On the probability of current and temperature overloading in power grids: A large deviations approach

Joost Bosman
Centrum Wiskunde & Informatica

Jayakrishnan Nair
IIT Bombay

Bert Zwart
Centrum Wiskunde & Informatica

ABSTRACT

The advent of renewable energy sources has huge implications for the design and control of power grids. On the engineering side, reliability is currently ensured by strict constraints on current, voltage and temperature. However, with growing supply-side uncertainty induced by renewables, these will need to be replaced by probabilistic guarantees, allowing constraints on a given line to be violated with a low probability, e.g., several minutes per year. In the present note we illustrate, using large deviations techniques, how replacing (probabilistic) current constraints by temperature constraints can lead to capacity gains in power grids.

1. INTRODUCTION

Operating a power grid entails matching supply and demand at all times, ensuring that line constraints are not violated. The system operator achieves this by making periodic control actions (typically every 5-10 minutes) that adapt the operating point of the grid in response to changing conditions [4]. A key assumption driving grid operation today is that the grid remains roughly static between control instants. In other words, it is assumed that the operating point does not change much between control instants. Thus, the operator simply ensures that line constraints are satisfied at each control instant. This assumption is of course reasonable when there is little short-term uncertainty in demand and supply.

However, with increasing penetration of renewable sources, supply-side uncertainty is set to grow dramatically going forward. Renewable energy sources like wind and solar can exhibit considerable variability in power generation in the short-term [6, 8]. Thus, in the near future, system operators will no longer be able to assume that the grid is static between control instants, and will have to set the operating point taking into account its variability in the short-term. This entails setting the operating point of the grid with stochastic guarantees on constraint satisfaction [5, 14]. In other words, the operating point must be set such that line constraint violation is a sufficiently rare event until the next control instant.

An important constraint is that the temperature of each line should be bounded, to avoid sag and loss of tensile strength [15]. The typical manner in which this constraint is met is by imposing a certain bound on the line current. However, since temperature responds gradually to current, from a rare events perspective, the constraint on current is much more conservative than the constraint on temperature. This is because a transient current overload does not necessarily imply an overload in temperature. Thus, imposing a constraint on the probability of current overload results in a smaller line capacity, as compared to the same constraint on the probability of temperature overload. This observation was noted via simulations in [13].

In this paper, we investigate this capacity gain analytically. Specifically, we analyse the probabilities of current and temperature overload from a rare events perspective. The dependence of temperature of a conductor to the current flowing through it is well understood [9]. We focus our attention on a single transmission line in a power grid. Modeling the current flowing through the line as an Ornstein-Uhlenbeck process, we analyse the probability of violation of current and temperature constraints in a finite time horizon.

Our main results are concerned with the probability of overheating. The decay rate for temperature constraint violation is the solution of a variational problem that is hard to solve in general. We develop a lower bound on the decay rate, and an approximation that is accurate when the thermal time constant of the conductor is small. We then demonstrate the capacity gain from relaxing the current constraint with the temperature constraint.

Much of the literature on power flow in electricity grids considers deterministic settings, focusing on computational and/or optimization issues. Power flow papers that analyse stochastic models include [5, 10, 12, 15]. One remark about these papers is that they model stochastic behavior at particular snapshots of time, as opposed to the 'process-level' model in this paper. Process-level models have been considered in simulation studies; see, for example [13, 14]. To the best of our knowledge, this is the first paper to analyse variability in power grids from a large deviations perspective.

We hope this type of problem will attract attention in the Performance community. In computer-communication networks, the notion of effective bandwidth has shown to be of fundamental importance and is based on large deviations techniques as presented here. Given the increased importance of uncertain and unreliable renewable energy generation, it is natural to develop similar concepts for energy networks. We envision more structural models involving user as well as physical behavior, leading to admission control schemes that ensure reliability of future power grids.

This is beyond the scope of the present paper, which makes a small step in this direction by showing how a less conservative approach, combined with large deviations theory, can lead to larger admissible capacity region. This may already be applied to loosen capacity constraints in planning problems such as optimal power flow [2].

