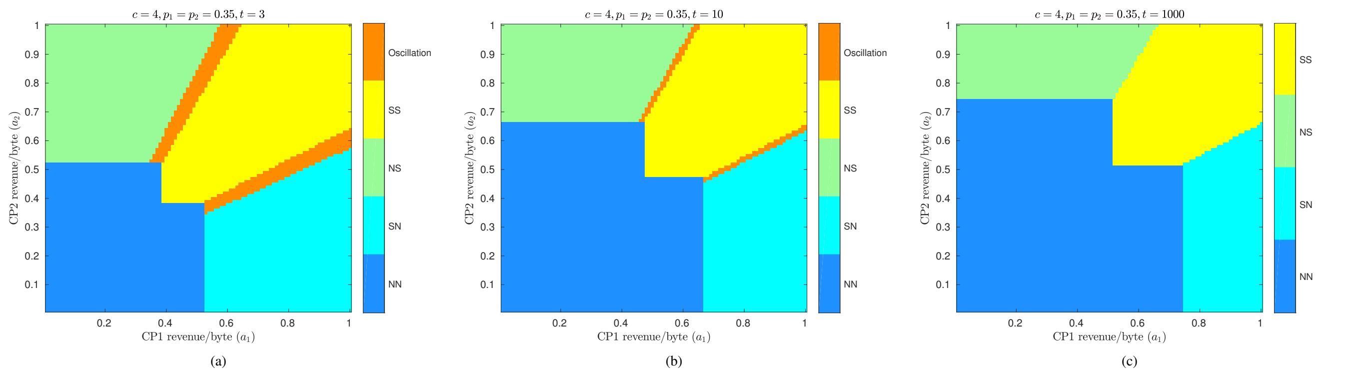

Fig. 2: Limiting sponsorship configurations as a function of a_1, a_2 with $c = 4$ and varying t. (a) $t = 3$ (b) $t = 10$ (c) $t = 1000$

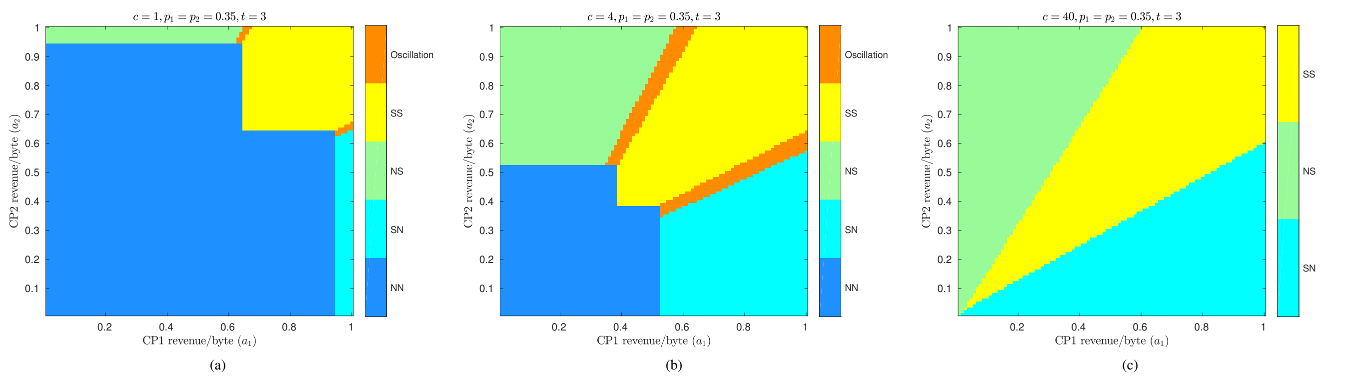

Fig. 3: Limiting sponsorship configurations as a function of a_1, a_2 with $t = 3$ and varying c. (a) $c = 1$ (b) $c = 4$ (c) $c = 40$

really large. On the other hand, when c is large, ISPs induce sponsoring even at moderate revenue rates to benefit from the increased data consumption from the users.

B. Surplus

Having explored the equilibrium sponsorship configurations that result from alternating best response dynamics between ISPs, we now consider the equilibrium surplus realized by the ISPs, the CPs, and the users. Since a 3-d visualization of surplus in the $a_1 \times a_2$ space is hard to interpret, we use the parameterization $(a_1, a_2) = (a, \rho a)$ for $\rho \in (0, 1)$ (as in Section III). We first consider the case when ρ is small (i.e., CP2's revenue/byte is much less than that of CP1) and then the case where ρ is close to 1 (i.e., the revenue rates of both CPs are comparable).

Small ρ : Figure 4 shows the surplus of the ISPs (recall that since the equilibria we observe are symmetric across the ISPs, both ISPs obtain the same surplus), CP1, CP2, and the user base as a function of a for $\rho = 0.1$ and $t = 3$. From Figure 2(a), it is clear that in this case, both ISPs induce an NN equilibrium for a less than a certain threshold, and an SN equilibrium beyond this threshold. We benchmark the equilibrium surplus under our model (ISP *duopoly*) with case where users are infinitely sticky (i.e., each ISP operates as a *monopoly*) and the case where neither ISP operates a zero-rating platform.

As was observed in Section IV-A, competition forces both ISPs to induce an SN configuration for smaller values of a as compared to the monopoly setting ($t \to \infty$). This is evident from the lower threshold (in a) for sponsorship as compared to the monopoly setting. This *prisoner's dilemma* between the ISPs causes both ISPs to obtain a smaller profit compared with the monopoly case for intermediate values of a; see Figure 4(a). For larger values of a however, the each ISP's surplus matches that in the monopoly case. The surplus of the CP 1 (the sponsoring CP) remains the same under all three models, in line with the conclusion of Theorem 2; see Figure 4(b). On the other hand, CP 2 (the non-sponsoring CP) is worse off due to zero-rating, also in line with Theorem 2; see Figure 4(c). Finally, we note that user surplus gets enhanced due to zero-rating, as expected; see Figure 4(d). To summarize, we observe that except for intermediate values of a, where competition forces both ISPs to induce sponsorship prematurely, the surplus of all parties matches that in the monopoly case: the ISPs are able to extract a considerable fraction of CP surplus, and neither CP benefits from zero-rating. Indeed, as we prove below, for large enough a and small enough ρ the monopoly configuration is indeed a system equilibrium in our duopoly model.

