Delay Constrained Scheduling over Wireless Channels

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Outline

- Wireless Channel, Efficient Cross Layer Design
- Providing QoS over Wireless Channel
- Uplink (Multiuser) Problem
- Point-to-Point (Single user) Analysis
- Uplink Solution
- Implementation Aspects
- Experimental Evaluation
- Conclusions and Future Work

Wireless Channel Characteristics

- Wireless Channel is characterized by
  - Signal strength variation over time, frequency and space
    - Small scale variation (Fading)
  - Limited battery life at hosts
- Physical Layer no longer a fixed rate bit pipe
- Resource allocation needs to take channel characteristics into account

Leads to Cross Layer View

Wireless Channel Characteristics

- Wireless Channel is characterized by
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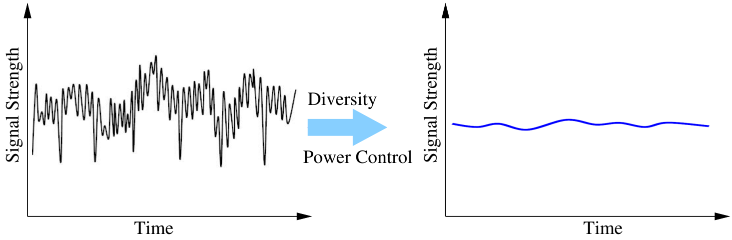
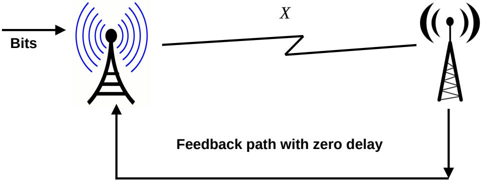
Leads to Cross Layer View

Traditional Approach to Counter Fading



- Diversity
- Channel inversion through power control

Capacity of Fading Channel-Point to Point



- Fading channel capacity with average power constraints,

$$C(\bar{P}) = \max \mathbf{E}\left[\log_2\left(1 + \frac{P(X)X}{N_0}\right)\right]$$

subject to,

$$\mathbf{E}\left[P(X)\right] \leq \bar{P}$$

$\bar{P}$=average transmit power, P= instantaneous transmit power, X= channel state

- Constrained optimization problem

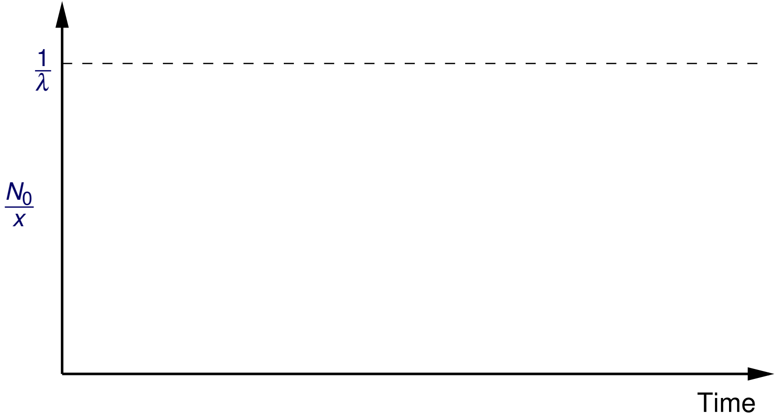
Solution

- With perfect CSI at the transmitter, Capacity maximized by water filling power allocation

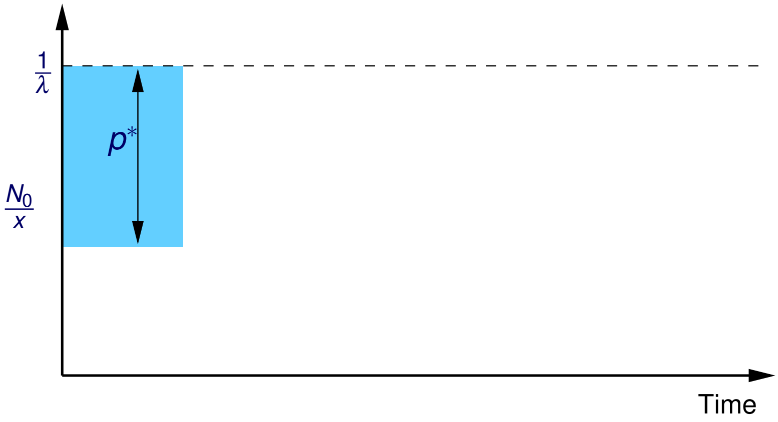
$$P^*(\gamma) = \left(\frac{1}{\lambda} - \frac{N_0}{X}\right)^+$$

