# Power Controlled FCFS Splitting Algorithm for Wireless Networks


Ashutosh Deepak Gore
Marvell India Private Limited
Pune, India
Email: adgore@gmail.com

Abhay Karandikar
Department of Electrical Engineering
Indian Institute of Technology - Bombay, India
Email: karandi@ee.iitb.ac.in



**Abstract**

We consider random access in wireless networks under the physical interference model, wherein the receiver is capable of power-based capture, i.e., a packet can be decoded correctly in the presence of multiple transmissions if the received Signal to Interference and Noise Ratio exceeds a threshold. We propose a splitting algorithm that varies the transmission powers of users on the basis of quaternary channel feedback (idle, success, capture, collision). We show that our algorithm achieves a maximum stable throughput of 0.5518. Simulation results demonstrate that our algorithm achieves higher throughput and lower delay than that of the First Come First Serve splitting algorithm with uniform transmission power.



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## I. INTRODUCTION

Medium Access Control (MAC) problem is present in all communication networks, both wired and wireless. Multiple nodes (users) can access a single channel simultaneously to communicate with each other or a common receiver – the challenge is to design efficient channel access algorithms to achieve the desired performance in terms of throughput and delay. Several solutions to the MAC problem have been proposed depending on source traffic characteristics, channel models and Quality of Service (QoS) requirements of the users.

MAC protocols can be broadly classified into two types: fixed resource allocation protocols and random access protocols. Fixed resource allocation protocols such as Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA) and Code Division Multiple Access (CDMA) assign orthogonal or near-orthogonal channels to every user and are mostly implemented in voice-dominant circuit switched networks. These protocols typically require the presence of a central entity, such as a base station in cellular networks, to perform channel allocation and admission control, i.e., they are highly centralized. Though fixed resource allocation protocols are contention-free and can multiplex users with similar traffic characteristics easily, they suffer from low throughput and high channel access delay when the traffic is bursty and there are large number of users. On the other hand, in random access protocols, users vary their transmission probabilities or transmission times based on limited channel feedback, i.e., random access protocols are highly distributed. Random access protocols are more suitable for scenarios wherein many users with varied traffic requirements have to be multiplexed.

Random access algorithms for satellite communications, multidrop telephone lines and multitap bus ("traditional random access algorithms") have been well studied for the past four decades. See [1] for a textbook treatment of traditional random access algorithms. These algorithms can be broadly classified into three categories: ALOHA [2], [3], Carrier Sense Multiple Access [4] and tree (or stack or splitting) algorithms [5]. Traditional random access algorithms have been implemented in practical systems. For example, ALOHA is used in most cellular networks **(for**

**example, GSM networks)** to request channel access and also used in satellite communication networks. Carrier Sense Multiple Access with Collision Detection (CSMA/CD) is used to resolve contentions in Local Area Networks (LANs) like IEEE 802.3 based Ethernet [6], while Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) is used to resolve contentions in IEEE 802.11 based Wireless LANs [7].

In First Come First Serve (FCFS) splitting algorithm [1], nodes involved in a collision split into two subsets based on the arrival times of collided packets. Using this approach, each subset consists of all packets that arrived in some given interval, and when a collision occurs, that interval will be split into two smaller intervals. By always transmitting packets that arrived in the earlier interval first, the algorithm transmits successful packets in the order of their arrival. The FCFS algorithm is stable for $\lambda < 0.4871$ [1]. Conflict resolution protocols based on tree algorithms have provable stability properties [8].

On the other hand, random access algorithms that incorporate physical layer characteristics such as Signal to Interference and Noise Ratio (SINR) and channel variations have only been studied recently. These algorithms, which have been primarily proposed for wireless networks, can be broadly classified into three categories: algorithms based on signal processing and diversity techniques, channel-aware ALOHA algorithms based on adapting the retransmission probabilities of contending users and "tree-like" algorithms based on adapting the set of contending users. Existing random access algorithms, such as CSMA/CA, are not channel-aware and can lead to low throughput. Thus, the design of physical layer aware random access algorithms can be a potential step towards achieving higher data rates in future wireless networks.

Existing research work on physical layer aware random access algorithms can be broadly classified into the following categories:

1) Techniques which employ signal processing and diversity techniques to correctly decode received packets, for example in [9], [10], [11], [12],
2) "Channel-aware ALOHA" techniques whose central theme is to adapt the retransmission probabilities of users, for example in [13], [14], [15], [16], [17], and
3) Splitting (or tree or stack) algorithms whose main idea is to adapt the set of contending users based on feedback from the channel or common receiver, as in [18], [19], [20].

**Note that some form on random access is necessary in a wireless network (say, for connection setup or topology discovery) before it is possible to employ more sophisticated**

**and computationally intensive scheduled channel access techniques. For example, in a GSM wireless cellular network, mobile stations transmit on the random access channel for sub-channel reservation and synchronization with the base station. In a WiMAX network, a subscriber station transmits randomly on pre-determined frequency bands (control channels) to set up its connection to the base station. In a WLAN, clients employ binary exponential backoff prior to sensing the channel and sending data to the access point.**

### *A. Related Work on Physical Layer Aware Splitting Algorithms*

In this section, we mainly review representative research work on random access algorithms whose main idea is to adapt the set of contending users based on feedback from the channel or the common receiver. In such work, the authors develop and analyze splitting algorithms for various models of the wireless channel and evaluate the performance of their algorithms via simulations.

In [18], the authors propose an opportunistic splitting algorithm for a multipoint to point wireless network. They assume a slotted system, block fading channel and $\{0, 1, e\}$ feedback, where $0$, $1$ and $e$ denote idle, success and error respectively. Assuming that each user only knows its own channel gain and the number of backlogged users, the authors propose a distributed splitting algorithm to determine the user with the best channel gain over a sequence of mini-slots. The algorithm determines a lower threshold $H_l$ and a higher threshold $H_h$ for each mini-slot, such that only users whose channel gains lie between between $H_l$ and $H_h$ are allowed to transmit their packets. Based on results from “partitioning a sample with binary type questions” [21], they show that the average number of mini-slots required to determine the user with the best channel is 2.5, independent of the number of users and the fading distribution. However, their algorithm is based on the assumption that every user can accurately estimate the number of backlogged users.

In [19], the authors consider a random access network with infinite users, Poisson arrivals and $\{0, k, e\}$ immediate feedback, where $k$ is any positive integer. In contrast to standard tree algorithms, such as Basic Tree Algorithms (BTA), Modified Tree Algorithm (MTA) [22] and FCFS that discard collided packets, they propose an algorithm that stores collided packets. The receiver extracts information from the collided packets by relying on successive interference cancellation techniques ([9], Chapter 7) and the tree structure of a collision resolution algorithm. Though their algorithm achieves a stable throughput of 0.693, it requires infinite storage and increased input voltage range at the receiver, which may not be feasible in practical systems.

In [20], the author considers a multipoint to point wireless channel with and without capture and Multiple Packet Reception (MPR). The channel provides Empty(E)/Non-Empty(NE) feedback to all active users and 'success' feedback to successful users only. The users do not need to know the starting times and ending times of collision resolution periods. For such a channel with E/NE binary feedback, the author proposes and analyzes a stack multiple access algorithm that is limited sensing and does not require any frame synchronization. The author considers two models for capture, namely Rayleigh fading with incoherent and coherent combining of joint interference power. For MPR, the author assumes a maximum of two successes during a collision. The maximum throughput of the algorithm is numerically evaluated to be 0.6548 when capture and MPR are present, and 0.2891 when both effects are absent.

**In [23], the authors consider a random access network with finite number of nodes (users) having limited energy, Poisson arrivals and $\{0, 1, e\}$ feedback under the physical interference model. They propose Random Energy Based Splitting (REBS) algorithm wherein the criterion for splitting the number of interfering users is based on the amount of residual energy at each node. In REBS, the probability that a packet in the left subtree belongs to a high-energy node is strictly greater than the probability that it originated from a low-energy node, leading to energy savings. Assuming that the initial energies of the nodes are uniformly distributed, they demonstrate the energy-efficacy of REBS algorithm compared to FCFS algorithm via simulations. Finally, they suggest hybrid algorithms for the cases where all nodes have equal energy or unlimited energy.**

Finally, we briefly review representative research papers which focus on power control techniques in random access wireless networks. We then motivate the use of variable transmission power to increase the throughput in random access wireless networks in Section I-B.

In [24], the author considers a time-slotted CDMA-based wireless network wherein a finite number of nodes communicate with a common receiver. The author formulates the problem of determining the set of nodes that can transmit in each slot along with their corresponding transmission powers, subject to constraints on maximum transmission power and the SINRs of all transmissions exceeding the communication threshold. Due to its NP-hard nature, the problem is relaxed to a case wherein a node transmits with a certain probability in each slot. Equivalently, the problem of joint power control and link scheduling is transformed to a problem of power controlled random access, wherein the objective is to determine the probability of transmission $\Delta_i$ and transmission power $P_i$ for each node $i$, subject to constraints on maximum transmission power and the "expected SINR" exceeding the communication threshold. The author seeks to minimize a weighted sum of the maximum transmission power and maximum reciprocal probability, i.e., minimize $(\max_i P_i + \lambda \max_i \frac{1}{\Delta_i})$. This convex optimization problem is solved using techniques from geometric programming [25]. Finally, the author derives the probability of outage[1] and delay distribution of buffered packets and demonstrates the efficacy of the schemes via simulations.

In [26], the authors investigate transmission power control and rate adaptation in random access wireless networks using game theoretic techniques. They consider multiple transmitters sharing a time-slotted channel to communicate equal-length packets with a common receiver. A user's packet is successfully received if the SINR at the receiver is no less than a certain threshold called the communication threshold, i.e., the authors employ the physical interference model [27]. The random access problem is formulated as a game wherein each user selects its strategy (transmit or wait) at each stage of the game in a non-cooperative (independent) or cooperative manner. The authors evaluate equilibrium strategies for non-cooperative and cooperative symmetric random access games. Finally, the authors describe distributed power control and rate adaptation games for non-cooperative users for a collision channel with power-based capture. Their numerical results demonstrate improved expected user utilities when power control and rate adaptation are incorporated, at the expense of increased computational complexity. Though the authors propose a distributed random access algorithm based on game theoretic techniques, it also assumes that every user knows $n$, the number of backlogged users, in each slot. In practice, $n$ can only be estimated using techniques such as Rivest's pseudo-Bayesian algorithm [28].

### B. Contributions of our Work

Though researchers have addressed the problem of random access in wireless networks by considering various channel models, different types of feedback and realistic criteria for suc-

[1]An outage occurs on a link if the received (actual) SINR on the link is less a certain threshold.

cessful packet reception, only few of them exploit the idea that throughput gains are achievable in a random access wireless network by varying transmission powers of users.

We envisage developing a power controlled random access algorithm for wireless networks under the physical interference model. We seek an algorithm that yields higher throughput than traditional random access algorithms. In cognizance of these requirements, we propose a power controlled splitting algorithm for wireless networks. The algorithm is so designed that successful packets are transmitted in the order of their arrivals, i.e., in an FCFS manner.

In the system model that we consider, if multiple transmissions occur, the receiver can decode a certain user's packet correctly only if the received SINR exceeds a threshold, i.e., we consider a channel with power-based capture. The notion of capture has been addressed previously, though in different contexts [15], [20], [29], [30]. However, in this paper, we motivate the idea that a user can transmit at variable power levels to increase the chances of capture. Moreover, unlike [15], [20], [29], [30], we assume $\{0, 1, c, e\}$ feedback, where $0$, $1$ and $e$ denote idle, success and error respectively, and $c$ denotes capture in the presence of multiple transmissions. Note that this system model is different from those considered in existing works on splitting algorithms for wireless networks. For example, in [20], the author proposes a novel splitting algorithm, but does not take into account throughput gains possible by varying the transmission power. Though the authors of [18] propose a splitting algorithm to determine the user with the best channel gain, their algorithm may not be practical because it assumes that every user can accurately estimate the number of backlogged users.