Theorem 3. *Let* $(a_1, a_2) = (a, \rho a)$ *for* $\rho \in (0, 1)$*. There exist thresholds* $a_{SN} > 0$ *and* $\rho_{SN} > 0$ *such that for* $a > a_{SN}$ *and* $\rho < \rho_{SN}$*, there exists* $q(a)$ *such that:*

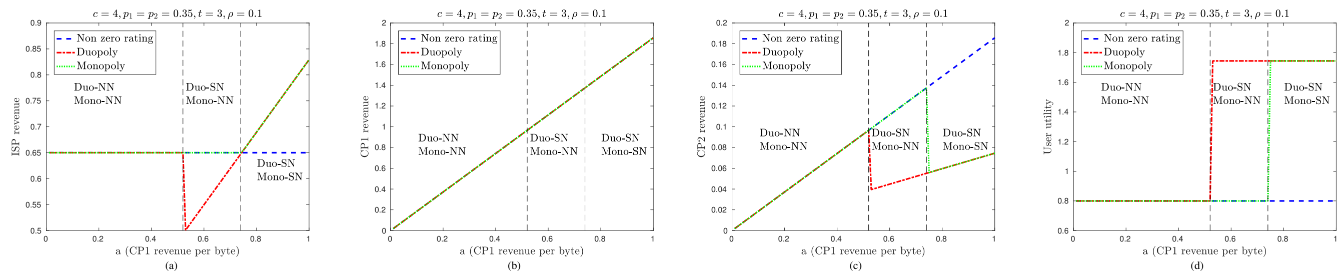

Fig. 4: Surplus of various entities for $c = 4, t = 3, p = 0.35$ and $\rho = 0.1$ as a function of a. (a) ISP revenue (b) CP1 revenue (c) CP2 revenue (d) User surplus

1) *$(q(a), SN, q(a), SN)$ is a system equilibrium. Under this configuration, neither CP has the unilateral incentive to reverse its sponsorship decision on one/both ISPs. Moreover, CP 1 makes the same profit as it would in the absence of the zero-rating platforms, whereas CP 2 makes a profit less than or equal to that it would in the absence of the zero-rating platforms.*
2) *In the monopoly setting ($t \to \infty$), it is optimal for each ISP to induce an SN equilibrium by setting its sponsorship charge equal to $q(a)$.*

Note that Theorem 3 does not prove that for large enough a and small enough ρ, the best response dynamics converge to the stated configuration. It merely establishes that the configuration that the best response dynamics converge to in our experiments is indeed a system equilibrium. In fact, it proves that the observed configuration is an equilibrium in a stronger sense, in that neither CP has the incentive to switch its sponsorship decisions across both ISP platforms. The proof of Theorem 3 is presented in the appendix.

Large ρ : Next, we consider the case where $\rho = 0.8$. Figure 5 shows the surplus of various entities as a function of a. From Figure 2(a), it is clear that in this case, both ISPs induce an NN equilibrium for a less than a certain threshold, and an SS equilibrium beyond this threshold.

As before, we observe a prisoner's dilemma between the ISPs for intermediate values of a, where the ISP's enter into a mutually sub-optimal sponsorship equilibrium; see Figure 5(a). However, for larger values of a, each ISP's surplus matches that in the monopoly setting. Interestingly, this case also represents a prisoner's dilemma between the CPs, wherein both CPs end up sponsoring for large enough a, and in the process end up worse off than if neither CP had sponsored; see Figures 5(b) and 5(c). Finally, we note as before that user surplus is enhanced by sponsorship, more so than in the SN configuration that emerges when ρ is small; see Figure 5(d). As before, we prove that when a and ρ are large enough, the observed equilibrium of the best response dynamics is indeed a system equilibrium in our duopoly model. Moreover, under this configuration, neither CP has the incentive to switch their sponsorship decision on both ISPs.

Theorem 4. *Let $(a_1, a_2) = (a, \rho a)$ for $\rho \in (0, 1)$. There exist thresholds a_{SS} and ρ_{SS} such that for $a > a_{SS}$ and $\rho > \rho_{SS}$, there exists $q(a, \rho)$ such that:*

1) *$q(a, \rho), SS, q(a, \rho), SS)$ is a system equilibrium. Under this configuration, neither CP has the unilateral incentive to reverse its sponsorship decision on one/both ISPs. Moreover, at least one CP makes a profit less than or equal to that it would in the absence of the zero-rating platforms.*
2) *In the monopoly setting ($t \to \infty$), it is optimal for each ISP to induce an SS equilibrium by setting its sponsorship charge equal to $q(a, \rho)$.*

The proof of Theorem 4 is presented in the appendix.

Intermediate ρ : So far, we have seen that:

- For small enough ρ and large enough a, the limiting configuration under alternating best response dynamics is SN on both ISPs, which matches configuration under the monopoly setting.
- For $\rho \approx 1$ and large enough a, the limiting configuration under alternating best response dynamics is SS on both ISPs, which also matches configuration under the monopoly setting.

It is thus natural to ask what happens for intermediate values of ρ. In this section, we show that for intermediate values of ρ, a different type of *prisoner's dilemma* can occur between the ISPs, where both ISPs arrive at an SS configuration, even though an SN configuration would be better for both ISPs.

To illustrate this most clearly, we set $c = 90$. Figure 6 shows the limiting ISP configurations for duopoly and monopoly. We observe that for a range of ρ, the limiting duopoly configuration is SS on both ISPs, whereas in the monopolistic setting, both ISPs prefer an SN equilibrium. This is a different type of *prisoner's dilemma* between the ISPs — it is optimal for both ISPs to operate an SN configuration. However, in this state, each ISP has a unilateral incentive to switch to SS, in order to gain a higher market share. However, once one ISP switches to SS, the other ISP is also incentivized to switch to SS to regain its lost market share, resulting in a mutually suboptimal equilibrium. Figure 7 shows surplus of various entities for $\rho = 0.8$. Note that ISP surplus is lower than that under the monopoly setting. Indeed, CP 1 actually benefits from this *prisoner's dilemma* between the ISPs.

To summarize the key takeaways from this section, we see that strategic interaction between ISPs, as captured by alternating best response dynamics, can result in:

- Identical configuration to the monopoly setting. In this

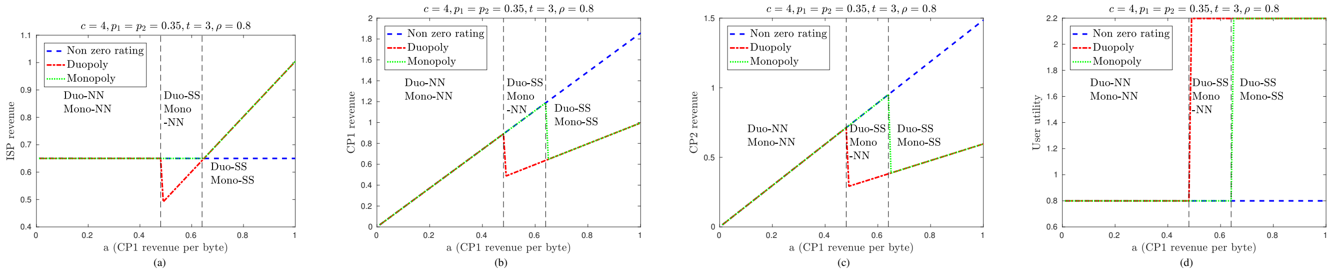

Fig. 5: Surplus of various entities for $c = 4, t = 3, p = 0.35$ and $\rho = 0.8$ as a function of a. (a) ISP revenue (b) CP1 revenue (c) CP2 revenue (d) User surplus