- λ chosen to satisfy the average power constraint

Power Allocation - “Water Filling”



Power Allocation - “Water Filling”



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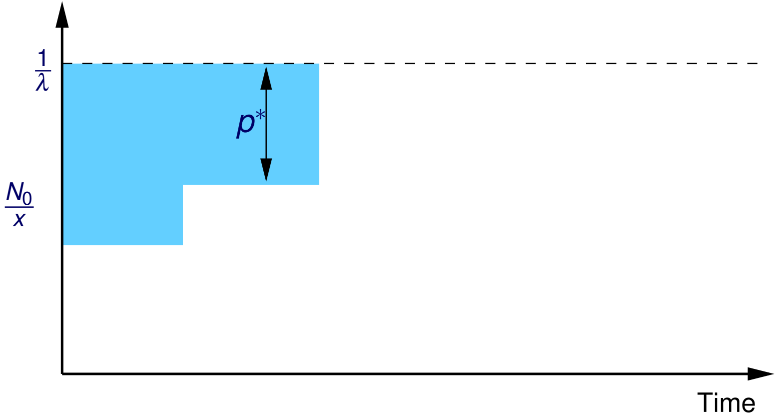
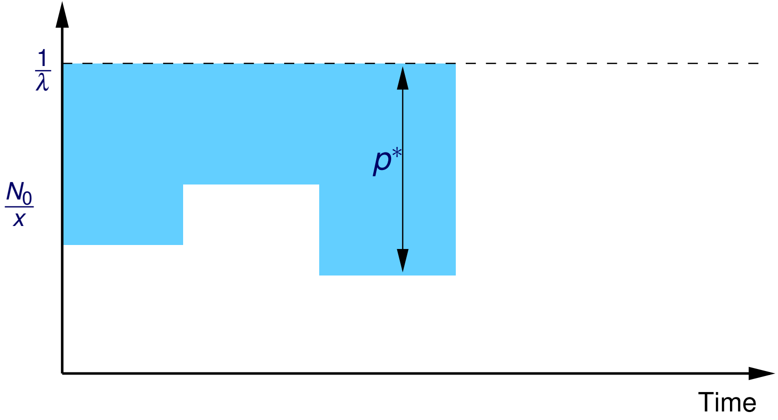


Power Allocation - "Water Filling"



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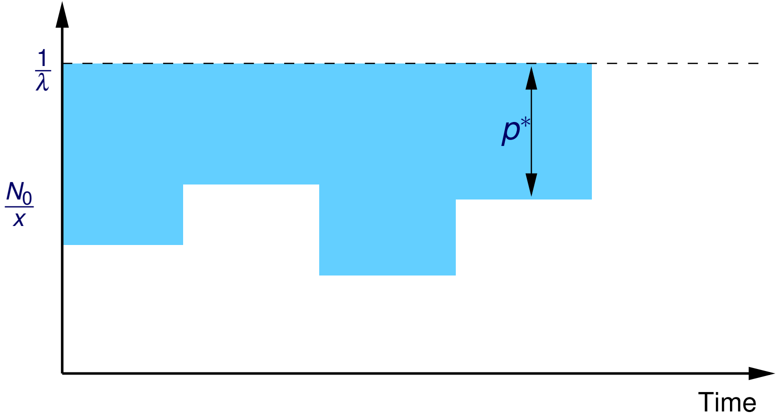
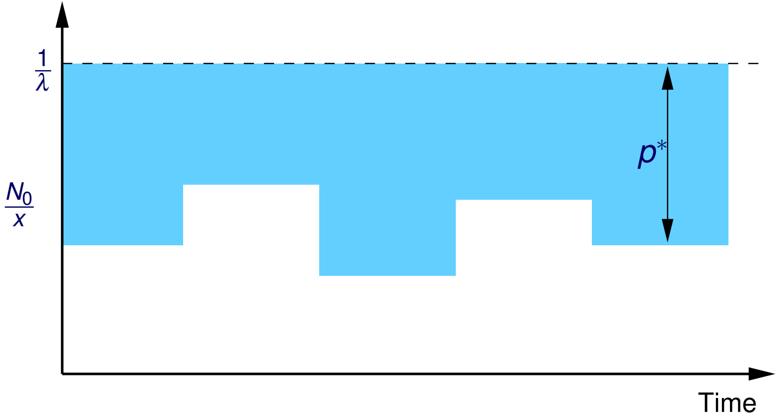
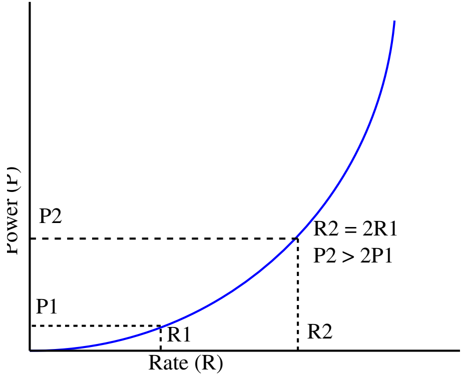


