Efficient Scheduling under Fading Channels

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Outline

Multipath Propagation

- Multiple copies of the same signal reach the receiver at different times and phases.

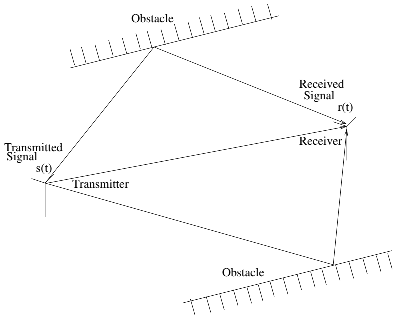

Figure: Multipath propagation

Multipath Propagation cont'd..

- $r(t)$ is a vector addition of all the received copies.
- The obstacles can move around, this leads to time varying received signal at the receiver.
- The channel variations are modeled as a multiplicative term $\gamma(t)$ [1],

$$r(t) = \gamma(t) * s(t) + n(t) \tag{1}$$

- $|\gamma(t)|$ is modeled as a random variable, typically with a *Rayleigh* or a *Ricean* PDF.

Rate-Power Relationship

- Transmissions under "good" channel conditions require much less energy than under "bad" channel conditions for the same BER at the receiver.
- Rate-power relationship is *convex*.

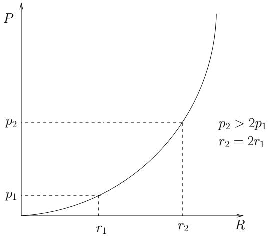

Figure: Rate-Power relationship

Energy Efficiency

- Energy efficiency is a primary concern while dealing with wireless devices.
- The rate-power relationship presents us with two kinds of strategies for saving energy,
  - Transmit when the channel is "good" [2].
  - For a delay constrained problem, transmit at rates just sufficient to meet the delay requirements, i.e. *lazy* schedules [3],[4],[5].
  - There is an *energy* vs. *delay* tradeoff.

"Perfect" power control

- Consider uplink transmissions in a single-cell communication system, with "perfect" power control.
- "Perfect" power control,
  - tries to *invert* the path-loss and fading effects of the channel.
  - transforms the channel into a Gaussian multi-user channel, whose capacity is known.

A "new" power control scheme

Fading as an opportunity

The new power control scheme in [6],

- allows the user with the largest instantaneous power to transmit in any time slot.
- works in exactly opposite sense to conventional power control.
- can be interpreted as an access control/scheduling scheme.
- allocates a user more/less power when its channel is relatively "good"/"bad".
- achieves higher capacity by making use of the channel fading to its advantage.

"Opportunistic" Scheduling

- Opportunistic scheduling exploits multiuser diversity and is effective when
  - there are large number of users.
  - users channel conditions are *diverse*, so that in each time slot there is atleast one relatively "strong" user.
  - fading occurs at a reasonable pace.
- Opportunistic schedulers may introduce fairness issues by favouring "stronger" users [7].
- To make the scheme effective, artificial diversity might be introduced by beam-forming mechanisms [1].

Fair Opportunistic Scheduling

- Fair scheduling problem is a resource allocation problem.
- Types of fairness
  - Long term fairness, time as the resource considered in [7].
  - Short term fairness, time as the resource considered in [8].
  - Long term fairness, power as the resource considered in [9].

Problem setting

What are we dealing with?

- We consider a cellular system.
- Users connect to a base station to receive and transmit *data*.
- User might impose Quality of Service (QoS) requirements like
  - *rate*
  - *delay*
  - *fairness*
- The users are mobile and are *power controlled* by the base station.

Energy efficient opportunistic scheduler

- Can we exploit the convex rate-power relationship to conserve energy?
- This has to be done while satisfying the rate/delay guarantees specified by the users.

Thus our problem becomes, design a scheduler such that it,

- opportunistically schedules the strongest user in a time slot.
- also determines the optimal transmission energy just sufficient to satisfy the rate/delay guarantees.

Rate constraints in a Nutshell [10]

- Design an energy efficient, opportunistic scheduler.
- The scheduler satisfies the rate guarantees given to the users.

System Model

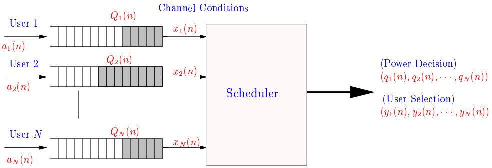

Figure: Single hop system model

We consider

- Slotted single-hop TDMA system

We assume that

- Scheduler has perfect channel state information
- Channel process is ergodic, IID.

Formulation as an optimization problem

- Minimize average power

$$\text{Minimize} \quad \limsup_{M\to\infty} \frac{1}{M}\sum_{n=1}^{M} q(n),$$

- Subject to average rate constraints C_i

$$\begin{aligned} \liminf_{M\to\infty} \frac{1}{M}\sum_{n=1}^{M}\sum_{i=1}^{N} U_i(q_i(n), x_i(n))) &\geq C_i \quad \forall i, \\ q(n) &\geq 0, \\ \sum_{i=1}^{N} y_i(n) &\leq 1 \quad \forall n \end{aligned} \tag{2}$$

  - U is concave differentiable function of x_i, q_i

    $U = \log(1 + x_i q_i)$
  - x and y are the channel gain and indicator vectors.