The rest of the paper is organized as follows. We describe our system model in Section II and motivate variable transmission powers of contending users in Section III. We describe the proposed random access algorithm and provide an illustrative example in Section IV. We model the algorithm dynamics by a Markov chain and derive its maximum stable throughput in Section V. The performance of the proposed algorithm is evaluated in Section VI. We conclude in Section VII.

## II. System Model

Consider a multipoint to point wireless network. We assume the following:

1) Slotted system: Users (nodes) transmit fixed-length packets to a common receiver over a time-slotted channel. All users are synchronized such that the reception of a packet starts

at an integer time and ends before the next integer time.

2) Poisson arrivals: The packet arrival process is Poisson distributed with overall rate $\lambda$, and each packet arrives to a new user that has never been assigned a packet before. After a user successfully transmits its packet, that user ceases to exist and does not contend for channel access in future time slots.
3) Channel model: The wireless channel is modeled by the path loss propagation model. The received signal power at a distance $D$ from the transmitter is given by $\frac{P}{D^\beta}$, where $P$ is the transmission power and $\beta$ is the path loss factor. We do not consider fading and shadowing effects.
4) **Equal distances: We assume that each user is at the same distance $D$ from the common receiver.**
5) **Power-based capture: According to the physical interference model [27], a packet transmission from $j^{th}$ transmitter, denoted by $t_{i,j}$, to receiver $r$ in $i^{th}$ time slot is successful if and only if the SINR at receiver $r$ is greater than or equal to the communication threshold $\gamma_c$[2], i.e.,**

$$\frac{\frac{P_{i,j}}{D^\beta}}{N_0 + \sum_{\substack{k=1\\k\neq j}}^{M_i} \frac{P_{i,k}}{D^\beta}} \geqslant \gamma_c, \tag{1}$$

where

$$\begin{aligned} M_i &= \text{number of concurrent transmitters in } i^{th} \text{ time slot,} \\ t_{i,j} &= j^{th} \text{ transmitter in } i^{th} \text{ time slot } \ (j = 1, 2, \ldots, M_i), \\ D &= \text{Euclidean distance between any transmitter } t_{i,j} \text{ and } r, \\ P_{i,j} &= \text{transmission power of } t_{i,j}, \\ N_0 &= \text{thermal noise power spectral density.} \end{aligned}$$

6) $\{0, 1, c, e\}$ immediate feedback: At the end of each slot, users are informed of the feedback from the receiver immediately and without any error. The feedback is one of:
   a) idle $(0)$: when no packet transmission occurs,

[2] In literature, $\gamma_c$ is also referred to as capture ratio [20], capture threshold [15] and power ratio threshold [10].

b) perfect reception $(1)$: when one packet transmission occurs and is received successfully,

c) capture $(c)$: when multiple packet transmissions occur and only one packet is received successfully, or

d) collision $(e)$: when multiple packet transmissions occur and no packet reception is successful.

The receiver can distinguish between $1$ and $c$ by using energy detectors [19], [31]. Thus, at the end of every slot, only two bits are required to provide feedback from the receiver to all users. Note that two bits are required to provide feedback even for the classical $\{0, 1, e\}$ feedback model. Thus, our $\{0, 1, c, e\}$ immediate feedback assumption does not increase the number of bits required for feedback..

7) **Window Channel Access Algorithm: We first define Collision Resolution Period (CRP) as the time interval from the slot where an initial collision occurs up to and including the slot in which all users recognize that all packets involved in the collision have been successfully received. In a window channel access algorithm, after the completion of the current CRP, only packets that arrive in a specific window on the time axis are allowed to join the new CRP. If we $\phi_0$ denote the maximum window duration and $\tau$ denote the time elapsed from the end of the current window till the end of the CRP it generates, then the next window size is $\min(\phi_0, \tau)$.**

The above assumptions are similar to the standard assumptions employed for the analysis of traditional random access algorithms such as ALOHA and FCFS.

## III. Motivation and Problem Formulation

The maximum stable throughput of the well-known FCFS splitting algorithm is 0.4871 [1], which is the highest throughput amongst a wide class of random access algorithms for wired networks. However, in a wireless network, transmission power of a node provides an extra degree of freedom, and higher throughputs are achievable.

Consider a scenario wherein all contending nodes transmit with equal power $P$ in a given time slot. When only one node transmits, its packet is successfully received if the SINR threshold condition (1) is satisfied, i.e.,

$$P \geqslant \gamma_c N_0 D^{\beta}. \tag{2}$$

When $M$ nodes transmit concurrently with equal power $P$, where $M \geqslant 2$, the SINR corresponding to $i^{th}$ transmission is given by

$$\text{SINR}_1 = \frac{\frac{P}{D^\beta}}{N_0 + (M-1)\frac{P}{D^\beta}}, \tag{3}$$

**Note that the signal attenuation model of (3) is valid only for the far field region, i.e., beyond a distance of 1 m from the transmitter[3]. For brevity in notation, we assume that** $D > 1$ **throughout this paper. More importantly, the right hand side of (3) is always less than 1.** Since $\gamma_c > 1$ for all practical narrowband communication receivers [10], $\text{SINR}_i < \gamma_c$ $\forall\ i$ and all $M$ transmissions are unsuccessful[4]. Thus, when multiple nodes transmit with equal power, a collision occurs irrespective of the transmission power $P$.

However, the above situation can be circumvented by varying transmission powers of users in some special cases. With relatively small attempt rates, when a collision occurs, it is most likely between only two packets [1]. In this case, if the receiver is capable of power-based capture, a collision between two nodes can be avoided by using different transmission powers. Specifically, one of the nodes, say $N_1$, transmits with minimum power $P_1$ such that, if it were the only node transmitting in that time slot, then its packet transmission will be successful. From (1), the required nominal power is

$$P_1 = \gamma_c N_0 D^\beta. \tag{4}$$

The other node, say $N_2$, transmits with minimum power $P_2$ such that if there is exactly one other node transmitting at nominal power $P_1$, then the packet transmitted by $N_2$ will be successful. From (1) and (4), we obtain

$$\begin{aligned} \frac{\frac{P_2}{D^\beta}}{N_0 + \frac{P_1}{D^\beta}} &= \gamma_c, \\ P_2 &= \gamma_c(1+\gamma_c)N_0 D^\beta. \end{aligned} \tag{5}$$

Note that $\frac{P_2}{P_1} = 1+\gamma_c$. We do not consider more than two power levels for the following reasons:

1) it complicates the power-control algorithm, and

[3]In general, (3) can be written as $\text{SINR}_1 = \frac{\frac{P}{\max(1,D)^\beta}}{N_0+(M-1)\frac{P}{\max(1,D)^\beta}}$.

[4]For a spread spectrum CDMA system with processing gain $L$, (3) gets modified to $\text{SINR}_1 = \frac{\frac{P}{D^\beta}}{N_0+\frac{1}{L}(M-1)\frac{P}{D^\beta}}$ [26]. For such a wideband system, $\gamma_c < 1$, and more than one packet can be decoded correctly in the presence of multiple transmissions. However, in this paper, we consider narrowband systems only.

2) most devices have constraints on peak transmission power.

Note that the above power control technique converts some collisions into "captures". Thus, it has the potential of increasing the throughput of random access algorithms employing uniform transmission power.

We seek to design a distributed algorithm incorporating this power control technique, while still ensuring that the algorithm transmits successful packets in the order of their arrival, i.e., in an FCFS manner[5].

## IV. PCFCFS Interval Splitting Algorithm

In this section, we present an algorithmic description of the proposed Power Controlled First Come First Serve (PCFCFS) splitting algorithm. We also explain the behavior of the proposed algorithm by providing an illustrative example.

### *A.* Notation and Terminology

We first describe the notation. Slot $k$ is defined to be the time interval $[k, k+1)$. At each integer time $k$ ($k \geqslant 1$), the algorithm specifies the packets to be transmitted in slot $k$ to be the set of all packets that arrived in an earlier interval $[T(k), T(k)+\phi(k))$, which is defined as the *allocation interval* for slot $k$. The maximum size of the allocation interval is denoted by $\phi_0$, a parameter which will be optimized for maximum throughput in Section V. Packets are indexed as $1, 2, \ldots$ in the order of their arrival. Since the arrival times are Poisson distributed with rate $\lambda$, the inter-arrival times are exponentially distributed with mean $\frac{1}{\lambda}$. Let $a_i$ denote the arrival time of $i^{th}$ packet. Using the memoryless property of the exponential distribution (and without loss of generality), we assume that $a_1 = 0$. The transmission power of $i^{th}$ packet in slot $k$ is denoted by $P_i(k)$, where $P_i(k) \in \{0, P_1, P_2\}$. Note that, if $P_i(k) = 0$, then $i^{th}$ packet is not transmitted in slot $k$.

The proposed Power Controlled First Come First Serve (PCFCFS) splitting algorithm is the set of rules by which the users compute allocation interval parameters $\{T(k+1), \phi(k+1), \sigma(k+1)\}$ and transmission power $P_i(k+1)$ for slot $k+1$ in terms of the feedback and allocation interval

[5]Since successful packets are transmitted in an FCFS manner, the delay experienced by a packet will not be significantly higher than the average packet delay. Thus, from a QoS perspective, FCFS transmission of packets not only guarantees average delay bounds, but also ensures fairness of users' packets.

parameters for slot $k$. In our algorithm, every allocation interval is tagged as a "left" ($\mathcal{L}$) or "right" ($\mathcal{R}$) interval. $\sigma(k)$ denotes the tag ($\mathcal{L}$ or $\mathcal{R}$) of allocation interval $[T(k), T(k)+\phi(k))$ in slot $k$. Moreover, whenever an allocation interval is split, it is always split into two equal-sized subintervals, and these subintervals $(\mathcal{L}, \mathcal{R})$ are said to *correspond* to each other.