Power Allocation - "Water Filling"



Energy Efficiency

- Rate-Power relationship is convex



Power Allocation - Multi-user Case

- Symmetric users
  - Same fading statistics
  - Equal average power constraints
- Solution - TDMA
  - Allow the user with the best channel condition to transmit-Opportunistic Scheduling
  - Transmit power determined by single user waterfilling power allocation

Lessons Learnt

- Transmit at higher power when channel is good
- Do not transmit when channel is poor -leads to Queuing Delays
- Allow the user with the best channel to transmit
- Transmit at lower rates to conserve power - leads to Queuuing Delays

Delay Constrained Scheduling Algorithms

- Average delay important for *data centric* applications such as `ftp` and `http` transfers
- Our objective: Design cross layer algorithm for providing average delay guarantees over fading channels
- Cross layer problems can be formulated as control problems where:
    - Scheduler is the controller
    - Objective is to minimize energy subject to delay
- MDP approach is a well known approach for addressing these problems

Issues with MDP Approach

- MDP solution techniques
  - Computationally infeasible for large problem dimensionality
  - Assume a knowledge of the underlying Markov chain which depends on knowledge of:
    - Channel statistics
    - Arrival statistics

Issues with MDP Approach Contd...

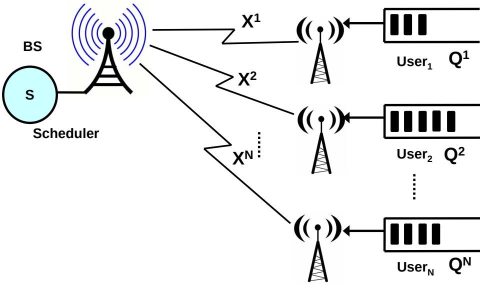
- Exact knowledge of channel statistics difficult to possess in practice
  - Location and topology dependent
  - Models like Rayleigh/Ricean model available
    - Accuracy?
- Knowledge of arrival statistics also difficult to possess
- Performance of schemes developed under assumed system model limited by the modeling assumptions

Summary of our Work

- Problem: For each user on the uplink, minimize average power subject to average delay constraint
- Key Contributions:
    - Design of an algorithm for addressing the above problem that
        - Does not need the knowledge of system model
        - Is computationally efficient
    - Analysis of proposed algorithm
    - Simulation analysis to demonstrate practical utility of the algorithm

System Model

- Single receiver (Base Station) and multiple transmitters
- Base Station is the centralized scheduler



Problem Formulation

- Queue transition, average queue length, average power for user i

$$
\begin{aligned}
Q^i_{n+1} &= Q^i_n - I^i_n R^i_n + A^i_{n+1}, \;\; R^i_n \leq Q^i_n \\
\bar{Q}^i &= \limsup_{M\to\infty} \frac{1}{M}\left[\sum_{n=1}^{M} Q^i_n\right] \\
\bar{P}^i &= \limsup_{M\to\infty} \frac{1}{M}\left[\sum_{n=1}^{M} P(X^i_n, I^i_n R^i_n)\right]
\end{aligned}
$$

- Problem: Minimize the power consumption of each user subject to delay constraint of each user

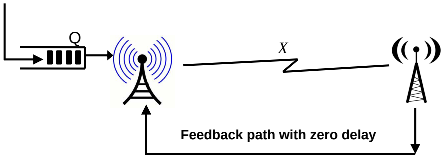
$$
\text{Minimize } \bar{P}^i \text{ subject to } \bar{Q}^i \leq \bar{\delta}^i, \; i = 1, \ldots, N \qquad (1)
$$