Multiuser Optimal Solution

Theorem

Optimal Policy for multiple users is to select k^{th} user and transmit with power $q_i^* = \left(\lambda_i - \frac{1}{x_i}\right)^+$.

Proof.

Sketch of Proof

- Use ergodicity of $x_i(n)$.
- Consider Lagrangian associated with (2).
- Minimize w.r.t. q first, then w.r.t. y.

□

Multiuser Optimal Solution

Proof.

Cont'd..

- Optimal power for single user,

$$q_i^* = \left(\lambda_i - \frac{1}{x_i}\right)^+, \text{where } \lambda \text{ is the Lagrange multiplier.}$$

- Minimizing w.r.t. y,

$$k = \arg\min_i \left(q_i^* - \lambda_i \left[\log(1 + q_i^* x_i) - C_i\right]\right)$$

□

Stochastic Approximation based Online Algorithm

- Estimate λ_i online

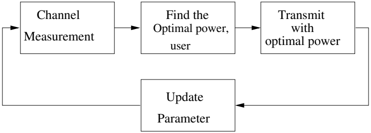

Figure: Block diagram for on-line policy

- Update Equation

$$\lambda_i(n+1) = \{\lambda_i(n) - \epsilon(n)\underbrace{[y_i(n)\log\left(1+\left(\lambda_i(n)-\frac{1}{x_i(n)}\right)^+ x_i(n)\right)]}_{h_i(\lambda)} - C_i\}^+ \quad \forall i, \tag{3}$$

View details

Simulations

- Rayleigh fading channel with parameter γ

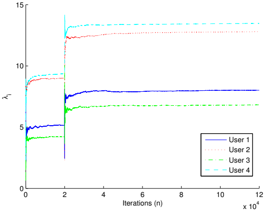

Figure: Behavior of trajectory because of addition in number of users N=4 to N=5

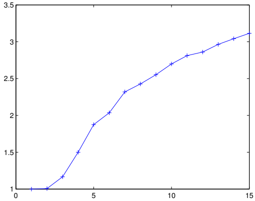

Figure: Gain of the optimal policy over variable power round robin policy, C=0.6, $\gamma = 0.7$

Delay Constraints in a Nutshell [11]

- Design an opportunistic scheduler.
- The scheduler satisfies the delay guarantees given to the users.
- To reduce the complexity of the problem, we ignore the energy minimization.

Formulation as an optimization problem

Variables

Let

- $y_i(n)$ be an indicator variable for user i in time slot n.
- $a_i(n)$ number of arrivals for user i in time slot n.
- $r_i(n)$ be the rate for user i in time slot n.
- $Q_i(n)$ be the queue length for user i in time slot n.
- Let there be N users in the system.

We want to maximize the average throughput given by,

$$T_{av}(N, \overline{D}) = \liminf_{M \to \infty} \frac{1}{M} \sum_{n=1}^{M} \sum_{i=1}^{N} y_i(n) r_i(n) \tag{4}$$

Formulation as an optimization problem
Constraints

- Using Little's law, we convert the delay constraints $\overline{D}$ into queue length constraints $\overline{Q}$.

We want to satisfy the constraints given by,

$$\limsup_{M\to\infty} \frac{1}{M} \sum_{n=1}^{M} Q_i(n) \leq \overline{Q}_i \quad i = 1, \ldots, N \tag{5}$$

Formulation as an optimization problem

The unconstrained problem

- Introduce Lagrange Multipliers (LMs), hence the problem becomes, maximize $L(\pi, \lambda)$, given by,

$$L(\pi, \lambda) = \liminf_{M \to \infty} \frac{1}{M} \sum_{n=1}^{M} \sum_{i=1}^{N} [y_i(n) r_i(n) - \lambda_i Q_i(n)] \qquad (6)$$

  where π is the policy.
- The objective is to find the *saddle* point of this Lagrangian function.
- π forms the primal variable while λ forms the dual variable.
- We use primal-dual approaches for solving the problem.

The Buffer evolution equation

- The buffer evolution equation for user i can be written as

$$Q_i(n+1) = Q_i(n) - y_i(n)r_i(n) + a_i(n) \tag{7}$$

  where the convention regarding time is shown in figure.
- We make use of the buffer evolution equation to track the average queue lengths of the users.

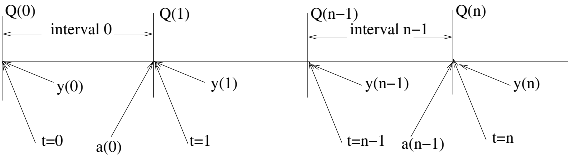

Figure: Buffer Evolution equation

Solution Methodologies

- The problem stated above is a Markov Decision Problem.
- Finding the optimal policy using value iteration has very high computational complexity.
- We suggest heuristic policies to solve the problem.