## *B.* Illustrative Example

**In this section, we elucidate the rules of the PCFCFS algorithm for a single CRP by providing an illustrative example. The algorithm will be formally described in Section IV-C.**

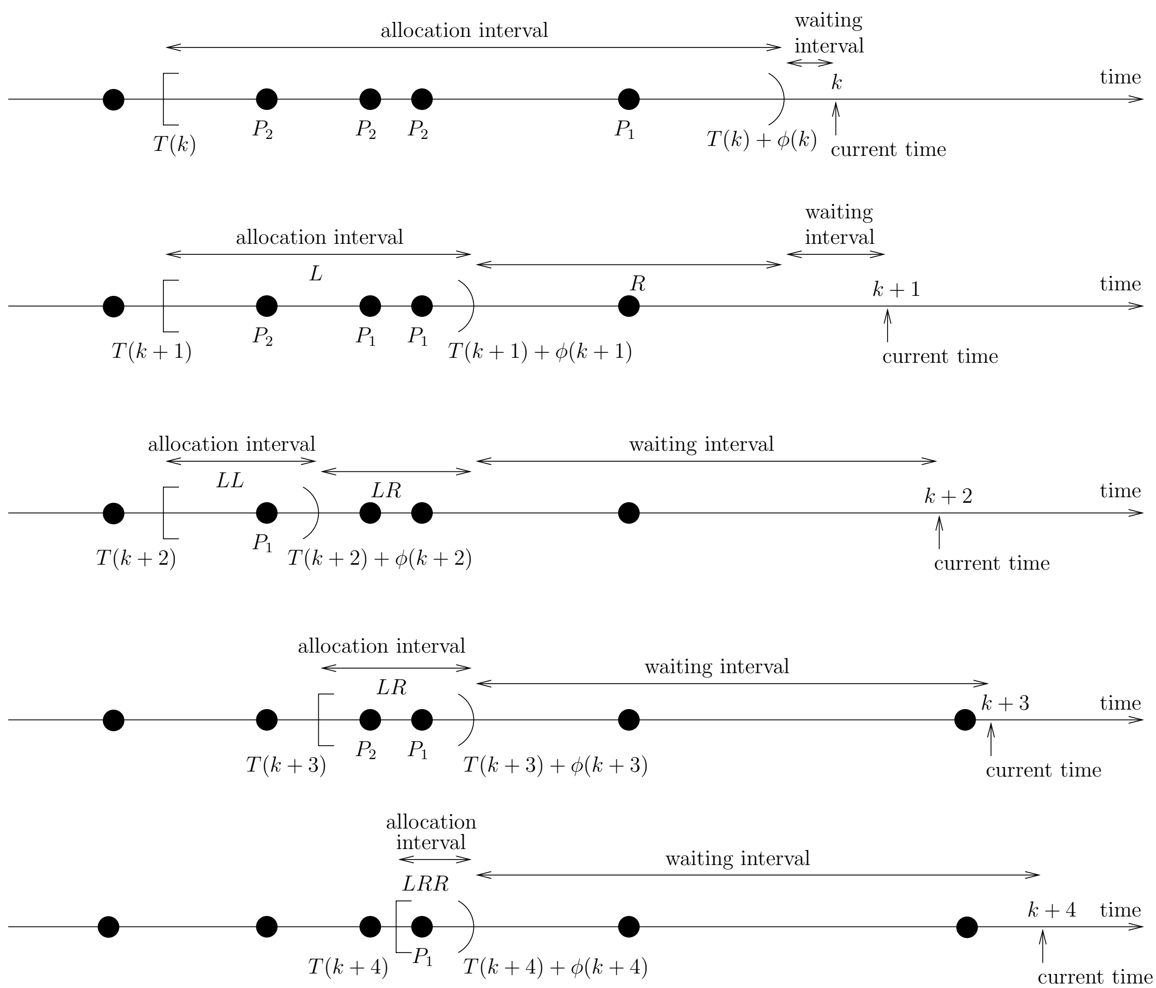


Fig. 1. PCFCFS splitting algorithm illustrating a collision followed by another collision.

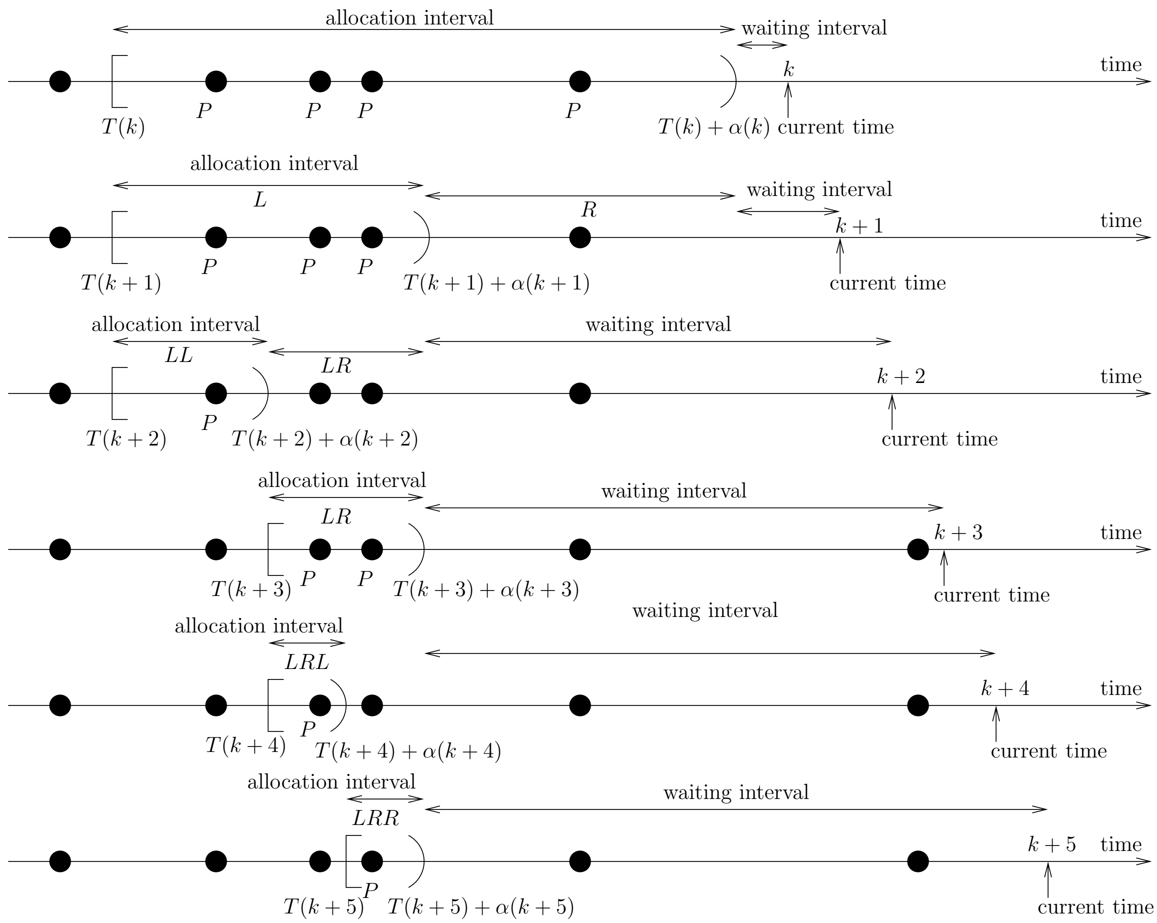


Fig. 2. FCFS splitting algorithm illustrating a collision followed by another collision.

Consider the scenario shown in Figure 1. In slot $k$, the allocation interval has three nodes in its left half and one node in its right half. All 'left half' nodes transmit with higher power $P_2$, while the 'right half' node transmits with nominal power $P_1$, leading to a collision. So, the allocation interval is split, with the left interval $L$ being the allocation interval for slot $k+1$. In slot $k+1$, the allocation interval has one node in its left half, which transmits with higher power $P_2$, and two nodes in its right half, which transmit with nominal power $P_1$. Hence, a collision occurs, and the allocation interval $L$ is split into two equal sized subintervals $LL$ and $LR$, with $LL$ being the allocation interval for slot $k+2$. Since a collision is followed by another collision, the right interval $R$ is returned to the waiting interval in slot $k+2$. In slot $k+2$, there is only one node in the allocation interval. Since this lone node lies in the right half of the allocation interval, it transmits with nominal power $P_1$, leading to a success. Thus, $LR$ becomes the allocation interval for slot $k+3$. For this allocation interval, the node in the left half transmits with higher power $P_2$ and the node in the right half transmits with nominal power $P_1$, resulting in a capture of the packet transmitted by the former node. Consequently, $LRR$ becomes the new allocation interval for slot $k+4$. Finally, in slot $k+4$, the lone node transmits with nominal power $P_1$, leading to a deterministic success and completing the CRP. For the same sequence of arrival times, the behavior of the FCFS algorithm with uniform transmission power $P$ is shown in Figure 2. Note that the FCFS algorithm requires 6 slots to resolve the collisions, while the proposed PCFCFS algorithm requires only 5 slots.

### *C.* **Algorithm Description**

Algorithm 1 describes the proposed PCFCFS splitting algorithm.

In Phase 1 of the algorithm, we initialize various quantities. $\tau$ denotes the number of slots for which the algorithm operates; ideally $\tau \to \infty$. By convention, the initial allocation interval is $[0, \min(\phi_0, 1))$, which is a right interval ($\mathcal{R}$). The initial channel feedback is assumed to be idle (0).

In Phase 2 of the algorithm, we determine power levels, obtain channel feedback and compute allocation interval parameters for each successive slot $k$. In Phase 2a, all users whose arrival times lie in the left half of the current allocation interval transmit with higher power $P_2$, while all users whose arrival times lie in the right half of the current allocation interval transmit with nominal power $P_1$. However, if a capture occurred in the previous slot $k-1$, all users in the current allocation interval transmit with nominal power $P_1$. Therefore, our algorithm always transmits successful packets in an FCFS manner. In Phase 2b, the allocation interval parameters are modulated based on the channel feedback. More specifically, if a collision occurs, then the left half of the current allocation interval becomes the new allocation interval. If a capture occurs, then the right half of the current allocation interval becomes the new allocation interval. If a success occurs and the current allocation interval is tagged as a left interval, then the corresponding right interval becomes the new allocation interval. If an idle occurs and the current allocation interval is tagged as a left interval, then the left half of the corresponding right interval becomes the new allocation interval. Otherwise, if a success or an idle occurs and the current allocation interval is tagged as a right interval, the waiting interval truncated to length $\phi_0$ becomes the new allocation interval, and a new Collision Resolution Period (CRP) begins in the next time slot $k+1$.

**Algorithm 1** PCFCFS splitting algorithm

**input:** $\phi_0$, $P_1$, $P_2$, arrivals $a_1, a_2, a_3, \ldots$ in $[0, \tau)$ {Phase 1 begins}
$T(1) \leftarrow 0$
$\phi(1) \leftarrow \min(\phi_0, 1)$
$\sigma(1) = \mathcal{R}$
feedback $= 0$ {Phase 1 ends}
**for** $k \leftarrow 1$ to $\tau$ **do** {Phase 2 begins}
  **if** feedback $\neq c$ **then** {Phase 2a begins}
    **for all** $i$ such that $T(k) \leqslant a_i < T(k) + \frac{\phi(k)}{2}$ **do**
      $P_i(k) = P_2$
    **end for**
    **for all** $i$ such that $T(k) + \frac{\phi(k)}{2} \leqslant a_i < T(k) + \phi(k)$ **do**
      $P_i(k) = P_1$
    **end for**
  **end if**{Phase 2a ends}
  transmit packets whose arrivals times lie in $[T(k), T(k) + \phi(k))$ and obtain channel feedback {Phase 2b begins}
  **if** feedback $= e$ **then**
    $T(k+1) \leftarrow T(k)$
    $\phi(k+1) \leftarrow \frac{\phi(k)}{2}$
    $\sigma(k+1) \leftarrow \mathcal{L}$
  **else if** feedback $= c$ **then**
    $T(k+1) \leftarrow T(k) + \frac{\phi(k)}{2}$
    $\phi(k+1) \leftarrow \frac{\phi(k)}{2}$
    $\sigma(k+1) \leftarrow \mathcal{R}$
  **else if** feedback $= 1$ and $\sigma(k) = \mathcal{L}$ **then**
    $T(k+1) \leftarrow T(k) + \phi(k)$
    $\phi(k+1) \leftarrow \phi(k)$
    $\sigma(k+1) \leftarrow \mathcal{R}$
  **else if** feedback $= 0$ and $\sigma(k) = \mathcal{L}$ **then**
    $T(k+1) \leftarrow T(k) + \phi(k)$
    $\phi(k+1) \leftarrow \frac{\phi(k)}{2}$
    $\sigma(k+1) \leftarrow \mathcal{L}$
  **else**
    $T(k+1) \leftarrow T(k) + \phi(k)$
    $\phi(k+1) = \min(\phi_0, k - T(k))$
    $\sigma(k+1) \leftarrow \mathcal{R}$
  **end if**{Phase 2b ends}
**end for**{Phase 2 ends}

## V. Throughput Analysis

**In this section, we derive the maximum stable throughput of PCFCFS. Recall that PCFCFS employs *Window Access* (WA) as its channel access algorithm (Assumption 7, Section II). However, for purposes of throughput analysis, we assume that PCFCFS employs *Simplified Window Access* (SWA), wherein the initial allocation interval (window size) after each CRP has the constant duration of $\phi_0$. It can be shown that SWA channel access algorithm has the same throughput as the WA channel access algorithm [8].**