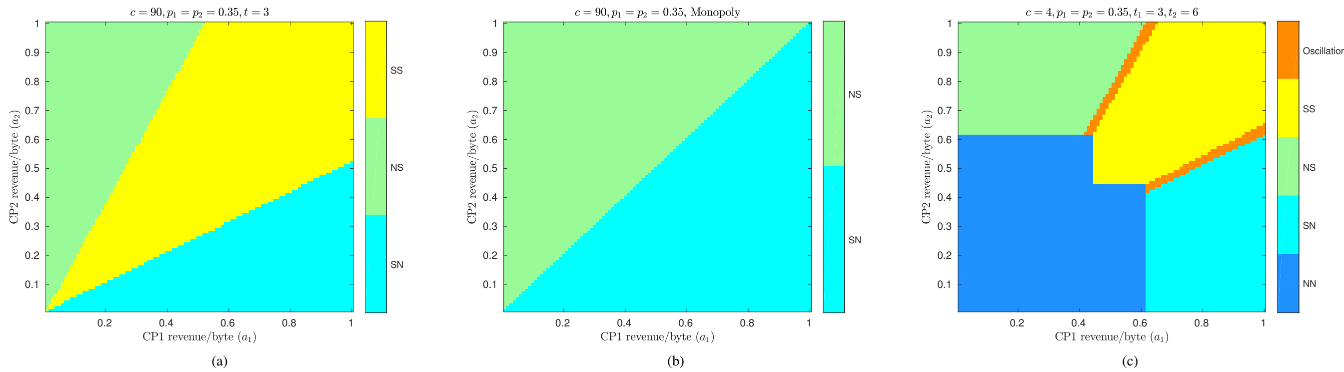

Fig. 6: ISP configurations as a function of a_1, a_2. (a) Duopoly with $t = 3$ for $c = 90$ (b) Monopoly for $c = 90$ (c) For asymmetric stickiness with $t_1 = 3, t_2 = 6, c = 4$. Initial ISP2 in NN.

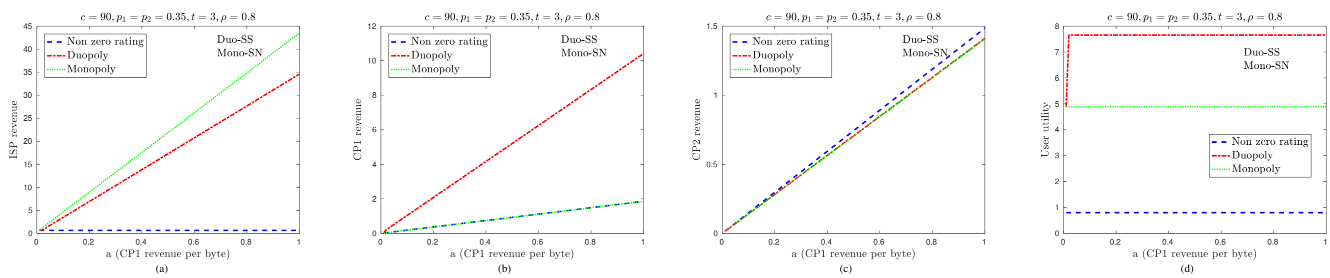

Fig. 7: Surplus of various entities for $c = 90, t = 3, p_1 = p_2 = 0.35$ and $\rho = 0.8$ as a function of a (a) ISP revenue (b) CP1 revenue (c) CP2 revenue (d) User surplus

case, the ISPs are not affected by inter-ISP competition. Both ISPs manage to extract a portion of CP-side surplus, and at least one CP is worse off due to zero-rating.

- Prisoner's dilemma between the ISPs. This occurs for (i) intermediate values of a, with small ρ or $\rho \approx 1$, and (ii) intermediate values of ρ. In this case, the ISPs are hurt by inter-ISP competition. However, the CPs do not necessarily benefit even in this case; at least one CP still ends up worse off due to zero-rating.

V. Asymmetric ISPs

In this section, we consider a generalization of our model, where the ISPs are asymmetric with respect to user stickiness, i.e., where one ISP enjoys higher customer loyalty than the other. We capture asymmetric user stickiness via asymmetric transportation costs in Hotelling model. We find that while the equilibria of best response dynamics between the ISPs under this generalization are qualitatively similar to those in Section IV, the ISP that enjoys a higher user stickiness benefits more from zero-rating than the other.

The generalized Hotelling model is parameterized by two parameters t_1 and t_2. Under this model, for sponsorship configuration M_j on ISP j, the fraction of subscribers of ISP 1, denoted $x_{M_1}^{M_2}$, is given by

$$u^{M_1} - t_1 x_{M_1}^{M_2} = u^{M_2} - t_2(1 - x_{M_1}^{M_2}).$$

To ensure a meaningful solution, we assume $t_1, t_2 > u^{SS} - u^{NN}$. Note that if $t_1 < t_2$, users incur a lower transportation cost to ISP 1 as compared to ISP 2, implying that ISP 1 enjoys a higher user stickiness than ISP 2.

We simulate the alternating best response dynamics for this setting, taking $t_1 = 3$, and $t_2 = 6$. As before, $\psi(\theta) = \log(1+\theta)$. Interestingly, the limiting sponsorship configurations that emerge from best response dynamics remain symmetric across the ISPs; see Figure 6c. Moreover, the limiting configurations are qualitatively identical to the case where user stickiness is symmetric. Indeed, asymmetry in user-stickiness primarily manifests in an asymmetry in the market shares of the two ISPs. To see this, we let $(a_1, a_2) = (a, \rho a)$, and plot the equilibrium surpluses of the ISPs, CPs, and the user base as a function of a for $\rho = 0.1$ (see Figure 8). We benchmark the observed surplus against the surplus when (i) neither ISP operates a zero-rating platform, and (ii) the monopoly setting where each ISP's market share is fixed to that under case (i). We observe that

1) As expected, ISP 1 enjoys a higher surplus than ISP 2, owing to its larger market share.
2) When ρ is small, both ISPs induce an SN equilibrium for large enough a.
3) When ρ is large, both ISPs induce an SS equilibrium for large enough a (figure omitted due to space constraints).
4) For intermediate values of a, there is a prisoner's dilemma between the ISPs, where they both induce sponsorship prematurely, resulting in a lower surplus. Except in this region, the equilibrium sponsorship configurations and surpluses match those in the monopoly model.
5) At-least one CP, and sometimes both CPs, end up worse off due to zero-rating.