2. MODEL

We consider a particular transmission line in a power grid. Let $K(t)$ denote the temperature of the line at time t, and let $I(t)$ denote the current flowing through the line

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at time t. For reliability, we require that $K(t) \leq K_{max}$. Define I_{max} such that if $|I(t)| = I_{max}$ at all times, then $\lim_{t\to\infty} K(t) = K_{max}$. The traditional approach of enforcing the temperature constraint is to impose, instead, the following constraint on the current: $|I(t)| \leq I_{max}$. Define the normalized current $Y(t) := \frac{I(t)}{I_{max}}$, and the normalized temperature $\Theta(t) := \frac{K(t)-K_{env}}{K_{max}-K_{env}}$. Here, K_{env} denotes the ambient temperature, which is assumed to be constant over the period of interest. Thus, at time t, the current constraint is violated if $|Y(t)| > 1$, and the temperature constraint is violated if $|\Theta(t)| > 1$.

As is well known, the transient relationship between current and temperature is given by the the following ordinary differential equation [9].

$$\tau \frac{d\Theta(t)}{dt} + \Theta(t) = Y^2(t)$$

Here, τ denotes the thermal time constant of the transmission line. Thus, the temperature of the line responds to the current as

$$\Theta(t) = \Theta(0)e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-s)/\tau} Y^2(s)ds.$$

We denote the above mapping from the (normalized) current $Y(\cdot)$ to the (normalized) temperature $\Theta(\cdot)$ via $\Theta = \xi(Y)$. Note that the parameter τ determines the time lag in the response of temperature to current. As τ becomes small, the dependence of the present temperature on past values of current becomes weaker. In the limit as $\tau \downarrow 0$, temperature responds instantaneously to current, i.e., $\Theta(t) = Y^2(t)$.

We model current Y as an Ornstein-Uhlenbeck process. That is, Y evolves according to the stochastic differential equation

$$dY(t) = \gamma(\mu - Y(t))dt + dW(t),$$

where $W(t)$ denotes the standard Brownian motion, and $\gamma > 0$. We also assume that $Y(0) = \mu < 1$. Thus, the process Y is stationary in the mean, and has a tendency to revert to its mean value μ. We may interpret μ as the value of the current as set by the system operator at control instant 0, with the current subsequently fluctuating randomly due to the variability of renewable sources attached to the grid. Also, we assume that $\Theta(0) = \mu^2$; this being the (normalized) steady state temperature corresponding to a constant current μ.

More structural physical models of current exist using wind speed models, and power flow equations, among others, see [13, 14]. Our modeling choice is parsimonious, enabling us to focus on the connection between current and temperature. In the journal version of this paper we also consider other models.

We are interested in estimating the probability that the current and temperature violate their constraints over a finite time interval $[0, T]$. Thus, we are interested in estimating the current and temperature violation probabilities

$$P\left(\max_{t\in[0,T]} |Y(t)| > 1\right), \quad P\left(\max_{t\in[0,T]} |\Theta(t)| > 1\right).$$

Our approach is to use the theory of large deviations to estimate these probabilities. In the standard manner (see, for example, [3][Chapter 5]), for $\epsilon > 0$, we define the scaled current process Y_ϵ as the solution of

$$dY_\epsilon(t) = \gamma(\mu - Y_\epsilon(t))dt + \sqrt{\epsilon}dW(t),$$

with $Y_\epsilon(0) = \mu$. Note that as ϵ decreases, we are scaling down the 'noise' in the process Y_ϵ around its mean value μ. The scaled temperature process $\Theta_\epsilon(t)$ is defined via $\Theta_\epsilon = \xi(Y_\epsilon)$. For a continuous function f over $[0, T]$, let $\|f\| := \max_{t\in[0,T]} |f(t)|$. Thus, we estimate the probabilities

$$P(\|Y_\epsilon\| > 1), \quad P(\|\Theta_\epsilon\| > 1)$$

as $\epsilon \downarrow 0$.

3. CURRENT PROCESS

$\{Y_\epsilon\}$ satisfies a sample path large deviations principle (SPLDP) with good rate function

$$\mathcal{I}_1(f) = \begin{cases} \frac{1}{2}\int_0^T (f'(s) + \gamma f(s) - \gamma\mu)^2 ds & \text{if } f \in H_\mu \\ \infty & \text{if } f \notin H_\mu \end{cases}$$

(see [3, Theorem 5.6.3]). Here,

$$H_x := \{f \in C_x[0, T] \; : \; f(t) = x + \int_0^t \phi(s)ds, \phi \in L_2[0, T]\}.$$

It then follows, using continuity properties of $\mathcal{I}_1$, that

$$\lim_{\epsilon\downarrow 0} -\epsilon \log P(\|Y_\epsilon\| > 1) = I_y := \inf_{f\in H_\mu : \|f\|=1} \mathcal{I}_1(f), \qquad (1)$$

yielding the approximation

$$P(\|Y_\epsilon\| > 1) \approx e^{-I_y/\epsilon}.$$

The following lemma gives an expression for the decay rate $I_y = I_y(\mu)$. The optimal path y^* is the most likely path followed by the current in the event of an overload.