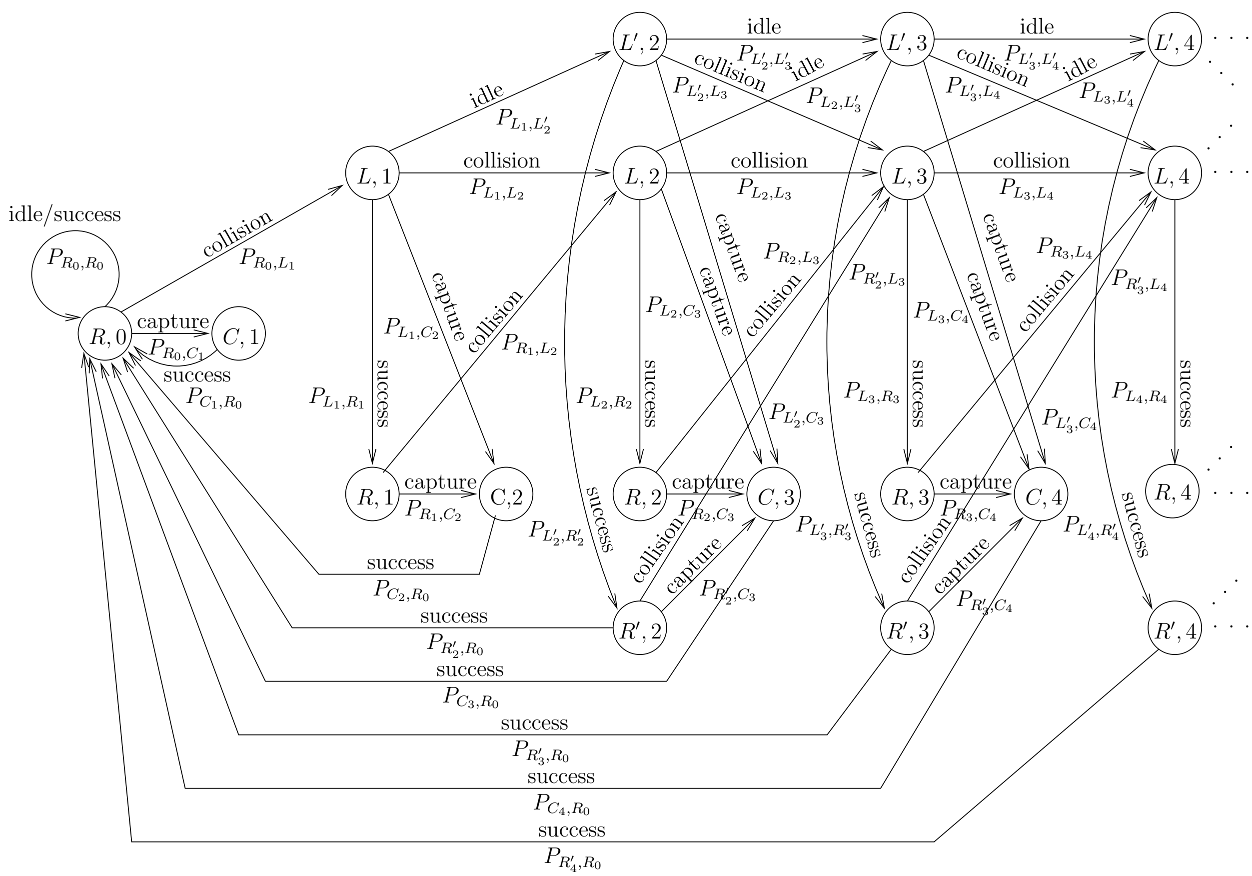


Fig. 3. Discrete Time Markov Chain representing a CRP of PCFCFS splitting algorithm.

The evolution of a CRP can be represented by the Discrete Time Markov Chain (DTMC) shown in Figure 3. Every state in the DTMC is a pair $(\sigma, i)$, where $\sigma$ is the status $\{L, L', R, R', C\}$ and $i$ is the number of times the original allocation interval (of length $\phi_0$) has been split. State $(R, 0)$ corresponds to the initial slot of a CRP. If an idle or a success occurs, the CRP ends immediately and a new CRP begins in the next slot. If a capture occurs, a transition occurs to state

$(C,1)$, where $C$ indicates that capture has occurred in the allocation interval. If a collision occurs in $(R,0)$, a transition occurs to state $(L,1)$. Each subsequent idle in a left allocation interval generates one additional split with a smaller left allocation interval, corresponding to a transition to $(L',i+1)$, where $L'$ indicates that the current left allocation interval has been reached after a collision (in some time slot) followed by one or more idles. A collision in an allocation interval generates one additional split with a smaller left allocation interval, corresponding to a transition to $(L,i+1)$, where $L$ indicates that the current left allocation interval has been reached just after a collision. A capture in an allocation interval generates an additional split with a smaller right allocation interval and corresponds to a transition to $(C,i+1)$. This is followed by a success from $(C,i+1)$ to $(R,0)$, thus ending the CRP. A success in a left allocation interval leads to the corresponding right allocation interval with no additional split, which causes a transition from $(L,i)$ to $(R,i)$, or $(L',i)$ to $(R',i)$. A success in $(R',i)$ causes a transition to $(R,0)$, thus ending the CRP. It can be easily verified that the states and transitions in Figure 3 constitute a Markov chain, i.e., each transition from every state is independent of the path used to reach the given state.

We now analyze a single CRP. Assume that the size of the initial allocation interval is $\phi_0$ (corresponding to state $(R,0)$). Each splitting of the allocation interval halves this, so that states $(L,i)$, $(L',i)$, $(R,i)$, $(R',i)$ and $(C,i)$ in Figure 3 correspond to allocation intervals of size $2^{-i}\phi_0$. Since the arrival process is Poisson with rate $\lambda$, the number of packets in the original allocation interval is a Poisson random variable (r.v.) with mean $\lambda\phi_0$. Consequently, the a priori distributions on the number of packets in disjoint subintervals are independent and Poisson. Define $G_i$ as the expected number of packets in an interval that has been split $i$ times. Thus

$$G_i \;=\; 2^{-i}\lambda\phi_0 \;=\; 2^{-i}G_0 \quad \forall \quad i \geqslant 0, \tag{6}$$

$$G_i \;=\; \frac{1}{2}G_{i-1} \quad \forall \quad i \geqslant 1. \tag{7}$$

We view $(R,0)$ as the starting state as well as the final state. For brevity in notation, the transition probability from state $(A,i)$ to state $(B,j)$ is denoted by $P_{A_i,B_j}$, where $A,B \in \{L,L',R,R',C\}$ and $i,j \in \{0\}\cup\mathbb{Z}^+$ (see Figure 3). For example, the transition probability from $(L,1)$ to $(C,2)$ is denoted by $P_{L_1,C_2}$.

$P_{R_0,R_0}$ is the probability of an idle or success in the first slot of the CRP. Since the number of packets in the initial allocation interval is Poisson with mean $G_0$, the probability of 0 or 1

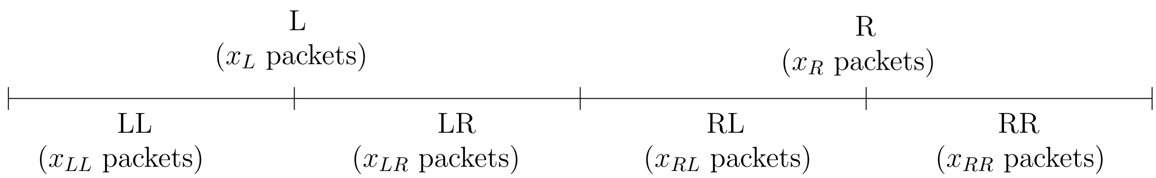


Fig. 4. Notation for number of packets in left and right subintervals of the original allocation interval.

packet is

$$P_{R_0,R_0} = (1+G_0)e^{-G_0}. \tag{8}$$

$P_{R_0,C_1}$ is the probability of capture in the first slot of a CRP. Let $x_L$ and $x_R$ denote the number of packets in the left and right halves of the original allocation interval respectively, as shown in Figure 4. Capture occurs if and only if $x_L = 1$ and $x_R = 1$. $x_L$ and $x_R$ are independent Poisson r.v.s of mean $G_1$ each. Thus

$$\begin{aligned} P_{R_0,C_1} &= \Pr(x_L = 1, x_R = 1), \\ &= \Pr(x_L = 1)\Pr(x_R = 1), \\ &= G_1^2 e^{-2G_1}, \\ P_{R_0,C_1} &= \frac{G_0^2}{4} e^{-G_0}. \end{aligned} \tag{9}$$

State $(L,1)$ is entered after collision in state $(R,0)$. Using (8) and (9), this occurs with probability

$$\begin{aligned} P_{R_0,L_1} &= 1 - P_{R_0,R_0} - P_{R_0,C_1}, \\ P_{R_0,L_1} &= 1 - \left(1 + G_0 + \frac{G_0^2}{4}\right)e^{-G_0}. \end{aligned} \tag{10}$$

Since a capture is always followed by a deterministic success,

$$P_{C_i,R_0} = 1 \ \forall \ i \geqslant 1. \tag{11}$$

*Lemma 1:* The outgoing transition probabilities from $(L, i)$, where $i \geqslant 1$, are given by

$$P_{L_i,R_i} = \frac{(1-e^{-G_i}-G_ie^{-G_i})G_ie^{-G_i}}{1-\left(1+G_{i-1}+\frac{G_{i-1}^2}{4}\right)e^{-G_{i-1}}}, \quad (12)$$

$$P_{L_i,L'_{i+1}} = \frac{(1-e^{-G_i}-G_ie^{-G_i})e^{-G_i}}{1-\left(1+G_{i-1}+\frac{G_{i-1}^2}{4}\right)e^{-G_{i-1}}}, \quad (13)$$

$$P_{L_i,C_{i+1}} = \frac{\frac{G_i^2}{4}e^{-G_i}}{1-\left(1+G_{i-1}+\frac{G_{i-1}^2}{4}\right)e^{-G_{i-1}}}, \quad (14)$$

$$P_{L_i,L_{i+1}} = \frac{1-(1+G_i+\frac{G_i^2}{4})e^{-G_i}}{1-\left(1+G_{i-1}+\frac{G_{i-1}^2}{4}\right)e^{-G_{i-1}}}. \quad (15)$$

*Proof:* Refer to Appendix A. ■

*Lemma 2:* The outgoing transition probabilities from $(R, i)$ are given by

$$P_{R_i,C_{i+1}} = \frac{\frac{G_i^2}{4}e^{-G_i}}{1-(1+G_i)e^{-G_i}} \;\; \forall \;\; i \geqslant 1, \quad (16)$$

$$P_{R_i,L_{i+1}} = \frac{1-\left(1+G_i+\frac{G_i^2}{4}\right)e^{-G_i}}{1-(1+G_i)e^{-G_i}} \;\; \forall \;\; i \geqslant 1. \quad (17)$$

*Proof:* Refer to Appendix B. ■

*Lemma 3:* The outgoing transition probabilities from $(L', i)$ are given by

$$P_{L'_i,R'_i} = \frac{(1-e^{-G_i})G_ie^{-G_i}}{1-(1+G_{i-1})e^{-G_{i-1}}} \;\; \forall \;\; i \geqslant 2, \quad (18)$$

$$P_{L'_i,L'_{i+1}} = \frac{(1-e^{-G_i}-G_ie^{-G_i})e^{-G_i}}{1-(1+G_{i-1})e^{-G_{i-1}}} \;\; \forall \;\; i \geqslant 2, \quad (19)$$

$$P_{L'_i,C_{i+1}} = \frac{\frac{G_i^2}{4}e^{-G_i}}{1-(1+G_{i-1})e^{-G_{i-1}}} \;\; \forall \;\; i \geqslant 2, \quad (20)$$

$$P_{L'_i,L_{i+1}} = \frac{1-\left(1+G_i+\frac{G_i^2}{4}\right)e^{-G_i}}{1-(1+G_{i-1})e^{-G_{i-1}}} \;\; \forall \;\; i \geqslant 2. \quad (21)$$

*Proof:* Refer to Appendix C. ■

*Lemma 4:* The outgoing transition probabilities from $(R', i)$ are given by

$$P_{R'_i,R_0} = \frac{G_i e^{-G_i}}{1-e^{-G_i}} \ \ \forall \ \ i \geqslant 2, \tag{22}$$

$$P_{R'_i,C_{i+1}} = \frac{\frac{G_i^2}{4} e^{-G_i}}{1-e^{-G_i}} \ \ \forall \ \ i \geqslant 2, \tag{23}$$

$$P_{R'_i,L_{i+1}} = \frac{1-\left(1+G_i+\frac{G_i^2}{4}\right)e^{-G_i}}{1-e^{-G_i}} \ \ \forall \ \ i \geqslant 2. \tag{24}$$

*Proof:* Refer to Appendix D. ■

In summary, Figure 3 is a DTMC and the transition probabilities are given by (8), (9), (10) and (11), and Lemmas 1, 2, 3 and 4.