VI. Extension to 2 ISPs and 3 CPs

In this section, we present some empirical results corresponding to a generalized model with 2 ISPs and 3 CPs. We find that our qualitative conclusions from Sections III and IV continue to hold under this generalization. As before, CP i, $i \in \{1, 2, 3\}$, derives a_i dollars per unit of data consumed by users, and user derive utility $\psi_i(z)$ by consuming z units of its content. It is easy to observe that in this case, each ISP would have 8 sponsorship configurations, namely $\{NNN, NNS, NSN, NSS, SNN, SNS, SSN, SSS\}$, leading to 64 possible system equilibria across ISPs 1 and 2.

First, we consider the case where CP 1 and CP 2 are substitutable from the standpoint of the user base, i.e., $\psi_1(\theta) = \psi_2(\theta) = \log(1+\theta)$. To understand the effect of CP revenue on the sponsorship configurations, we simulate the best response dynamics by varying a_1 and a_2 independently, fixing $a_3 = a_1$. Figures 9(a) and 9(b) show the limiting equilibrium configurations when $\psi_3(\theta) = 2\log(1+\theta)$, and $\psi_3(\theta) = 0.5\log(1+\theta)$, respectively. Note that the behavior of the substitutable CP pair (CPs 1&2) is similar to our earlier observations for the two CP case. Between CPs 1 and 3, we observe that when $\psi_3 > \psi_1$, as $a_1 = a_3$ is increased for fixed a_2, CP 1 sponsors for smaller values of a_1 as compared to CP 3, to compensate for its smaller content consumption. A reciprocal observation can be made when $\psi_3 < \psi_1$ from Figure 9(b). We also find that in this case, our earlier observations that the ISPs extracts a significant portion of the CPs surplus, and that at least one CP is worse off due to zero-rating, also hold true. Formal results similar to the ones presented in Sections III and IV can be proved for this case as well. Next, we consider the setting where the CPs can approximately be divided into two classes. Specifically, we simulate the best response dynamics when $\psi_1 = \psi_2 = \log(1+\theta)$ and $a_2 = 0.9a_1$, such that CP 1 and CP 2 can be considered (approximately) as being in the same class. Figures 10(a), 10(b) show the limiting ISP configurations obtained by the best response dynamics for different choices of ψ_3. Figure 11 shows the surplus of various entities for the latter setting. Our main observations are as follows:

1) CPs of the same 'class' make the same sponsorship decisions at equilibrium. This can be observed from Figures 10(a), 10(b) where the observed equilibrium configurations are only $\{NNN, NNS, SSN, SSS\}$.
2) As observed earlier, in this case when $\psi_3 < \psi_1$ CP 3 is forced to sponsor at smaller revenue rates as compared to CP 1 to compensate for its smaller content consumption.
3) When a CP sponsors and other CPs do not, then the sponsoring CP makes the same surplus as it would in the absence of zero-rating; see Figure 11. But the non-sponsoring CPs lose surplus. These observations are same as in the two CP case (Lemma 7 and Theorem 3). In the SSS configuration, all the three CPs lose surplus relative to the setting wherein zero-rating is disallowed.
4) As in the 2 CPs case, for intermediate revenue rates, the ISPs experience a prisoner's dilemma and enforce a sub-optimal sponsorship configuration. Except for this region, the ISPs benefit from zero-rating, whereas the CPs do not (see Figure 11).

The empirical results presented in this section show that our insights from the 2 CP case apply even when the number of CPs is increased.

VII. Policy Implications and Discussion

The key takeaway from our analyses is the following. When ISPs lead in setting sponsorship prices, they do so in such a way that a significant fraction of the CP surplus gets paid to the ISPs in the form of sponsorship costs. This reduces the CP surplus significantly. Further, if one of the CPs is more profitable than the other, then ISPs force a configuration in which the more profitable CP sponsors and the other does not, skewing the consumption profile of the user base (Lemma 7 and Theorem 3). This fact does not change with increased capacity of the users to consume, or when the users are not sticky in their choice of ISP. Therefore the ISPs have an interest in picking winners from among the CPs. Interestingly, even the 'winner' CP does not typically benefit in this process! On the other hand, less profitable CPs can suffer and be eliminated from the market (Theorems 3 and 4). In other words, data sponsorship practices grant ISPs considerable market power—indeed, our analysis highlights that this power is *not* diminished by inter-ISP competition. Thus the meta message from our analysis is that the zero rating, although good for the consumers in the short term because of the increase in their surplus, could in the long run have negative consequences on the CP marketplace.

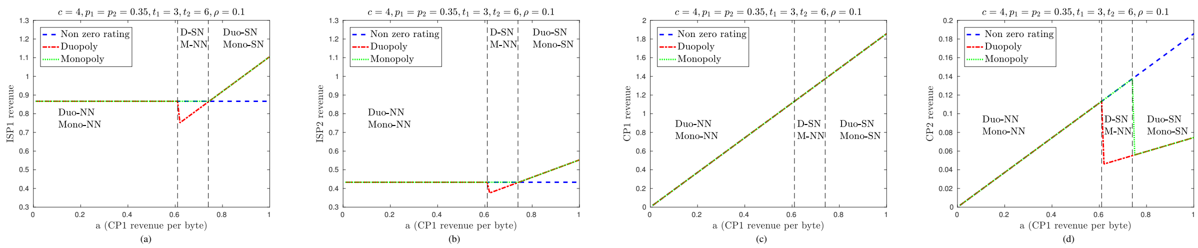

Fig. 8: Surplus of various entities for $c = 4, t_1 = 3, t_2 = 6, p_1 = p_2 = 0.35$ and $\rho = 0.1$ as a function of a (a) ISP1 revenue (b) ISP2 revenue (c) CP1 revenue (d) CP2 revenue

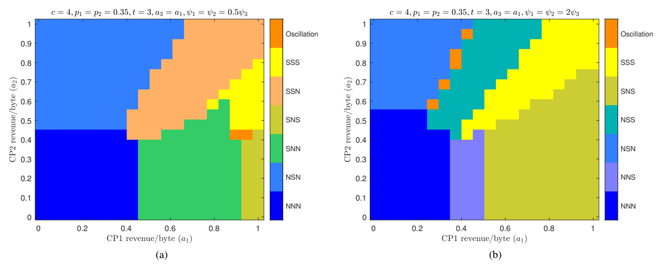

Fig. 9: Limiting sponsorship configurations as a function of a_1, a_2 with $c = 4, t = 3, p = 0.35, a_3 = a_1, \psi_1 = \psi_2 = \log(1+\theta)$ for different utility functions for CP 3 (a) $\psi_3 = 2\log(1+\theta)$ (b) $\psi_3 = 0.5\log(1+\theta)$