LEMMA 1. $I_y(\mu) = \frac{\gamma(1-\mu)^2}{1-e^{-2\gamma T}}$. *The optimal path for the variational problem* (1) *is*

$$y^*(t) = \mu + \frac{1-\mu}{e^{\gamma T} - e^{-\gamma T}}(e^{\gamma t} - e^{-\gamma t}).$$

PROOF. It is easy to show that in solving (1), it suffices to infimize over paths $f \in H_\mu$ satisfying $f(T) = 1$. The statement then follows from Proposition 2.2 in [7]. $\square$

4. TEMPERATURE PROCESS

$\{\Theta_\epsilon\}$ satisfies a SPLDP with good rate function $\mathcal{I}_2(\cdot)$:

$$\mathcal{I}_2(f) = \frac{1}{2}\int_0^T \left(\frac{\tau f'' + f'}{2\sqrt{\tau f' + f}} + \gamma\sqrt{\tau f' + f} - \gamma\mu\right)^2 ds$$

for $f \in \xi(H_\mu)$, $f(0) = \mu^2$, and $\mathcal{I}_2(f) = \infty$ otherwise. This follows from the contraction principle [3]. It then follows, using continuity properties of $\mathcal{I}_2$, that

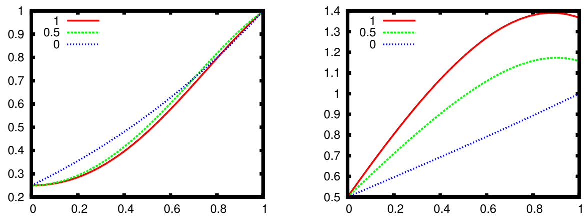
$$\lim_{\epsilon\downarrow 0} -\epsilon \log P(\|\Theta_\epsilon\| > 1) = I_\theta := \inf_{f:\|f\|=1} \mathcal{I}_2(f). \qquad (2)$$

This yields the approximation

$$P(\|\Theta_\epsilon\| > 1) \approx e^{-I_\theta/\epsilon}.$$

The variational problem (2) for the decay rate $I_\theta = I_\theta(\tau, \mu)$ is difficult to solve in general. We are able to solve (2) numerically by using Euler's criterion [11, Theorem C.13]. We fix $\mu = 0.5$, $\gamma = 0.5$, $T = 1$. The decay rates equal 0.1977, 0.4336, and 0.7901 for $\tau \downarrow 0$, $\tau = 0.5$, and $\tau = 1$ respectively. The corresponding optimal paths for the temperature and current are depicted in Fig. 1. Note that as τ is increased, the decay rate for temperature overflow increases, i.e., the overflow becomes increasingly rare. Moreover, it takes a stronger current to produce the temperature overflow.

As $\tau \downarrow 0$, it can be shown that $I_\theta(\tau, \mu) \to I_y(\mu)$. This is intuitive, since in the limit $\tau \downarrow 0$, $\Theta_\epsilon(t) = Y_\epsilon(t)^2$, making temperature and current overload equally likely. The following proposition gives a lower bound on $I_\theta(\tau, \mu)$.



(a) (b)

Figure 1: Optimal paths for temperature (a) and current (b) corresponding to the variational problem (2) **for different values of** τ.

PROPOSITION 2.

$$I_\theta(\tau,\mu) \geq \frac{\gamma}{1-e^{-2\gamma T}} \left(\sqrt{\frac{1-\mu^2 e^{-T/\tau}}{1-e^{-T/\tau}}} - \mu \right)^2$$

This proposition is proved using the fact that $\|\Theta_\epsilon\| > 1$ implies $\|Y_\epsilon\| > \sqrt{1-\mu^2 e^{-T/\tau}}/\sqrt{1-e^{-T/\tau}}$. We omit the proof. Note that the above lower bound is tight as $\tau \downarrow 0$. Moreover, the lower bound is strictly increasing in τ, approaching ∞ as $\tau \to \infty$. This shows that temperature overload becomes considerably less likely than current overload for large τ, suggesting a substantial capacity gain.