Heuristic Policy

- Intuitively, some weighted combination of queue length and channel rate should decide the user who is scheduled.
- We suggest policies of the type, Schedule a user j such that

$$j = \arg\max_i \{Q_i + \theta_i r_i\} \quad (8)$$

An approach based on parameterized policy iterations [12]

- We try to find out the best policy from within a subset of policies described by a parameter θ.
- The transition probabilities and reward functions are dependent on parameter θ.
- Start with an initial policy based on some initial value of θ.
- Improve the policy by improving the value of θ in the direction of gradient of the reward function.
- At the same time, adjust the LMs so that the resultant policy is constraint satisfying.

View details

Summary

We have looked at

- Use of Opportunism and energy efficiency while designing scheduling algorithms.
- Providing rate and delay guarantees to the users.

Future work includes

- Designing proper admission control schemes for such scheduling algorithms.
- Extending the work to OFDM channels.
- Designing a *distributed* scheduler based on this framework.

References I

[1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*.
Cambridge University Press, 2005.

[2] B. E. Collins and R. L. Cruz, “Transmission policies for time varying channels with average delay constraints,” in *Proc. 1999 Allerton Conf. on Commun., Control, and Comp.*, (Monticello, IL, USA), 1999.

[3] B. Prabhakar, E. U. Biyikoglu, and A. E. Gamal, “Energy-efficient transmission over a wireless link via lazy packet scheduling,” in *Proc. IEEE INFOCOM’2001*, vol. 1, pp. 386–394, Apr. 2001.

[4] R. A. Berry and R. G. Gallager, “Communication over fading channels with delay constraints,” *IEEE Trans. Inform. Theory*, vol. 48, May 2002.

[5] D. Rajan, A. Sabharwal, and B. Aazhang, “Delay-bounded packet scheduling of bursty traffic over wireless channels,” *IEEE Trans. Inform. Theory*, vol. 50, pp. 125–144, Jan. 2004.

[6] R. Knopp and P. A. Humblet, “Information capacity and power control in single-cell multiuser communications,” in *Proc. IEEE ICC’1995)*, vol. 1, (Seattle, USA), pp. 331–335, June 1995.

[7] X. Liu, E. Chong, and N. Shroff, “Opportunistic transmission scheduling with resource-sharing constraints in wireless networks,” *IEEE J. Select. Areas Commun.*, vol. 19, pp. 2053–2064, Oct. 2001.

[8] S. Kulkarni and C. Rosenberg, “Opportunistic scheduling policies for wireless systems with short term fairness constraints,” in *Proc. IEEE GLOBECOM’2003*, vol. 1, pp. 533–537, Dec. 2003.

[9] Y. Liu and E. Knightly, “Opportunistic fair scheduling over multiple wireless channels,” in *Proc. IEEE INFOCOM’2003*, vol. 2, (San Francisco, USA), pp. 1106–1115, Mar. 2003.

References II

[10] A. Bhorkar, A. Karandikar, and V. Borkar, "Power optimal opportunistic scheduling," submitted to IEEE Communications Letters.

[11] N. Salodkar, A. Karandikar, and V. Borkar, "Efficient scheduling in fading channels," work in progress.

[12] P. Marbach and J. N. Tsitsiklis, "Simulation-based optimization of markov reward processes," *IEEE Trans. Automat. Contr.*, vol. 46, no. 2, pp. 191–209, 2001.

Thank you.

Optimality and Stability of Update Equation 3

- After minimizing over the primal variables q,y , optimal value of Lagrangian is,

$$F(\lambda) = [E(\min_i(q_i^* - \lambda_i \log(1 + q_i^* x_i(n)) - C_i)]$$

- $F(\lambda)$ strictly concave $\rightarrow$ unique maximum
- Need to find saddle point
- Iterations 3, a supergradient ascent scheme, converge to the differential inclusion a.s.

$$\dot{\lambda}(t) \in \partial F(\lambda(t)) + z(t)$$

  ∂F supergradient of F

  $z(t)$ boundary correction term
-

$$E[h_i(\lambda)] \in \partial F(\lambda(t)) + z(t)$$

  Hence the iterations converge to the optimal value.

Back

An approach based on parameterized policy iterations

$$
\begin{aligned}
p_{ij}(\theta) &= P(i_n = j | i_{n-1} = i, \theta) \\
L(\theta) &= \liminf_{M \to \infty} \frac{1}{M} \sum_{n=0}^{M-1} g(i_n, \theta) \\
\theta(n+1) &= \theta(n) + b(n) * h(n) \\
J_{n+1}(i, \theta(n)) &= J_n(i, \theta(n)) + a(n) * \\
\{g(i, \theta(n)) - J_n(i, \theta(n)) &- J_n(i_0, \theta(n)) + J_n(i_{n+1}, \theta(n))\} \\
\lambda(n+1) &= [\lambda(n) + c(n)(x(n) - \bar{x})]^+
\end{aligned} \tag{9}
$$

where $h(n)$ is an estimate of $\nabla_\theta(J(.,\theta(n),n))$ and $a(n), b(n)$ and $c(n)$ are positive step size sequences.

Back