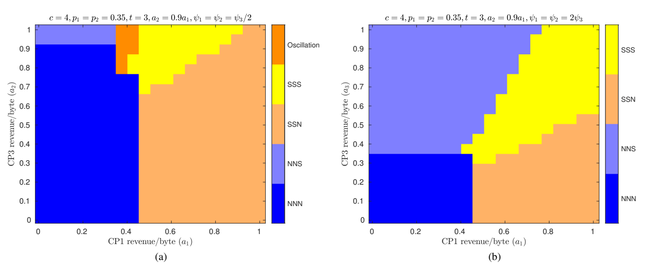

Fig. 10: Limiting sponsorship configurations as a function of a_1, a_3 with $c = 4, t = 3, p = 0.35, \psi_1 = \psi_2 = \log(1+\theta), a_2 = 0.9a_1$ for different utility functions for CP 3 (a) $\psi_3 = 2\log(1+\theta)$ (b) $\psi_3 = 0.5\log(1+\theta)$

An important observation from our analysis is that zero rating drives consumption away from non-sponsored content.[13] Indeed, even when the CP profitability is small, ISP competition induces sponsorship at smaller values of CP-profitability than in the monopoly case. Since this skew of user consumption in favor of sponsored content lies at the heart of the ISP market power, a possible regulatory intervention (other than disallowing data sponsorship entirely) could be to limit zero-rated content so as to leave room for non zero-rated content to also contend for user attention.

It is important at this point to clarify the scope of our model and our conclusions. Our leader-follower interaction model assumes the ISP as the leader and the CPs as followers. This is natural when a 'large' ISP operates a zero-rating platform for 'smaller' CPs. For example, *Sponsored Data* from AT&T and *FreeBee Data* from Verizon. However, it should be noted that there are also situations where the dominance is reversed, e.g., the interaction between small ISPs and large CPs like Google and Facebook. Such interactions are typically based not on data sponsorship, but on peering arrangements, and would require very different models. Early works on the economics of Internet peering are [19], [20] while [21]–[24] are some recent works analyzing paid peering.

Extension to non-substitutable CPs: We conclude with some remarks on how our analytical results easily generalize to the case where the CPs are not substitutable, i.e., $\psi_1(\cdot) \neq \psi_2(\cdot)$. In this case, in all our results of Section III, the set of possible sponsorship configurations induced by ISP 1 includes NS. The extension of Theorem 1 would thus state that ISP 1 enforces an NN equilibrium if $a \leq a_s$, and otherwise, it enforces an SS/NS/SN equilibrium. Lemmas 5 and 6 also hold under non-substitutable CPs. The CP surplus statements of Lemma 7 also generalize naturally, in that under an NS equilibrium, the sponsoring CP derives the same surplus as it would under NN, while the non-sponsoring CP is worse off due to zero-rating. For best response dynamics, Theorem 2 generalizes as above, and a statement similar to Theorem 3 can be derived for $(q(a), NS, q(a), NS)$ system equilibrium as well.

[13] This has also been verified empirically. dflmonitor.eu has reported that the ISPs that provide zero rated content actually sell significantly less bandwidth to end users than those that do not zero-rate.

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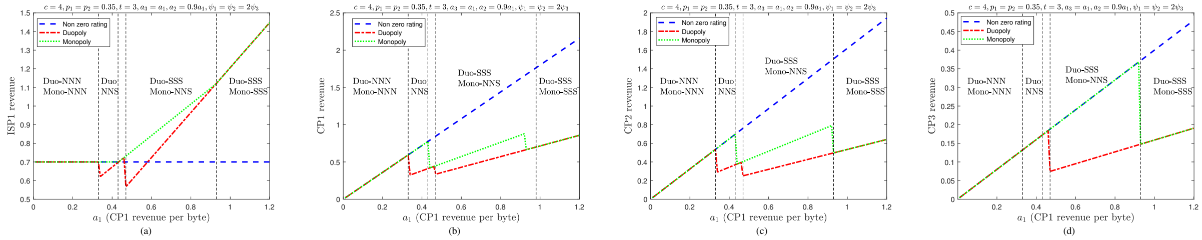

Fig. 11: Surplus of various entities with 2 ISPs and 3 CPs when CP 1 and CP 2 are belong to same class and CP 3 is non subsitutable as function of a_1. Here parameters are $c = 4, t = 3, p = 0.35, \psi_1 = \psi_2 = \log(1+\theta), \psi_3 = 0.5\log(1+\theta), a_2 = 0.9a_1, a_3 = a_1$. (a) ISP revenue (b) CP1 revenue (c) CP2 revenue (d) CP3 revenue

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APPENDIX A
PROOFS OF RESULTS IN SECTION II

The proof of Lemma 1 is trivial and therefore omitted.

A. Proof of Lemma 2

Recall that user utility for both CPs is $\psi_1(\cdot) = \psi_2(\cdot) = \psi(\cdot)$. It is easy to verify that $\theta_1^{NS} = \theta_2^{SN}$. We will prove the lemma for θ_1^{NS}. It is easy to observe that θ_1^{NS} is the maximum solution of the following concave function over $[0, c]$: $f(x) = \psi(x) + \psi(c-x) - px$. We consider the following three cases:

Case 1: $\theta_1^{NS} < \theta_1^{SS}$. Observe that $\theta_1^{SS} = c/2$ is the maximum of the concave function $g(x) = \psi(x) + \psi(c-x)$ over $[0, c]$. Observe that $f(x) = g(x) - px$. At θ_1^{SS}, $f'(\theta_1^{SS}) = g'(\theta_1^{SS}) - p = -p$, which implies that $\theta_1^{NS} < \theta_1^{SS}$.

Case 2: $\theta_1^{NS} < \theta_1^{SN}$. We will show that $\theta_1^{SN} > \theta_1^{SS}$ which combined with Case 1 statement will prove the result. It is easy to observe that θ_1^{SN} is the maximum solution of concave function $h(x) = \psi(x) + \psi(c-x) - p(c-x)$. Writing it in terms of optimization function for θ_1^{SS} we get $h(x) = g(x) - p(c-x)$. Thus $h'(\theta_1^{SS}) = p > 0$ implying that $\theta_1^{SN} > \theta_1^{SS}$.

Case 3: $\theta_1^{NS} < \theta_1^{NN}$. Note that θ_1^{NN} is the maximum solution of the concave function $d(x) = \psi(x) + \psi(c-x) - px - p(c-x)$. By definition of $f(x)$, we get $f(x) = d(x) + p(c-x)$ By similar arguments as Case 1, $\theta_1^{NS} < \theta_1^{NN}$.