We now analyze the DTMC in Figure 3. Observe that no state can be entered more than once before the return to $(R, 0)$. Let $Q_{X_i}$ denote the probability that state $(X, i)$ is entered before returning to $(R, 0)$, where $X \in \{L, L', R, R', C\}$ and $i \in \mathbb{Z}^+$. In other words, $Q_{X_i}$ denotes the probability of hitting $(X, i)$ in a CRP given that we start from $(R, 0)$. Note that $Q_{C_1} = P_{R_0,C_1}$ and $Q_{L_1} = P_{R_0,L_1}$. The probabilities $Q_{X_i}$ can be calculated iteratively from the initial state $(R, 0)$ as follows:

$$Q_{C_1} = \frac{G_0^2}{4} e^{-G_0}, \tag{25}$$

$$Q_{L_1} = 1-\left(1+G_0+\frac{G_0^2}{4}\right)e^{-G_0}, \tag{26}$$

$$Q_{C_2} = Q_{L_1}P_{L_1,C_2} + Q_{R_1}P_{R_1,C_2}, \tag{27}$$

$$Q_{L'_2} = Q_{L_1}P_{L_1,L'_2}, \tag{28}$$

$$Q_{L_2} = Q_{L_1}P_{L_1,L_2} + Q_{R_1}P_{R_1,L_2}, \tag{29}$$

$$\begin{aligned} Q_{L_i} = \ & Q_{L'_{i-1}}P_{L'_{i-1},L_i} + Q_{L_{i-1}}P_{L_{i-1},L_i} + Q_{R_{i-1}}P_{R_{i-1},L_i} \\ & + Q_{R'_{i-1}}P_{R'_{i-1},R_i} \ \ \forall \ \ i \geqslant 3, \end{aligned} \tag{30}$$

$$Q_{L'_i} = Q_{L'_{i-1}}P_{L'_{i-1},L'_i} + Q_{L_{i-1}}P_{L_{i-1},L'_i} \ \ \forall \ \ i \geqslant 3, \tag{31}$$

$$Q_{R_i} = Q_{L_i}P_{L_i,R_i} \ \ \forall \ \ i \geqslant 1, \tag{32}$$

$$Q_{R'_i} = Q_{L'_i}P_{L'_i,R'_i} \ \ \forall \ \ i \geqslant 2, \tag{33}$$

$$Q_{C_i} = Q_{L'_{i-1}}P_{L'_{i-1},C_i} + Q_{L_i}P_{L_i,C_i} + Q_{R_{i-1}}P_{R_{i-1},C_i} + Q_{R'_{i-1}}P_{R'_{i-1},C_i} \ \ \forall \ \ i \geqslant 3. \tag{34}$$

Let random variable $K$ denote the number of slots in a CRP. Thus, $K$ equals the number of states visited in the Markov chain, including the initial state $(R, 0)$, before the return to $(R, 0)$. Thus

$$E[K] \;=\; 1 + \sum_{i=1}^{\infty}(Q_{L_i} + Q_{L'_i} + Q_{R_i} + Q_{R'_i} + Q_{C_i}), \tag{35}$$

where we assume $Q_{L'_1} = Q_{R'_1} = 0$.

We evaluate the change in $T(k)$ from one CRP to the next, i.e., we evaluate the difference in left endpoints of initial allocation intervals of successive CRPs. For the assumed initial interval of size $\phi_0$, this change is at most $\phi_0$. However, if left allocation intervals have collisions or captures (e.g., $L$ in Figure 1), then the corresponding right allocation intervals (e.g., $R$ in Figure 1) are returned to the waiting interval, and the change is less than $\phi_0$. Let random variable $F$ denote the fraction of $\phi_0$ returned in this manner over a CRP, so that $\phi_0(1 - F)$ is the change in $T(k)$. We distinguish between two cases:

1) If a left allocation interval of type $(L, i)$ has a collision or a capture, then the corresponding right allocation interval $(R, i)$ is returned to the waiting interval. Let $U_{L_i}$ denote the probability that $(L, i)$ has a collision or a capture. Hence, $U_{L_i}$ denotes the probability that $(L, i)$ has two or more packets. Thus, $U_{L_i} = P_{L_i,L_{i+1}} + P_{L_i,C_{i+1}}$. Using (14) and (15), we obtain

$$U_{L_i} \;=\; \frac{1 - (1 + G_i)e^{-G_i}}{1 - \left(1 + G_{i-1} + \frac{G_{i-1}^2}{4}\right)e^{-G_{i-1}}} \;\;\forall\;\; i \geqslant 1. \tag{36}$$

2) If a left allocation interval of type $(L', i)$ has a collision or a capture, then the corresponding right allocation interval $(R', i)$ is returned to the waiting interval. Let $U_{L'_i}$ denote the probability that $(L', i)$ has a collision or a capture. Hence, $U_{L'_i}$ denotes the probability that $(L', i)$ has two or more packets. Thus, $U_{L'_i} = P_{L'_i,L_{i+1}} + P_{L'_i,C_{i+1}}$. Using (20) and (21), we obtain

$$U_{L'_i} \;=\; \frac{1 - \left(1 + G_i\right)e^{-G_i}}{1 - (1 + G_{i-1})e^{-G_{i-1}}} \;\;\forall\;\; i \geqslant 2. \tag{37}$$

In either case, the fraction of the original allocation interval returned on such a collision or a capture is $2^{-i}$. Therefore, the expected value of $F$ is

$$E[F] \;=\; \sum_{i=1}^{\infty}(Q_{L_i}U_{L_i} + Q_{L'_i}U_{L'_i})2^{-i}, \tag{38}$$

where we assume $U_{L'_1} = 0$.

From (6), (35) and (38), we observe that $E[K]$ and $E[F]$ are functions only of the product $\lambda\phi_0$. Note that as $i \to \infty$, $G_i = 2^{-i}\lambda\phi_0 \to 0$. Using the Taylor series expansion for $e^x$ or L'Hôpital's Rule, we can easily prove that:

1)

$$\lim_{i\to\infty} P_{L'_i,R'_i} = \frac{1}{2}, \tag{39}$$

$$\lim_{i\to\infty} P_{L'_i,L'_{i+1}} = \frac{1}{4}, \tag{40}$$

$$\lim_{i\to\infty} P_{L'_i,C_{i+1}} = \frac{1}{8}, \tag{41}$$

$$\lim_{i\to\infty} P_{L'_i,L_{i+1}} = \frac{1}{8}, \tag{42}$$

2)

$$\lim_{i\to\infty} P_{R'_i,R_0} = 1, \tag{43}$$

$$\lim_{i\to\infty} P_{R'_i,C_{i+1}} = 0, \tag{44}$$

$$\lim_{i\to\infty} P_{R'_i,L_{i+1}} = 0, \tag{45}$$

3)

$$\lim_{i\to\infty} P_{L_i,R_i} = 0, \tag{46}$$

$$\lim_{i\to\infty} P_{L_i,L'_{i+1}} = \frac{1}{2}, \tag{47}$$

$$\lim_{i\to\infty} P_{L_i,C_{i+1}} = \frac{1}{4}, \tag{48}$$

$$\lim_{i\to\infty} P_{L_i,L_{i+1}} = \frac{1}{4}, \tag{49}$$

4)

$$\lim_{i\to\infty} P_{R_i,C_{i+1}} = \frac{1}{2}, \tag{50}$$

$$\lim_{i\to\infty} P_{R_i,L_{i+1}} = \frac{1}{2}. \tag{51}$$

The proofs of these results are given in Appendix E. Hence, $Q_{L_i}$, $Q_{L'_i}$, $Q_{R'_i}$ and $Q_{C_i}$ tend to zero with increasing $i$ as $2^{-i}$, while $Q_{R_i}$ tends to zero with increasing $i$ as $4^{-i}$. Thus, $E[K]$ and $E[F]$ can be easily evaluated numerically as functions of $\lambda\phi_0$.

Define the time backlog to be the difference between the current time and the left endpoint of the allocation interval, i.e., $k - T(k)$. Note that all packets that arrived in the interval $T(k), k$ have not yet been successfully transmitted, i.e., they are backlogged. Moreover, we define the drift $D$ to be the expected change in time backlog, $k - T(k)$, over a CRP, assuming an initial allocation interval of $\phi_0$. Thus, $D$ is the expected number of slots in a CRP less the expected change in $T(k)$, and is given by

$$D \;=\; E[K] - \phi_0(1 - E[F]). \tag{52}$$

The drift is negative if $E[K] < \phi_0(1 - E[F])$. Equivalently, the drift is negative if

$$\lambda \;<\; \frac{\lambda\phi_0(1 - E[F])}{E[K]} =: \zeta. \tag{53}$$

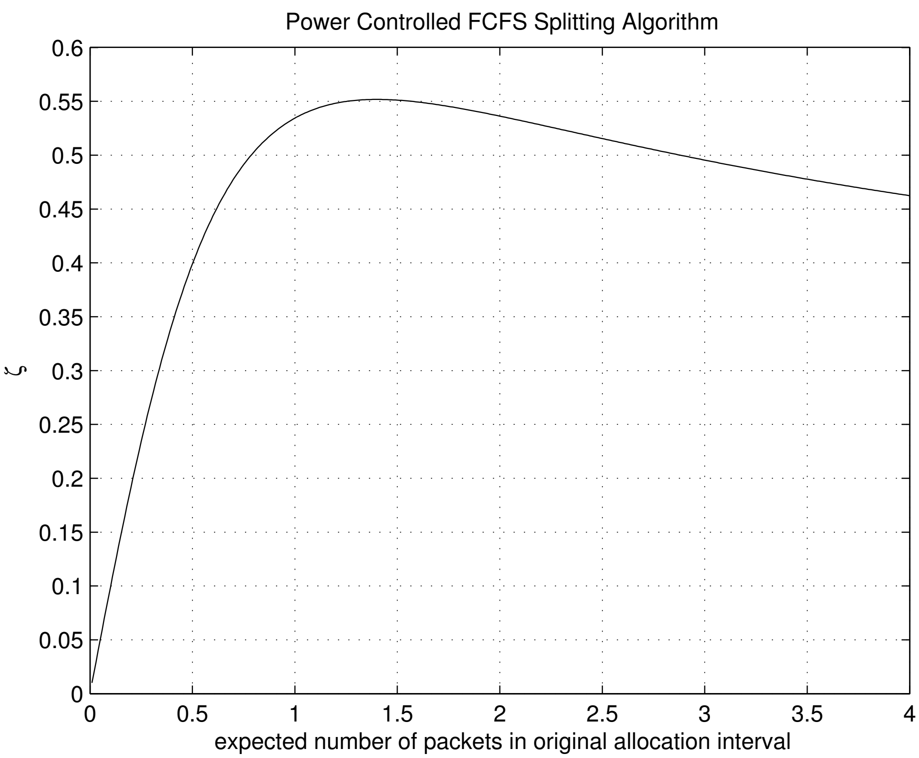


Fig. 5. Plot of $\zeta$ versus $\lambda\phi_0$.