Finally, it is possible to obtain a first order approximation of $I_\theta(\tau,\mu)$ around $\tau = 0$, which can be utilized to give a first idea about the potential gain in capacity when the thermal constant τ is small:

PROPOSITION 3.

$$I_\theta(\tau,\mu) = \frac{\gamma(1-\mu)^2}{1-e^{-2\gamma T}}(1+2\gamma\tau) + o(\tau), \qquad \tau \to 0$$

PROOF. The proof is based on an infinite-dimensional version of Danskin's theorem [1, Proposition 4.12]. For fixed f we write $\mathcal{I}_2(f) = G_f(\tau,\mu)$ and analyze this as a function of τ. Set $g_\tau = \sqrt{\tau f' + f} - \mu$, which allows us to write

$$G_f(\tau,\mu) = \frac{1}{2}\int_0^T (g_\tau'(s) + \gamma g_\tau(s))^2 ds. \tag{3}$$

Let $G_f^{(\tau)}(\tau,\mu)$ be the derivative of G w.r.t. τ. Set $h = g_0 = \sqrt{f} - \mu$. A lengthy but straightforward computation gives

$$G_f^{(\tau)}(0,\mu) = \int_0^T (h'(s) + \gamma h(s))(h''(s) + \gamma h'(s)) ds.$$

Recall that y^* is the optimal current path for $\tau = 0$. Thus, $f_0^* = (y^*)^2$ is the optimal path to overflow of the temperature process if $\tau = 0$. Setting $K = (1-\mu)/(e^{\gamma T} - e^{-\gamma T})$ and $f = f_0^*$ we get $h(s) = K(e^{\gamma s} + e^{-\gamma s})$. Differentiating yields

$$h'(s) = \gamma h(s) - 2\gamma K e^{-\gamma s}, \quad h''(s) = \gamma^2 h(s).$$

Some additional straightforward computations yield

$$G_{f_0^*}^{(\tau)}(0,\mu) = 4\gamma^3 \int_0^T (h'(s) - K e^{-\gamma s}) ds = \frac{2\gamma^2(1-\mu)^2}{1-e^{-2\gamma T}}.$$

Proposition 4.12 of [1] yields

$$\frac{d}{d\tau} I_\theta(\tau,\mu)|_{\tau=0} = G_{f_0^*}^{(\tau)}(0,\mu).$$

Combine the last two displays, and invoke Lemma 1. $\square$

5. CAPACITY PROVISIONING

The maximum current $\bar{\mu}$, which could serve as input to an optimal power flow problem should be set such that the reliability constraint $P(\|\Theta_\epsilon\| > 1) \leq p$ is valid. Given the large deviations estimates developed before, we approximate $P(\|\Theta_\epsilon\| > 1) \approx e^{-I_\theta(\tau,\mu)/\epsilon}$, so that $\bar{\mu} = \bar{\mu}(\tau)$ solves $I_\theta(\tau,\mu) = \epsilon \log(1/p)$.

Set μ_0 to be the conservative current capacity constraint, based on the traditional approach of enforcing $P(\|Y_\epsilon\| > 1) \leq p$. This implies, via the same approximations as above, that μ_0 satisfies $I_y(\mu) = \epsilon \log(1/p)$. Thus, our capacity gain of interest equals $\bar{\mu}(\tau) - \mu_0$.

How much capacity could be gained with the more refined approach, using the more complicated temperature process directly? Note that $\bar{\mu}(\tau) - \mu_0 \to 0$ when $\tau \downarrow 0$. Proposition 3 enables us to quantify the capacity gain when τ is small. Equating $I_\theta(\tau,\bar{\mu}) = I_\theta(0,\mu_0) = I_y(\mu_0)$ and solving leads to $(1-\mu_0)^2 = (1-\bar{\mu})^2(1+2\gamma\tau) + o(\tau)$, which for small τ is asymptotically equivalent to

$$\bar{\mu}(\tau) - \mu_0 = \gamma\tau(1-\mu_0) + o(\tau).$$

Similarly, Proposition 2 can be used to prove a lower bound on the capacity gain, which is strictly increasing in τ. We omit the details.

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