B. Nash equilibrium between CPs

We require the following notation.

$$\alpha_1 = 1 - \frac{(x_{SN}^{M_2} - x_{NN}^{M_2})\theta_1^{M_2} + x_{NN}^{M_2}\theta_1^{NN}}{x_{SN}^{M_2}\theta_1^{SN}}$$
$$\alpha_2 = \frac{(x_{SN}^{M_2} - x_{NN}^{M_2})\theta_1^{M_2}}{x_{SN}^{M_2}\theta_1^{SN}}\mathbb{1}_{\{(M_2=SN)||(M_2=SS)\}}$$
$$\beta_1 = 1 - \frac{(x_{NS}^{M_2} - x_{NN}^{M_2})\theta_2^{M_2} + x_{NN}^{M_2}\theta_2^{NN}}{x_{NS}^{M_2}\theta_2^{NS}}$$
$$\beta_2 = \frac{(x_{NS}^{M_2} - x_{NN}^{M_2})\theta_2^{M_2}}{x_{NS}^{M_2}\theta_2^{NS}}\mathbb{1}_{\{(M_2=NS)||(M_2=SS)\}}$$
$$\gamma_1 = 1 - \frac{(x_{SS}^{M_2} - x_{NS}^{M_2})\theta_1^{M_2} + x_{NS}^{M_2}\theta_1^{NS}}{x_{SS}^{M_2}\theta_1^{SS}}$$
$$\gamma_2 = \frac{(x_{SS}^{M_2} - x_{NS}^{M_2})\theta_1^{M_2}}{x_{SS}^{M_2}\theta_1^{SS}}\mathbb{1}_{\{(M_2=SN)||(M_2=SS)\}}$$

$$\delta_1 = 1 - \frac{(x_{SS}^{M_2} - x_{SN}^{M_2})\theta_2^{M_2} + x_{SN}^{M_2}\theta_2^{SN}}{x_{SS}^{M_2}\theta_2^{SS}}$$
$$\delta_2 = \frac{(x_{SS}^{M_2} - x_{SN}^{M_2})\theta_2^{M_2}}{x_{SS}^{M_2}\theta_2^{SS}}\mathbb{1}_{\{(M_2=NS)||(M_2=SS)\}}$$

It is not hard to show that $\alpha_i, \beta_i, \gamma_i, \delta_i \in [0,1]$ for $i = 1, 2$. We are now ready to state the conditions for each sponsorship configuration on ISP 1 to be a Nash equilibrium.

Lemma 8. *Given a sponsorship configuration M_2 on ISP 2, the conditions for the different sponsorship configurations on ISP 1 to be Nash equilibrium between the CPs are:*

1) *NN is Nash equilibrium on ISP 1 if and only if*
$$q_1 \geq \max(a_1\alpha_1 + q_2\alpha_2, a_2\beta_1 + q_2\beta_2)$$
2) *SN is Nash equilibrium on ISP 1 if and only if*
$$a_2\delta_1 + q_2\delta_2 \leq q_1 \leq a_1\alpha_1 + q_2\alpha_2$$
3) *NS is Nash equilibrium on ISP 1 if and only if*
$$a_1\gamma_1 + q_2\gamma_2 \leq q_1 \leq a_2\beta_1 + q_2\beta_2$$
4) *SS is Nash equilibrium on ISP 1 if and only if*
$$q_1 \leq \min(a_1\gamma_1 + q_2\gamma_2, a_2\delta_1 + q_2\delta_2)$$

Appendix B
Proof of Theorem 1

[Proof of Statement 1] For fixed a, ρ and p ISP1 will set q_1 which maximizes its revenue. Let the maximum revenue of ISP1 in M_1 configuration be $r_I^{M_1}(a)$. By Lemma 8, maximum revenue of ISP1 in various configurations is:

$$r_I^{NN}(a) = x_{NN}p(\theta_1^{NN} + \theta_2^{NN}) \tag{4}$$
$$r_I^{SN}(a) = x_{SN}p\theta_2^{SN} + x_{SN}a\alpha\theta_1^{SN} \tag{5}$$
$$r_I^{SS}(a) = x_{SS}ac\min(\gamma, \rho\delta). \tag{6}$$

Note that $r_I^{NN}(a)$ is constant with respect to a while $r_I^{SN}(a)$ and $r_I^{SS}(a)$ are linearly increasing functions of a. Thus there exists a $a = a'$ such that $r_I^{NN}(a') = r_I^{SN}(a')$. Similarly there exists $a = a''$ such that $r_I^{NN}(a'') = r_I^{SS}(a'')$. Therefore for $a < \min(a', a'')$ we get $\max(r_I^{SN}(a), r_I^{SS}(a)) \leq r_I^{NN}(a)$. Then we set $a_s = \min(a', a'')$ and for any $a \leq a_s$ ISP1 will enforce NN equilibrium by setting $q_1 \geq \max(a\alpha, a\rho\beta)$.

[Proof of Statement 2] For $a > a_s$, ISP1 will maximize its revenue as:

$$\begin{aligned} r_I(a) &= \max(r_I^{SN}(a), r_I^{SS}(a)) \\ &= \max(x_{SN}p\theta_2^{SN} + x_{SN}a\alpha\theta_1^{SN}, x_{SS}ac\min(\gamma, \rho\delta)). \end{aligned}$$

As both the terms are increasing functions of a, $r_I(a)$ is also an increasing function of a. In this case, ISP1 will select SN over SS if $r_I^{SN}(a) \geq r_I^{SS}(a)$. In other words, if $x_{SN}p\theta_2^{SN} + x_{SN}a\alpha\theta_1^{SN} \geq x_{SS}ac\min(\gamma, \rho\delta)$ then ISP1 will set $q_1 = a\alpha$ to get SN equilibrium else it will set $a_1 = a\min(\gamma, \rho\delta)$ to get SS equilibrium. Observe that when $\rho < \frac{\gamma}{\delta}$ ISP1 will set SN equilibrium if $\rho \leq \frac{x_{SN}p\theta_2^{SN} + x_{SN}a\alpha}{x_{SS}ac} = y$. Thus, when $\rho \leq \min(\frac{\gamma}{\delta}, y)$ and $a > a_s$ ISP1 will set SN equilibrium.

A. Proof of Lemma 5

When ISP2 is in NN state, for any transportation cost t, the market share of ISP 1 in NN state is $x_{NN} = 0.5$ for any t. Moreover, by Lemma 1, x_{SN} and x_{SS} are both decreasing functions of t. Recall the revenue of ISP 1 in NN, SN and SS state (see (4)-(6)). Hence ISP 1 revenue in NN state r_I^{NN} is independent of t while r_I^{SN} and r_I^{SS} are decreasing functions of t. Thus a' such that $r_I^{NN}(a') = r_I^{SN}(a')$ is an increasing function of t. Similarly a'' such that $r_I^{NN}(a'') = r_I^{SS}(a'')$ is an increasing function of t. This shows that the threshold $a_S = \min(a', a'')$ is an increasing function of t.