The right hand side of (53), $\zeta$, is a function of $\lambda\phi_0$ and is plotted in Figure 5. We observe that $\zeta$ takes its maximum value at $\lambda\phi_0 = 1.4$. More precisely, $\zeta$ has a numerically evaluated maximum of $0.5518$ at $\lambda\phi_0 = 1.4$. If $\phi_0$ is chosen to be $\frac{1.4}{0.5518} = 2.54$, then (53) is satisfied for all $\lambda < 0.5518$. Thus, the expected time backlog decreases whenever it is initially larger than $\phi_0$, and we infer that the algorithm is stable for $\lambda < 0.5518$. We have therefore proved the following result.

*Proposition 1:* The maximum stable throughput of the PCFCFS algorithm is 0.5518.

## VI. Numerical Results

In our numerical experiments, we use values of system parameters that are commonly encountered in wireless networks [32]. We compare the performance of the following algorithms:

1) FCFS with uniform power $P_1$,
2) **REBS [23] with uniform power** $P_1$, and
3) PCFCFS.

For each algorithm, the value of the initial allocation interval is chosen so as to achieve maximum stable throughput. For FCFS, maximum stable throughput occurs when its initial allocation interval, $\alpha_0 = 2.6$ [1]. From Section V, the maximum throughput of PCFCFS occurs at $\phi_0 = 2.54$. **For REBS, we use a fixed temporal allocation interval** $\chi_0 = 2$ **[23].** Let $n$ **denote the number of arrivals in** $[0, \tau)$**,** $n_{suc}$ denote the number of successful packets in $[0, \tau)$ and $d_i$ denote the departure time of $i^{th}$ packet.

For a given set of system parameters, we compute the following performance metrics:

$$\text{Throughput} = \frac{n_{suc}}{\tau}, \tag{54}$$

$$\text{Average Delay} = \frac{\sum_{i=1}^{n}(d_i - a_i)}{n}, \tag{55}$$

$$\text{Average Power} = \frac{\sum_{i=1}^{n}\sum_{k=\lceil a_i \rceil}^{d_i} P_i(k)}{n}. \tag{56}$$

Keeping all other parameters fixed, we observe the effect of increasing the arrival rate on the throughput, average delay and average power.

The system parameters for our numerical experiments are shown in Table I. From (4) and (5), we obtain $P_1 = 0.2$ mW and $P_2 = 0.6$ mW. We vary the arrival rate $\lambda$ from $0.40$ to $0.60$ packets/second in steps of $0.01$. Figure 6 plots the throughput versus arrival rate for the FCFS, REBS and PCFCFS algorithms. Figure 7 plots the average delay per successful packet versus

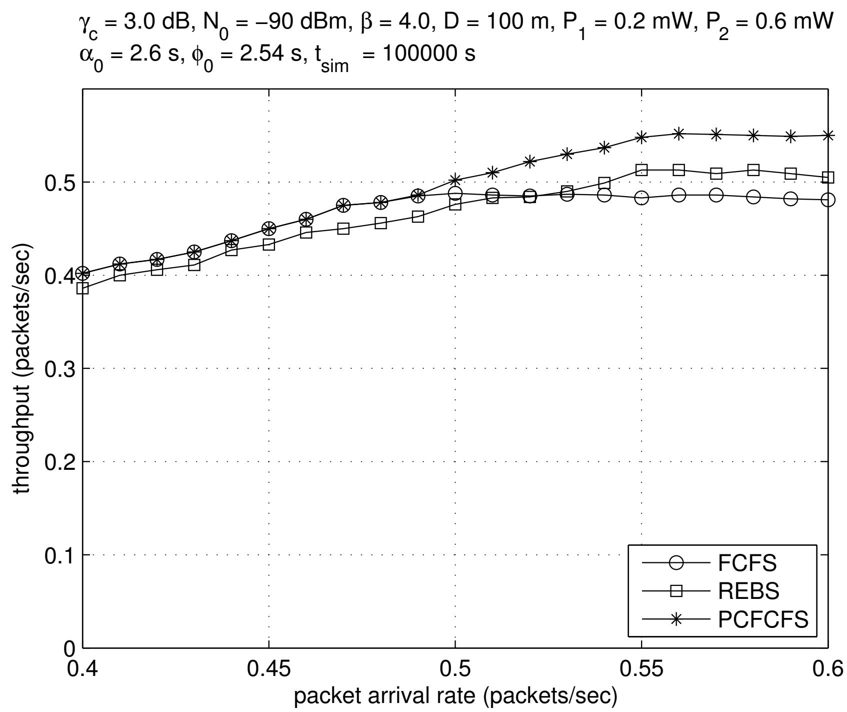


Fig. 6. Throughput versus arrival rate for FCFS, REBS and PCFCFS algorithms.

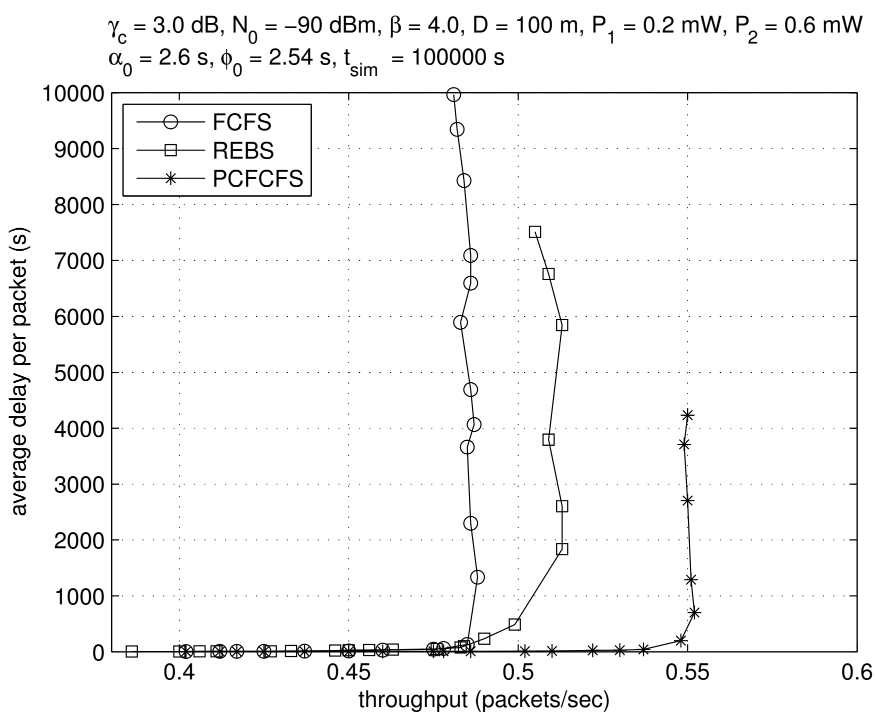


Fig. 7. Average delay versus throughput for FCFS, REBS and PCFCFS algorithms.

| Parameter | Symbol | Value |
|---|---|---|
| communication threshold | $\gamma_c$ | 3 dB |
| noise power spectral density | $N_0$ | -90 dBm |
| path loss exponent | $\beta$ | 4 |
| transmitter-receiver distance | $D$ | 100 m |
| initial allocation interval of FCFS | $\alpha_0$ | 2.6 s |
| initial allocation interval of PCFCFS | $\phi_0$ | 2.54 s |
| algorithm operation time | $\tau$ | $10^5$ s |

TABLE I

SYSTEM PARAMETERS FOR PERFORMANCE EVALUATION OF PCFCFS AND FCFS ALGORITHMS.

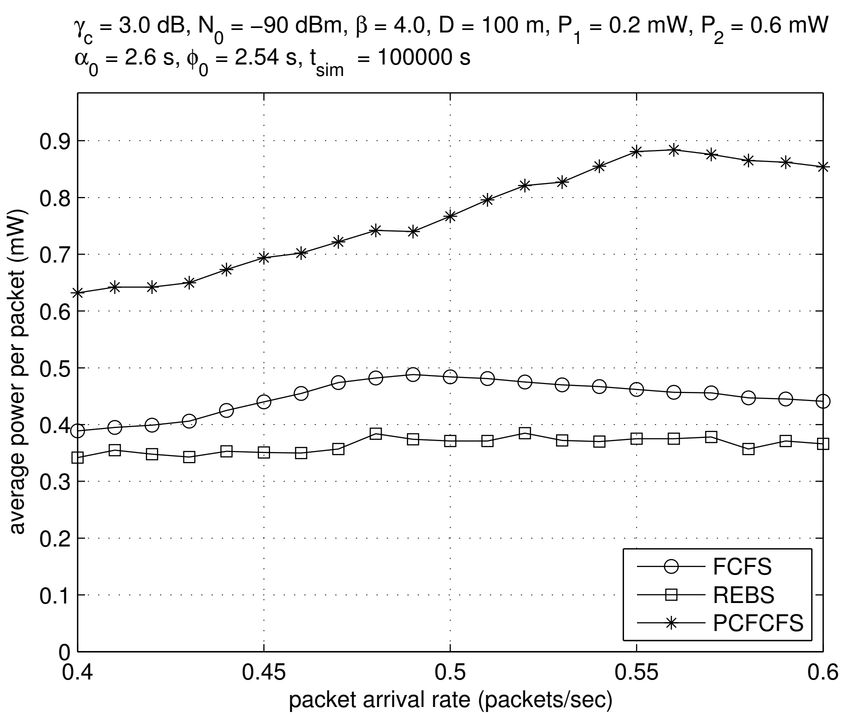


Fig. 8. Average power versus arrival rate for FCFS, PCFCFS and REBS algorithms.

arrival rate for all algorithms. Finally, Figure 8 plots the average power per successful packet versus arrival rate for all algorithms.

For arrival rates exceeding $0.56$, the throughput of PCFCFS is less than the arrival rate (Figure 6) and the average delay of PCFCFS increases rapidly (Figure 7), which leads to a substantial increase in the number of backlogged packets and system instability. Hence, the maximum stable throughput of PCFCFS is between $0.55$ and $0.56$. Thus, Figures 6 and 7 corroborate our result that the maximum stable throughput of PCFCFS is $0.5518$ (see Section V).

For both PCFCFS and FCFS, the departure rate (throughput) equals the arrival rate for all arrival rates up to 0.487 (Figure 6). Hence, both these algorithms are stable for arrival rates below 0.487. For arrival rates exceeding 0.487, the departure rate of FCFS is strictly lower than its arrival rate, leading to packet backlog and system instability. On the other hand, for PCFCFS, the departure rate still equals its arrival rate for arrival rates between 0.487 and 0.5518. In other words, the PCFCFS algorithm is stable for a higher range of arrival rates compared to FCFS algorithm. However, the PCFCFS algorithm becomes unstable for arrival rates exceeding $0.5518$.