- Multi-objective optimization problem
- A related problem - minimize a weighted sum of power expenditures
  - Can be formulated as a CMDP with state $\mathbf{S}_n = [\mathbf{Q}_n, \mathbf{X}_n]$

Key Issue in Determining an Optimal Solution and Contribution

- State space large even for moderate number of users
  - Example: A system with 4 users, buffer of 50 packets and 4 channel states has $\sim 10^{10}$ states
  - Prohibits use of CMDP solution techniques based on LP
  - RL over the state space would take prohibitively long time to converge to optimal values
- Contribution: Novel use of single user algorithm for obtaining multiuser solution

Point-to-Point System Model and Problem Formulation



Problem: Minimize average power subject to average delay (queue length) constraint, i.e.,

$$\text{Minimize} \quad \bar{P} \quad \text{subject to} \quad \bar{Q} \leq \bar{\delta}$$

Formulation as a CMDP

- Well known that problem has the structure of a CMDP with state $S_n = (Q_n, X_n)$
- Define two costs:
  - Power cost $c_p(S_n, U_n) \triangleq P(X_n, U_n)$
  - Queue cost $c_q(S_n, U_n) \triangleq Q_n$
- Under a randomized policy μ,

$$\bar{P}^{\mu} \triangleq \mathbf{E}^{\mu}\left[c_p(S_n, \mu(S_n))\right]$$

$$\bar{Q}^{\mu} \triangleq \mathbf{E}^{\mu}\left[c_q(S_n, \mu(S_n))\right]$$

- Scheduler objective:

$$\text{Minimize } \bar{P}^{\mu} \text{ subject to } \bar{Q}^{\mu} \leq \bar{\delta} \quad (2)$$

Towards a Solution Technique

Key Issue: Determine an optimal solution in absence of system model

- Address the constraint using the Lagrangian approach by introducing Lagrange Multiplier (LM)
- For a particular LM λ, determine $L(\mu^*, \lambda) \Rightarrow$ unconstrained MDP
- Relative Value Iteration Algorithm (RVIA) for unconstrained MDP

$$V_{n+1}(s) = \min_{u \in U} \left\{ c(\lambda, s, u) + \sum_{s'} p(s, u, s') V_n(s') - V_n(s^0) \right\}$$

- s : System state, $s \in \mathscr{S}$
- u : Action taken, $u \in \mathscr{U}$
- c : Immediate cost
- s' : Next state, $s' \in \mathscr{S}$
- V : Value function
- $p(s, u, s')$: Transition kernel
- s^0 : fixed state, $s^0 \in \mathscr{S}$

The Post Decision State

- Introduce a virtual state: post decision state
  - System state just after taking an action but just before the influence of noise
  - Example: System state just after transmitting the packets but before the arrivals
- Let $\tilde{S} = (\tilde{Q}, \tilde{X})$ denote the post decision state

Reformulation of RVIA based on Post Decision State

- ζ : the unknown law for arrivals
- $\kappa(x'|x)$: the unknown law for channel state
- RVIA based on post decision state,

$$\begin{aligned}\tilde{V}_{n+1}(\tilde{s}) &= \sum_{a,x'} \zeta(a)\kappa(x'|x)(\min_{u \le q+a}[c(\lambda,(q+a,x'),u)+\tilde{V}((q+a-u,x'))]) - \\ &\quad \tilde{V}_n(\tilde{s}^0) \\ \tilde{V}_{n+1}(\tilde{s}'') &= \tilde{V}_n(\tilde{s}'') \ \forall\, \tilde{s}'' \neq \tilde{s}\end{aligned}$$

Online Algorithm

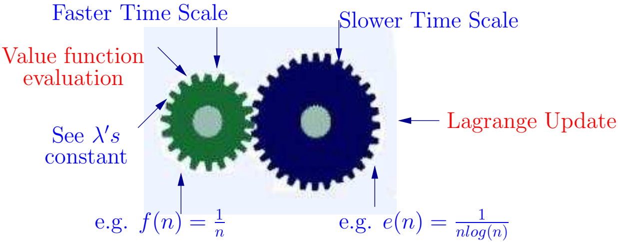
- Online RVIA employing SA:

$$
\begin{aligned}
\tilde{V}_{n+1}(\tilde{s}) &= (1-f_n)\tilde{V}_n(\tilde{s}) + f_n\Big\{\min_u [c(\lambda,(q+A_{n+1},X_{n+1}),u) \\
&\quad + \tilde{V}_n((q+A_{n+1}-u,X_{n+1}))] - \tilde{V}_n(\tilde{s}^0)\Big\}, \\
&= \tilde{V}_n(\tilde{s}) + f_n\Big\{\min_u [c(\lambda,(q+A_{n+1},X_{n+1}),u) \\
&\quad + \tilde{V}_n((q+A_{n+1}-u,X_{n+1}))] - \tilde{V}_n(\tilde{s}) - \tilde{V}_n(\tilde{s}^0)\Big\}, \\
\tilde{V}_{n+1}(\tilde{s}'') &= \tilde{V}_n(\tilde{s}'') \ \forall \ \tilde{s}'' \neq \tilde{s}
\end{aligned}
$$

- Dual evaluation on a slower time scale:

$$\lambda_{n+1} = \Lambda[\lambda_n + e_n\left(Q_n - \bar{\delta}\right)]$$

Two Time Scale Update



Value function update: Faster time scale

$$\begin{aligned}\tilde{V}_{n+1}(\tilde{s}) &= \tilde{V}_n(\tilde{s}) + f_n\Big\{\min_u[c(\lambda_n,(q+A_{n+1},X_{n+1}),u) \\ &\quad + \tilde{V}_n((q+A_{n+1}-u,X_{n+1}))] - \tilde{V}_n(\tilde{s}) - \tilde{V}_n(\tilde{s}^0)\Big\}\end{aligned}$$

Lagrangian update: Slower time scale

$$\lambda_{n+1} = \Lambda[\lambda_n + e_n\left(Q_n - \bar{\delta}\right)]$$

Convergence Analysis

Theorem

For the online algorithm, the value function and LM iterates converge to their optimal values, i.e., $(\tilde{V}_n, \lambda_n) \rightarrow (\tilde{V}, \lambda^*)$

Proof.

- For $\lambda_n = \lambda$, $\tilde{V}_n^\lambda \rightarrow \tilde{V}^\lambda$
- The value function iterates $\tilde{V}_n$ remain bounded a.s.
- $\tilde{V}_n - \tilde{V}^{\lambda_n} \rightarrow 0$ a.s.
  - This follows from two time scale SA theory
    - The value function iterates perceive the LM iterates as almost constant
    - The LM iterates perceive the value function iterates as equilibriated
- The λ_n iterates are bounded
- The coupled iterates converge to their respective optimal values
  - This follows from the envelop theorem

Towards an Uplink Solution Technique

- Consider the uplink scenario with power minimization objective modified as:

$$\text{Minimize } \bar{P}^i = \limsup_{M\to\infty} \frac{1}{M}\left[\sum_{n=1}^{M} P(X_n^i, R_n^i)\right] \quad (3)$$

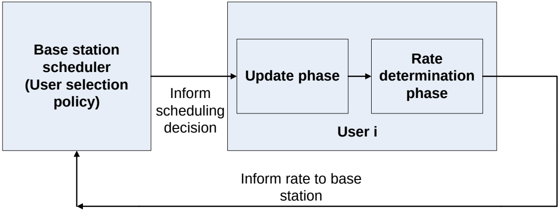
- Queue transition same as that of the original uplink scenario
- Employ Point-to-Point solution technique
- Scheduler determines the transmission rate using Point-to-Point algorithm:

$$\begin{aligned} r_{n+1}^i &= \arg\min_{v\in\mathscr{F}_{n+1}} \Big\{(1-f_n)\tilde{V}_n^i(\tilde{q},\tilde{x}) + f_n \times \{c(\lambda_n^i, \tilde{q}+A_{n+1}^i, X_{n+1}^i, v) \\ &\quad + \tilde{V}_n^i(\tilde{q}+A_{n+1}^i - v, X_{n+1}^i) - \tilde{V}_n^i(\tilde{q}^0, \tilde{x}^0)\}\Big\} \quad (4) \end{aligned}$$

- Value function and LM update same as that for the Point-to-Point scenario

Uplink Solution

- Visualize a link between user and base station as a Point-to-Point scenario
- Each user
  - Determines its transmission rate using the Point-to-Point algorithm
  - Informs this rate to the base station
- The base station schedules the user with the highest rate
- Queue transitions for a user who is scheduled and not for others
- Value function and LM at each user are appropriately updated