B. Proof of Lemma 6

[Proof of statement 1] For $a \leq a_s$, by Theorem 1 ISP1 sets NN equilibrium by appropriately choosing q_1. Thus ISP1's revenue in this case (see (4)) is constant with respect to a.

[Proof of statement 2] For $a > a_s$, by Theorem 1 ISP1 sets SN or SS equilibrium and revenue in both equilibriums is an increasing function of a (see (5) and (6)).

C. Proof of Lemma 7

[Proof of statement 1] For $a > a_s$, ISP1 sets $q = a\alpha$ to get SN equilibrium. In this case profit of CP1 is: $r_1^{SN} = x_{SN}(a - q_1)\theta_1^{SN} + (1 - x_{SN})a\theta_1^{NN}$. By substituting value of α and simplifying we get, $r_1^{SN} = a(x_{NN}\theta_1^{NN} + \theta 1^{NN}(1 - x_{NN})) = r_1^{NN}$. Thus CP1 profit is unaffected while going from NN to SN. CP2's profit when ISP1 is in SN configuration is: $r_2^{SN} = x_{SN}a\rho\theta_2^{SN} + (1 - x_{SN})a\rho\theta_2^{NN}$. Similarly CP2's profit when ISP1 is in NN configuration is: $r_2^{NN} = x_{NN}a\rho\theta_2^{NN} + (1 - x_{NN})a\rho\theta_2^{NN}$. By subtracting r_2^{NN} from r_2^{SN} and simplifying we get, $r_2^{SN} - r_2^{NN} = a\rho\left[x_{SN}(\theta_2^{SN} - \theta_2^{NN})\right] \leq 0$. The last inequality follows from Lemma 2. Thus,

$$r_2^{SN} \leq r_2^{NN} \tag{7}$$

This proves that CP2 is worse off in SN configuration of ISP1 compared to non-zero rating setting.

[Proof of statement 2] By Lemma 8 to get SS configuration, ISP1 sets $q_1 = a\min(\gamma, \rho\delta)$. Thus we will analyze profits of CPs in the following two cases:

Case 1: For $q_1 = a\rho\delta$, CP2 profit is: $r_2^{SS} = x_{SS}(\rho a - \delta\rho a)\theta_2^{SN} + (1 - x_{SS})\rho a\theta_2^{NN} = a\rho(x_{SN}\theta_2^{SN} + (1 - x_{SN})\theta_2^{NN}) = r_2^{SN}$. By (7) we know $r_2^{SN} \leq r_2^{NN}$. Thus, $r_2^{SS} \leq r_2^{NN}$ making CP2 worse-off than in NN configuration.

Case 2: For $q_1 = a\gamma$ it is easy to prove that $r_1^{SS} = r_1^{NS}$. Now we will prove that $r_1^{NS} \leq r_1^{NN}$. We know that $r_1^{NS} = x_{NS}a\theta_1^{NS} + (1-x_{NS})a\theta_1^{NN}$, and $r_1^{NN} = x_{NN}a\theta_1^{NN} + (1-x_{NN})a\theta_1^{NN}$. By subtracting r_1^{NN} from r_1^{NS} we get, $r_1^{NS} - r_1^{NN} = ax_{NS}(\theta_1^{NS} - \theta_1^{NN}) \leq 0$. Here last inequality follows from Lemma 2. Thus $r_1^{SS} \leq r_1^{NN}$ which shows that CP1 is worse-off than in non zero rating setting in this case.

APPENDIX C
PROOF OF THEOREM 2

Consider first the case $M = SN$; the case $M = NS$ follows via a symmetric argument. If $SN - SN$ is a system equilibrium, it can be shown that $a_1 \geq a_2$; indeed, if not, any ISP has the incentive to switch to an NS configuration. From the proof of Theorem 3, it then follows that under an SN-SN equilibrium, both ISPs would set their sponsorship price as $q = a_1\left(1 - \frac{\theta_1^{NN}}{\theta_1^{SN}}\right)$. The profit of the sponsoring CP in this case equals $x_{SN}^{SN}\theta_1^{SN}(a-q)+(1-x_{SN}^{SN})\theta_1^{SN}(a_1-q) = a_1\theta_1^{NN}$, which equals its profit in the absence of zero-rating. Now, profit of the non-sponsoring CP under this configuration equals $x_{SN}^{SN}a_2\theta_2^{SN} + (1-x_{SN}^{SN})a_2\theta_2^{SN} = a_2\theta_2^{SN} < a_2\theta_2^{NN}$. Thus, CP 2 is worse off under this configuration, relative to the scenario where zero-rating is not permitted.

We now consider $M = SS$. WLOG assume $a_1 \geq a_2$. It is easy to show that under a symmetric equilibrium, both ISPs would set $q = a_2\left(1 - \frac{\theta_2^{SN}}{c/2}\right)$ and revenue of CP2 is $x_{SS}^{SS}(a\rho-q)\theta_2^{SS} + (1-x_{SS}^{SS})(a\rho-q)\theta_2^{SS} = a\rho\theta_2^{SN} < a_2\theta_2^{NN}$. Thus, when ISPs are in SS-SS configuration, CP2 is worse off relative to when zero-rating is not permitted.