**For REBS, the throughput is about $5 - 10\%$ lower than the arrival rate. However, the throughput of REBS is higher than that of FCFS for arrival rates exceeding 0.52. Moreover, REBS achieves a maximum throughput of 0.513. This is because REBS has been designed to optimize on power rather than throughput. This is corroborated by Figure 8, which shows that REBS expends about $50\%$ power per packet compared to PFCFS.**

**In summary, from a throughput and delay perspective, PCFCFS outperforms both FCFS and REBS algorithms, as corroborated by Figures 6 and 7. However, this is achieved at the cost of expending higher power per packet. PCFCFS algorithm can be potentially applicable to wireless networks wherein users do not have stringent requirements on transmit power but expect high data rates from the service provider.**

## VII. Discussions and Conclusions

**In this section, we relax some of the assumptions of Section II and discuss their impact on the design of the PCFCFS algorithm. Specifically, we consider realistic scenarios such as unequal distances of transmitters (users) from the receiver and slow fading in wireless channels. Finally, we conclude the paper.**

**In general, transmitters (users) can be located at unequal distances from the receiver. For such a scenario, it is reasonable to assume that each user can estimate the distance to the receiver (instead of the exact geographical location). A possible solution to the distance estimation problem would be to use global positioning system or ranging techniques. Once a user $i$ has estimated its distance $D_i$ to the receiver, it can compute its nominal and**

**higher transmission power levels $P_{1,i}$ and $P_{2,i}$ using (4) and (5) respectively. Note that even in the case of unequal distances, one packet is received successfully in a slot if one user $i$ transmits with power $P_{1,i}$ or two users $i$ and $j$ transmit with powers $P_{1,i}$ and $P_{2,j}$. Since the receiver does not require knowledge of users' transmission powers, the PCFCFS algorithm can still be employed in its original form at the receiver.**

**In realistic wireless channels, the received signal power varies not only due to distance-dependent propagation loss (Assumption 3 in Section II), but also due to time-varying channel gain termed as fading. Since the receiver in PCFCFS algorithm employs energy detectors to distinguish between success $(1)$ and capture $(c)$, fading effects can lead to some incorrect decisions. A possible modification to the PCFCFS algorithm to alleviate this problem is as follows. Under the assumption that channel gain remains constant in a slot but can vary from slot to slot (slow fading), the receiver can adaptively change the threshold used by the energy detector to distinguish between $1$ and $c$. A possible method would be to compute a weighted combination of channel gains in the last few slots to estimate the channel gain (and thus the energy detector threshold) in the current slot, using an algorithm like Least Mean Squares (LMS).**

**In this paper, we have considered random access in wireless networks under the physical interference model. By recognizing that the receiver can successfully decode the strongest packet in presence of multiple transmissions, we have proposed PCFCFS, a splitting algorithm that modulates transmission powers of users based on observed channel feedback. PCFCFS achieves higher throughput and lower delay than those of FCFS and REBS algorithms with uniform transmission power. We show that the maximum stable throughput of PCFCFS is 0.5518. PCFCFS can be implemented in those scenarios where users are willing to trade some power for a substantial gain in throughput. Moreover, if users can estimate the arrival rate of packets, then they can employ FCFS algorithm for arrival rates up to 0.4871 and PCFCFS algorithm for higher arrival rates, thus leading to further reduction in average transmission power.**

## ACKNOWLEDGMENT

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# Appendix A
## Proof of Lemma 1

Refer to Figure 3. For $i = 1$, $(L, i)$ is entered only via a collision in $(R, i-1)$. For $i = 2$, $(L, i)$ is entered only via a collision in $(L, i-1)$ or $(R, i-1)$. For $i \geqslant 3$, $(L, i)$ is entered only via a collision in $(L', i-1)$, $(L, i-1)$, $(R, i-1)$ or $(R', i-1)$. In every case, a subinterval $Y$ is split into $YL$ and $YR$, and $YL$ becomes the new allocation interval. Let $x_{YL}$ and $x_{YR}$ denote the number of packets in $YL$ and $YR$ respectively. A priori, $x_{YL}$ and $x_{YR}$ are independent Poisson r.v.s of mean $G_i$ each. The event that a collision occurred in the previous state is $\{x_{YL} + x_{YR} \geqslant 2\} \cap \{x_{YL} = x_{YR} = 1\}^c =: \mathcal{C}_Y$. Note that $x_{YL} + x_{YR} = x_Y$ is a Poisson r.v. of mean $G_{i-1}$. From (7), $G_i = \frac{1}{2} G_{i-1} \; \forall \; i \geqslant 1$. The probability of success in $(L, i)$ is the probability that $x_{YL} = 1$ conditional on $\mathcal{C}_Y$, i.e.,

$$
\begin{aligned}
P_{L_i,R_i} &= \Pr(x_{YL} = 1 | \mathcal{C}_Y), \\
&= \frac{\Pr(\mathcal{C}_Y | x_{YL} = 1) \Pr(x_{YL} = 1)}{\Pr(\mathcal{C}_Y)}, \\
&= \frac{\Pr(\{x_{YR} = 1\} \cap \{x_{YR} = 1\}^c) \Pr(x_{YL} = 1)}{\Pr(\mathcal{C}_Y)}, \\
&= \frac{\Pr(x_{YR} \geqslant 2) \Pr(x_{YL} = 1)}{\Pr(\{x_{YL} + x_{YR} \geqslant 2\} \cap \{x_{YL} = x_{YR} = 1\}^c)}, \\
&= \frac{\Pr(x_{YR} \geqslant 2) \Pr(x_{YL} = 1)}{\Pr(x_Y \geqslant 2) - \Pr(x_{YL} = 1) \Pr(x_{YR} = 1)}, \\
&= \frac{(1 - e^{-G_i} - G_i e^{-G_i}) G_i e^{-G_i}}{1 - e^{-G_{i-1}} - G_{i-1} e^{-G_{i-1}} - G_i^2 e^{-2G_i}}, \\
P_{L_i,R_i} &= \frac{(1 - e^{-G_i} - G_i e^{-G_i}) G_i e^{-G_i}}{1 - \left(1 + G_{i-1} + \frac{G_{i-1}^2}{4}\right) e^{-G_{i-1}}}. \qquad (57)
\end{aligned}
$$

The probability of idle in $(L,i)$ is the probability that $x_{YL}=0$ conditional on $\mathcal{C}_Y$, i.e.,

$$\begin{aligned}
P_{L_i,L'_{i+1}} &= \Pr(x_{YL}=0|\mathcal{C}_Y), \\
&= \frac{\Pr(\mathcal{C}_Y|x_{YL}=0)\Pr(x_{YL}=0)}{\Pr(\mathcal{C}_Y)}, \\
&= \frac{\Pr(\{x_{YR}\geqslant 2\}\cap\{x_{YR}=1\}^c)\Pr(x_{YL}=0)}{\Pr(\mathcal{C}_Y)}, \\
&= \frac{\Pr(x_{YR}\geqslant 2)\Pr(x_{YL}=0)}{\Pr(\{x_{YL}+x_{YR}\geqslant 2\}\cap\{x_{YL}=x_{YR}=1\}^c)}, \\
&= \frac{\Pr(x_{YR}\geqslant 2)\Pr(x_{YL}=0)}{\Pr(x_Y\geqslant 2)-\Pr(x_{YL}=1)\Pr(x_{YR}=1)}, \\
&= \frac{(1-e^{-G_i}-G_ie^{-G_i})e^{-G_i}}{1-e^{-G_{i-1}}-G_{i-1}e^{-G_{i-1}}-G_i^2e^{-2G_i}}, \\
P_{L_i,L'_{i+1}} &= \frac{(1-e^{-G_i}-G_ie^{-G_i})e^{-G_i}}{1-\left(1+G_{i-1}+\frac{G_{i-1}^2}{4}\right)e^{-G_{i-1}}}. \qquad (58)
\end{aligned}$$

Let $x_{YLL}$ and $x_{YLR}$ denote the number of packets in $YLL$ and $YLR$ respectively. $x_{YLL}$ and $x_{YLR}$ are independent Poisson r.v.s of mean $G_{i+1}$ each, and $x_{YLL}+x_{YLR}=x_{YL}$. The probability of capture in $(L,i)$ is the probability that $x_{YLL}=1$ and $x_{YLR}=1$ conditional on $\mathcal{C}_Y$, i.e.,

$$\begin{aligned}
P_{L_i,C_{i+1}} &= \Pr(x_{YLL}=1,x_{YLR}=1|\mathcal{C}_Y), \\
&= \frac{\Pr(\mathcal{C}_Y|x_{YLL}=1,x_{YLR}=1)\Pr(x_{YLL}=1,x_{YLR}=1)}{\Pr(\{x_{YL}+x_{YR}\geqslant 2\}\cap\{x_{YL}=x_{YR}=1\}^c)}, \\
&= \frac{\Pr(\mathcal{C}_Y|x_{YL}=2)\Pr(x_{YLL}=1)\Pr(x_{YLR}=1)}{\Pr(\{x_{YL}+x_{YR}\geqslant 2\}\cap\{x_{YL}=x_{YR}=1\}^c)}, \\
&= \frac{\Pr(x_{YR}\geqslant 0)\Pr(x_{YLL}=1)\Pr(x_{YLR}=1)}{\Pr(x_Y\geqslant 2)-\Pr(x_{YL}=1)\Pr(x_{YR}=1)}, \\
&= \frac{1.G_{i+1}^2e^{-2G_{i+1}}}{1-e^{-G_{i-1}}-G_{i-1}e^{-G_{i-1}}-G_i^2e^{-2G_i}}, \\
P_{L_i,C_{i+1}} &= \frac{\frac{G_i^2}{4}e^{-G_i}}{1-\left(1+G_{i-1}+\frac{G_{i-1}^2}{4}\right)e^{-G_{i-1}}}. \qquad (59)
\end{aligned}$$

From (57), (58) and (59), we obtain

$$\begin{aligned}
P_{L_i,L_{i+1}} &= 1-P_{L_i,R_i}-P_{L_i,L'_{i+1}}-P_{L_i,C_{i+1}}, \\
P_{L_i,L_{i+1}} &= \frac{1-\left(1+G_i+\frac{G_i^2}{4}\right)e^{-G_i}}{1-\left(1+G_{i-1}+\frac{G_{i-1}^2}{4}\right)e^{-G_{i-1}}}. \qquad (60)
\end{aligned}$$

# Appendix B
# Proof of Lemma 2

Refer to Figure 3. For $i \geqslant 1$, $(R, i)$ is entered only via a success in $(L, i)$. Recall that $(L, i)$ was entered only via a collision from a previous state. We use the notation introduced in the proof of Lemma 1. Define the event

$$\begin{aligned}
\mathcal{S}_{YL} &:= \mathcal{C}_Y \cap \{x_{YL} = 1\}, \\
&= \{x_{YL} + x_{YR} \geqslant 2\} \cap \{x_{YL} = x_{YR} = 1\}^c \cap \{x_{YL} = 1\}, \\
&= \{x_{YR} \geqslant 1\} \cap \{x_{YR} = 1\}^c \cap \{x_{YL} = 1\}, \\
\mathcal{S}_{YL} &= \{x_{YR} \geqslant 2\} \cap \{x_{YL} = 1\}. \qquad (61)
\end{aligned}$$