Uplink Solution - Auction Interpretation

- The uplink solution strategy can be interpreted as an auction
- The base station auctions each time slot
- The user quoting the highest rate wins the bid
- Users quote rates that are just sufficient to satisfy their delay constraints
- Quoting unnecessarily higher rates not favorable since power minimization is the objective

Uplink Solution (Auction Algorithm) - Properties

- Satisfies delay constraints of users
- Does not need knowledge of system model
- Low communication overhead
- Linear time complexity in the number of users

Convergence Analysis

Theorem

For the Auction Algorithm (rate determination part), assuming stability of queues, the value function and LM iterates for each user converge to their equilibrium values, i.e., $(\tilde{V}_n^i, \lambda_n^i) \rightarrow (\tilde{V}^i, \lambda^i)$*. This implies that the delay constraints are satisfied.*

Proof.

- Assume stability of queues under the closed loop scheme.
- Analyze convergence of value function for an almost constant value of LM for each user.
- Value function of each user is updated in each slot regardless of scheduling decision, it is decoupled across users.
- Finally, prove LMs and coupled iterates converge – implies that delay constraints are satisfied.

Practical Implementation in WiMAX

- Assumptions: TDD and Single carrier system; channel remains constant for the duration of a frame
- Scheduling done on a frame-by-frame basis
- Schedule informed to the users using Downlink Map (DL-MAP) and Uplink Map (UL-MAP)
- Ranging Request (RNG-REQ) message for conveying CSI for downlink transmissions
- Base station can use Channel Measurement Report Request (REP-REQ) to obtain CSI
- Mobile station can respond using Channel Measurement Report Response (REP-RSP) messages

Simulation Setup

- Simulation within IEEE 802.16 framework with 20 users
- Divide users into 2 groups - Group 1 and Group 2 of 10 users each
- M-Pareto arrival process - (shape factor - 1.2, mode - 2000 bits, cutoff threshold = 10000 bits)
- Average packet size = 3860 bits, fragment of size 2000 bits
- Rayleigh fading with channel divided into 8 bins and discretized

Simulation Parameters

- Fix the arrival rate at 0.3860 Mbits/sec/user
- Fix the mean of X^i at 0.9817
- Delay constraints of users in Group 1 fixed at 100 msec
- Delay constraints of users in Group 2 varied as $25 - 175$ in 7 steps

Simulation Results

Achieved Delay

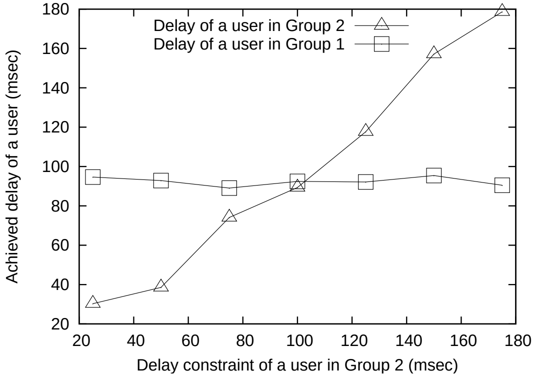


Figure: Achieved delay of a user with specified delay constraints

Simulation Results

Power Expended

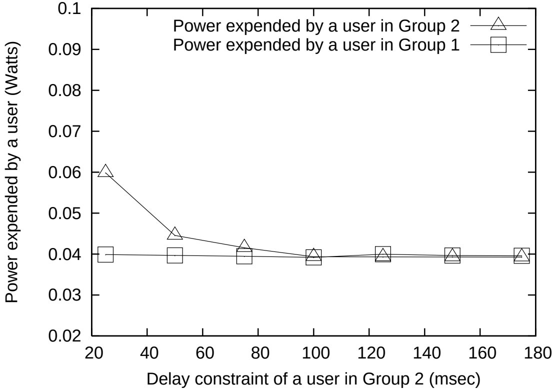


Figure: Power expended with specified delay constraints

Conclusions

Proposed a novel uplink algorithm that

- Satisfies delay constraints
- Has low communication overhead
- Has low computational complexity

Future Work

- Convergence rate analysis for the algorithm
- Implementation of the algorithm in actual testbeds and tuning the parameters
- Extension to the multihop case
- Other QoS considerations

Thank you