APPENDIX D
PROOF OF THEOREM 3

Suppose that ISP 1 and ISP 2 have both induced an SN state with equal $q = q_1 = q_2$. By Lemma 8, to maximize profit in SN state, ISP 1 would set $q_1 = a\alpha_1 + q_2\alpha_2$. Similarly ISP 2 would set $q_2 = a\alpha_1 + q_1\alpha_2$. For $q = q_1 = q_2$ we get, $q = \frac{a\alpha_1}{1-\alpha_2}$. Substituting $x_{SN}^{SN} = 0.5$ in the expressions for α_1, α_2 assuming other ISP is in SN state we get, $\alpha_1 = \frac{x_{NN}^{SN}(\theta_1^{SN}-\theta_1^{NN})}{0.5\theta_1^{SN}}, \alpha_2 = \frac{0.5-x_{NN}^{SN}}{0.5}$. Substituting these values in expression for q we get, $q = q(a) := a\left(1 - \frac{\theta_1^{NN}}{\theta_1^{SN}}\right)$. We now show that $(q(a), \text{SN}, q(a), \text{SN})$ is a system equilibrium. The revenue of ISP 1 in this configuration is: $r_1^{SN} = x_{SN}^{SN}(q\theta_1^{SN} + p\theta_2^{SN})$. Noting that $\theta_2^{SN} = c - \theta_1^{SN}$ and substituting the value of q in above expression we get, $r_1^{SN} = 0.5(a(\theta_1^{SN} - \theta_1^{NN}) + p(c - \theta_1^{SN}))$. By Lemma 2, for $a > \frac{p\theta_1^{SN}}{(\theta_1^{SN}-\theta_1^{NN})} = a_{SN}$, $r_1^{SN} > 0.5pc$. Now we will show that ISP 1 cannot increase its profit by switching to a different sponsorship configuration. For this we need to show that the if ISP 1 moves to NN or SS configuration given ISP 2 is in SN, then ISP 1's revenue will not increase from r_1^{SN}. Let $\hat{r}_1^{NN}$ be ISP 1's revenue if it moves to NN configuration given ISP 2 is in SN. Then, $\hat{r}_1^{NN} = x_{NN}^{SN}p(\theta_1^{NN}+\theta_2^{NN}) < 0.5pc$, where the last inequality follows from $x_{NN}^{SN} \leq x_{SN}^{SN} = 0.5$ and $\theta_1^{NN} + \theta_2^{NN} \leq c$. Thus for $a > a_s$, $r_1^{SN} > \hat{r}_1^{NN}$ hence ISP 1 will not switch to NN.

Let $\hat{r}_1^{SS}$ be the ISP 1 revenue if it moves to SS configuration given that ISP 2 is in the SN state. Then, we know that $\hat{r}^{SS} = x_{SS}^{SN}c\min(a\gamma_1 + \gamma_2, a\rho\delta_1)$. Note that $\delta_2 = 0$ as ISP2 is in SN state. For small enough ρ, $\hat{r}^{SS} = x_{SS}^{SN}ca\rho\delta_1$. Substituting the appropriate terms for δ_1 we get, $\hat{r}_1^{SS} = x_{SS}^{SN}ca\rho\left(1 - \frac{2\theta_{22}^{SN}}{c}\right)$. As $\theta_2^{SN} \in [0, c/2)$ we get $\hat{r}_1^{SS} \leq x_{SS}^{SN}a\rho c$. Also $c - \theta_1^{SN} \geq 0$ implying $r_1^{SN} \geq 0.5(a(\theta_1^{SN} - \theta_1^{NN}))$ thus for $\rho < \frac{0.5(\theta_1^{SN}-\theta_1^{NN})}{x_{SS}^{SN}c} = \rho_{SN}$ we get $r_1^{SN} > \hat{r}_1^{SS}$.

This proves that $(q(a), \text{SN}, q(a), \text{SN})$ is a system equilibrium. It is easy to show that $(q(a), \text{SN})$ is the optimal configuration for each ISP in the monopoly setting. Finally, we note that if any CP is to switch its sponsorship decision on both ISPs, the market split would remain equal. Therefore, SN being Nash equilibrium between CPs in the monopoly setting implies that neither CP has the incentive to switch its sponsorship decision on both ISPs.

APPENDIX E
PROOF OF THEOREM 4

Suppose that ISP 1 and ISP 2 both induce a SS configuration with $q_1 = q_2 = q$. Then by Lemma 8, $q_1 \leq \min(a\gamma_1 + q_2\gamma_2, a\rho\delta_1 + q_2\delta_2)$. First we will prove that $q_1 = a\rho\delta_1 + q_2\delta_2$ when both ISPs are in SS. Recall γ_1 and δ_1 :

$$\gamma_1 = 1 - \frac{(x_{SS}^{SS} - x_{NS}^{SS})\theta_1^{SS} + x_{NS}^{SS}\theta_1^{NS}}{x_{SS}^{SS}\theta_1^{SS}}$$

$$\delta_1 = 1 - \frac{(x_{SS}^{SS} - x_{SN}^{SS})\theta_2^{SS} + x_{SN}^{SS}\theta_2^{SN}}{x_{SS}^{SS}\theta_2^{SS}}.$$

As utility derived from both CPs is same for a user, $x_{NS}^{SS} = x_{SN}^{SS}$, and $\theta_1^{SS} = \theta_2^{SS} = c/2$, thus $\gamma_1 = \delta_1$. Similarly, $\gamma_2 = \delta_2$. As $\rho < 1$, we get $q_1 = a\rho\delta_1 + q_2\delta_2$. Simplifying this equation for $q = q_1 = q_2$ we get $q = q(a, \rho) := a\rho\left(1 - \frac{\theta_2^{SN}}{c/2}\right)$. Revenue of ISP1 in this state is $r_1^{SS} = x_{SS}^{SS}cq = 0.5ca\rho\left(1 - \frac{\theta_2^{SN}}{c/2}\right)$ which is a function of a and ρ. Now we will prove that ISP1 will not move to NN or SN state from here so as to increase its revenue. Let $\hat{r}_1^{NN}$ be ISP1's revenue when ISP2 is in SS which can be written as $\hat{r}_1^{NN} = x_{NN}^{SS}p(\theta_1^{NN} + \theta_1^{NN})$. Thus $\hat{r}_1^{NN}$ is independent of a or ρ. Thus there exists $a > a_n$ and $\rho > \rho_n$ such that $\hat{r}_1^{NN} < r_1^{SS}$ in which case ISP1 will not move to NN from SS. Let $\hat{r}_1^{SN}$ be the ISP1 revenue if it moves to SN given ISP2 is in SS. Then, $\hat{r}_1^{SN} = x_{SN}^{SS}(q_{SN}\theta_1^{SN} + p\theta_2^{SN})$ where q_{SN} is value of q_1 for SN state given ISP2 is in SS. By Lemma 8 $q_{SN} = a\alpha_1 + q\alpha_2$. Substituting values of q_{SN}, q and simplifying we get, $\hat{r}_1^{SN} = x_{SN}^{SS}(c(a\alpha_1+q\alpha_2)+\theta_2^{SN}(p-a\alpha_1-q\alpha_2)) < 0.5(c(a\alpha_1+q\alpha_2)) = 0.5ac\left(\alpha_1 + \rho\alpha_2\left(1 - \frac{\theta_2^{SN}}{c/2}\right)\right)$, where inequality holds for any $a > p/\alpha_1 = a_s$. Comparing this with r_1^{SS} we can say that if $\rho > \frac{\alpha_1}{1-\alpha_2}\left(1 - \frac{\theta_2^{SN}}{c/2}\right)^{-1} = \rho_n$ then $\hat{r}_1^{SN} < r_1^{SS}$. Thus if $a_{SS} = \max(a_s, a_n)$ and $\rho_{SS} = \max(\rho_s, \rho_n)$, ISP1 will not move away from SS. The rest of the argument is identical to that in the proof of Theorem 3.