Let $x_{YRL}$ and $x_{YRR}$ denote the number of packets in $YRL$ and $YRR$ respectively. $x_{YRL}$ and $x_{YRR}$ are independent Poisson r.v.s of mean $G_{i+1}$ each. Since $x_{YR} \geqslant 2$, a success or an idle can never occur in state $(R, i)$. Note that $x_{YR} = x_{YRL} + x_{YRR}$. The probability of capture in state $(R, i)$ is the probability that $x_{YRL} = 1$ and $x_{YRR} = 1$ conditional on $\mathcal{S}_{YL}$, i.e.,

$$\begin{aligned}
P_{R_i, C_{i+1}} &= \Pr(x_{YRL} = 1, x_{YRR} = 1 | x_{YR} \geqslant 2, x_{YL} = 1), \\
&= \Pr(x_{YRL} = 1, x_{YRR} = 1 | x_{YR} \geqslant 2), \\
&= \frac{\Pr(x_{YR} \geqslant 2 | x_{YRL} = 1, x_{YRR} = 1) \Pr(x_{YRL} = 1, x_{YRR} = 1)}{\Pr(x_{YR} \geqslant 2)}, \\
&= \frac{\Pr(x_{YRL} + x_{YRR} \geqslant 2 | x_{YRL} = 1, x_{YRR} = 1) \Pr(x_{YRL} = 1, x_{YRR} = 1)}{\Pr(x_{YR} \geqslant 2)}, \\
&= \frac{1. \Pr(x_{YRL} = 1) \Pr(x_{YRR} = 1)}{\Pr(x_{YR} \geqslant 2)}, \\
&= \frac{G_{i+1}^2 e^{-2G_{i+1}}}{1 - e^{-G_i} - G_i e^{-G_i}}, \\
P_{R_i, C_{i+1}} &= \frac{\frac{G_i^2}{4} e^{-G_i}}{1 - (1 + G_i) e^{-G_i}}. \qquad (62)
\end{aligned}$$

From (62), we obtain

$$\begin{aligned}
P_{R_i, L_{i+1}} &= 1 - P_{R_i, C_{i+1}}, \\
&= 1 - \frac{\frac{G_i^2}{4} e^{-G_i}}{1 - (1 + G_i) e^{-G_i}}, \\
P_{R_i, L_{i+1}} &= \frac{1 - \left(1 + G_i + \frac{G_i^2}{4}\right) e^{-G_i}}{1 - (1 + G_i) e^{-G_i}}. \qquad (63)
\end{aligned}$$

## APPENDIX C
## PROOF OF LEMMA 3

Refer to Figure 3. For $i = 2$, $(L', i)$ is entered only by an idle in $(L, i-1)$. For $i \geqslant 3$, state $(L', i)$ is entered by an idle in $(L', i-1)$ or an idle in $(L, i-1)$. In every case, a residual right subinterval, say $Z$, is split into $ZL$ and $ZR$, and $ZL$ becomes the new allocation interval. Note that $(L', i)$ can be entered if and only if there is a collision (in some time slot) followed by one or more idles. Therefore, $Z$ must contain at least two packets. Let $x_{ZL}$ and $x_{ZR}$ denote the number of packets in $ZL$ and $ZR$ respectively. A priori, $x_{ZL}$ and $x_{ZR}$ are independent Poisson r.v.s of mean $G_i$ each. Let $x_Z$ denote the number of packets in $Z$. Thus $x_Z = x_{ZL} + x_{ZR}$, $x_Z$ is a Poisson r.v. of mean $G_{i-1}$ and $x_Z \geqslant 2$.

The probability of success in $(L', i)$ is the probability that $x_{ZL} = 1$ conditional on $x_Z \geqslant 2$, i.e.,

$$\begin{aligned}
P_{L'_i,R'_i} &= \Pr(x_{ZL} = 1 | x_Z \geqslant 2), \\
&= \frac{\Pr(x_Z \geqslant 2 | x_{ZL} = 1)\Pr(x_{ZL} = 1)}{\Pr(x_Z \geqslant 2)}, \\
&= \frac{\Pr(x_{ZL} + x_{ZR} \geqslant 2 | x_{ZL} = 1)\Pr(x_{ZL} = 1)}{\Pr(x_Z \geqslant 2)}, \\
&= \frac{\Pr(x_{ZR} \geqslant 1)\Pr(x_{ZL} = 1)}{\Pr(x_Z \geqslant 2)}, \\
P_{L'_i,R'_i} &= \frac{(1 - e^{-G_i})G_i e^{-G_i}}{1 - (1 + G_{i-1})e^{-G_{i-1}}}. \qquad (64)
\end{aligned}$$

The probability of idle in $(L', i)$ is the probability that $x_{ZL} = 0$ conditional on $x_Z \geqslant 2$, i.e.,

$$\begin{aligned}
P_{L'_i,L'_{i+1}} &= \Pr(x_{ZL} = 0 | x_Z \geqslant 2), \\
&= \frac{\Pr(x_Z \geqslant 2 | x_{ZL} = 0)\Pr(x_{ZL} = 0)}{\Pr(x_Z \geqslant 2)}, \\
&= \frac{\Pr(x_{ZL} + x_{ZR} \geqslant 2 | x_{ZL} = 0)\Pr(x_{ZL} = 0)}{\Pr(x_Z \geqslant 2)}, \\
&= \frac{\Pr(x_{ZR} \geqslant 2)\Pr(x_{ZL} = 0)}{\Pr(x_Z \geqslant 2)}, \\
P_{L'_i,L'_{i+1}} &= \frac{(1 - e^{-G_i} - G_i e^{-G_i})e^{-G_i}}{1 - (1 + G_{i-1})e^{-G_{i-1}}}. \qquad (65)
\end{aligned}$$

Let $x_{ZLL}$ and $x_{ZLR}$ denote the number of packets in $ZLL$ and $ZLR$ respectively. A priori,$x_{ZLL}$ and $x_{ZLR}$ are independent Poisson r.v.s of mean $G_{i+1}$ each. The probability of capture in $(L', i)$

is the probability that $x_{ZLL} = 1$ and $x_{ZLR} = 1$ conditional on $x_Z \geqslant 2$, i.e.,

$$\begin{aligned} P_{L'_i,C_{i+1}} &= \Pr(x_{ZLL} = 1, x_{ZLR} = 1 | x_Z \geqslant 2), \\ &= \frac{\Pr(x_Z \geqslant 2 | x_{ZLL} = 1, x_{ZLR} = 1) \Pr(x_{ZLL} = 1, x_{ZLR} = 1)}{\Pr(x_Z \geqslant 2)}, \\ &= \frac{1. \Pr(x_{ZLL} = 1) \Pr(x_{ZLR} = 1)}{\Pr(x_Z \geqslant 2)}, \\ &= \frac{G_{i+1}^2 e^{-2G_{i+1}}}{1 - e^{-G_{i-1}} - G_{i-1} e^{-G_{i-1}}}, \\ P_{L'_i,C_{i+1}} &= \frac{\frac{G_i^2}{4} e^{-G_i}}{1 - (1 + G_{i-1}) e^{-G_{i-1}}}. \end{aligned} \tag{66}$$

From (64), (65) and (66), we obtain

$$P_{L'_i,L_{i+1}} = 1 - P_{L'_i,R'_i} - P_{L'_i,L'_{i+1}} - P_{L'_i,C'_{i+1}}, \tag{67}$$

$$P_{L'_i,L_{i+1}} = \frac{1 - \left(1 + G_i + \frac{G_i^2}{4}\right) e^{-G_i}}{1 - (1 + G_{i-1}) e^{-G_{i-1}}}. \tag{68}$$

## Appendix D
## Proof of Lemma 4

Refer to Figure 3. For $i \geqslant 2$, state $(R', i)$ is entered if and only if a success occurs in state $(L', i)$. When $(L', i)$ was entered, a residual right subinterval $Z$ was split into $ZL$ and $ZR$, and $ZL$ became the new allocation interval. Recall that $x_Z \geqslant 2$, since $(L', i)$ can only be entered after a collision followed by one or more idles. A success in $(L', i)$ implies $x_{ZL} = 1$. Hence, $(R', i)$ is entered if and only if both these events occurs, i.e., $x_Z \geqslant 2$ and $x_{ZL} = 1$. Therefore, $(R', i)$ can be entered if and only if $x_{ZR} \geqslant 1$. Note that there can never be an idle from $(R', i)$.

The probability of success in $(R', i)$ is the probability that $x_{ZR} = 1$ conditional on $x_{ZR} \geqslant 1$, i.e.,

$$\begin{aligned} P_{R'_i,R_0} &= \Pr(x_{ZR} = 1 | x_{ZR} \geqslant 1), \\ &= \frac{\Pr(x_{ZR} \geqslant 1 | x_{ZR} = 1) \Pr(x_{ZR} = 1)}{\Pr(x_{ZR} \geqslant 1)}, \\ P_{R'_i,R_0} &= \frac{G_i e^{-G_i}}{1 - e^{-G_i}}. \end{aligned} \tag{69}$$

Let $x_{ZRL}$ and $x_{ZRR}$ denote the number of packets in $ZRL$ and $ZRR$ respectively. Note that $x_{ZR} = x_{ZRL} + x_{ZRR}$. $x_{ZRL}$ and $x_{ZRR}$ are independent Poisson r.v.s of mean $G_{i+1}$ each. The probability of capture in state $(R', i)$ is the probability that $x_{ZRL} = 1$ and $x_{ZRR} = 1$ conditional on $x_{ZR} \geqslant 1$, i.e.,

$$\begin{aligned} P_{R'_i,C_{i+1}} &= \Pr(x_{ZRL} = 1, x_{ZRR} = 1 | x_{ZR} \geqslant 1), \\ &= \frac{\Pr(x_{ZR} \geqslant 1 | x_{ZRL} = 1, x_{ZRR} = 1) \Pr(x_{ZRL} = 1, x_{ZRR} = 1)}{\Pr(x_{ZR} \geqslant 1)}, \\ &= \frac{1.\Pr(x_{ZRL} = 1)\Pr(x_{ZRR} = 1)}{\Pr(x_{ZR} \geqslant 1)}, \\ &= \frac{G_{i+1}^2 e^{-2G_{i+1}}}{1 - e^{-G_i}}, \\ P_{R'_i,C_{i+1}} &= \frac{\frac{G_i^2}{4} e^{-G_i}}{1 - e^{-G_i}}. \end{aligned} \tag{70}$$

From (69) and (70), we obtain

$$\begin{aligned} P_{R'_i,L_{i+1}} &= 1 - P_{R'_i,R_0} - P_{R'_i,C_{i+1}}, \\ P_{R'_i,L_{i+1}} &= \frac{1 - \left(1 + G_i + \frac{G_i^2}{4}\right) e^{-G_i}}{1 - e^{-G_i}}. \end{aligned} \tag{71}$$

## Appendix E
## Proofs of Limiting Transition Probabilities

According to L'Hôpital's Rule, if $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ are both zero or are both $\pm\infty$ and, if $\lim_{x \to c} \frac{f(x)}{g(x)}$ has a finite value or if the limit is $\pm\infty$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}. \tag{72}$$

We will employ L'Hôpital's Rule to prove (39) in this appendix.

The proofs of (40) - (51) are similar to those of (39) and omitted for brevity.

### A. *Proof of (39)*

*Proof:* In (18), substitute $G_i = x$. From (7), $G_{i-1} = 2G_i = 2x$. As $i \to \infty$, $G_i = 2^{-i}\lambda\phi_0 \to 0$. Thus, using L'Hôpital's Rule successively, we obtain

$$\begin{aligned}
\lim_{i\to\infty} P_{L'_i,R'_i} &= \lim_{x\to 0} \frac{(1-e^{-x})xe^{-x}}{1-(1+2x)e^{-2x}}, \\
&= \lim_{x\to 0} \frac{\frac{d}{dx}(xe^{-x}-xe^{-2x})}{\frac{d}{dx}(1-e^{-2x}-2xe^{-2x})}, \\
&= \lim_{x\to 0} \frac{e^{-x}+xe^{-x}-e^{-2x}}{4xe^{-2x}}, \\
&= \lim_{x\to 0} \frac{\frac{d}{dx}(e^{-x}+xe^{-x}-e^{-2x})}{\frac{d}{dx}(4xe^{-2x})}, \\
&= \lim_{x\to 0} \frac{2e^{-2x}-xe^{-x}}{4e^{-2x}-8xe^{-2x}}, \\
\lim_{i\to\infty} P_{L'_i,R'_i} &= \frac{1}{2}.
\end{aligned}$$

■

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