Power Optimal Opportunistic Scheduling

Abhijeet Bhorkar, Abhay Karandikar
Department of Electrical Engineering,
Indian Institute of Technology - Bombay,
Powai, Mumbai 400076, India.
{bhorkar, karandi}@ee.iitb.ac.in

Vivek S. Borkar
School of Technology and Computer Science,
Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400005, India.
borkar@tifr.res.in

Abstract— **In this paper, we propose a power optimal opportunistic scheduling scheme for a multiuser single hop Time Division Multiple Access (TDMA) system. We formulate the problem of minimizing average transmission power with different minimum information theoretic rate constraints for individual users. We suggest a stochastic approximation based scheme to implement the policy and argue the convergence of this algorithm. Finally, we extend the power optimal scheduling algorithm for providing temporal fairness among users.**

I. Introduction

Wireless channels exhibit time varying fading characteristics, which vary from user to user. This multiuser diversity can be exploited by *opportunistically* scheduling the user with the best channel condition. Multiuser diversity has been explored in the pioneering work of Knopp and Humblet [1], where the problem of maximizing the information capacity of the uplink in a single cell environment under an average power constraint has been addressed. Opportunistic scheduling, however, introduces the issue of fairness among users. Proportional fairness in multiuser diversity has been investigated in [2].

In wireless systems, battery and transmission power constraints mandate conservative energy expenditure during transmission. Hence, resource allocation policies have to optimize energy resources subject to Quality of Service (QoS) constraints like minimum rate, delay and fairness. Most of the research work in energy efficient scheduling has focused on a point to point wireless link scenario. [3] provides an overview of energy efficient scheduling under delay constraints.

In this paper, we consider energy optimal opportunistic control strategies for a multiuser TDMA system subject to minimum rate constraints. Moreover, we propose an online algorithm based on stochastic approximation and argue that this method achieves optimality. We also extend the algorithm to consider "temporal" long term fairness as well as short term fairness[1]. [4] [5] [6] have investigated opportunistic scheduling under various types of fairness constraints. However, they do not consider variation in transmission power for energy efficient scheduling.

In [7], the authors have considered an interference-based joint scheduling and power allocation scheme for a multicellular environment. Though their problem formulation is mathematically similar to ours, issues such as the convergence, optimality and stability of the iterative algorithm have not been addressed. Moreover, we have validated our algorithm for independent and identically distributed (i.i.d.) as well as Markovian channel fading. Further, our formulation also incorporates temporal fairness.

In Section II, we describe our system model and derive the optimal scheduling policy. In Section III, we describe the stochastic approximation method used to solve the joint opportunistic and power optimal scheduling problem. In Section IV, we introduce a temporal short term fair power optimal scheme, which is based on the long term fair scheduler in Section IV-A.

II. Optimal scheduling

Consider a multiuser TDMA system with the base station as the centralized scheduler. Time is divided into slots of equal duration. The channel is time varying with slow fading[2]. The channel state at the beginning of slot n is denoted by the vector $(x_1(n), x_2(n), \cdots, x_N(n))$, where $x_i(n)$ denotes the channel gain for user i at slot n and N is the number of users. We assume that the channel state (channel gain) changes only at slot boundaries and perfect channel state information (CSI).

The channel state process $(x_1(n), x_2(n), \cdots, x_N(n))$ is assumed to be $\mathbb{R}^d-$valued and ergodic with marginal distribution ν, where $N \geq d \geq 1$. Channel gains experienced by different users are i.i.d. The channel state evolution with time can be either i.i.d. or Markovian. The rate requirement of each user is known apriori at the base station.

In any given time slot, only one user is allowed to transmit. The scheduler determines the user who can transmit and its transmission power subject to that user's rate constraint. For each user i, we associate an indicator function $y_i(n)$ which is 1 if user i is scheduled at time slot n, otherwise it is 0. Let $q(n)$ be the actual transmission power of the scheduled user at time slot n. Let C_i be the time-average minimum rate requirement for user i. We consider the rate to be information theoretic rate U_i, where U_i is increasing and concave in channel gain x_i and power q and is given by $U_i(q, x_i) = \log(1 + qx_i)$. Our objective is to minimize average power subject to average rate constraints, which can be expressed as:

$$\min \limsup_{M\to\infty} \frac{1}{MN} \sum_{n=1}^{M} \sum_{i=1}^{N} q(n) y_i(n),$$

Karandikar's research is supported by Department of Science and Technology grant (05DS025) and Borkar's research is supported in part by grant (2900-IT-1) from the *Centre Franco-Indien pour la Promotion de la Recherche Avancee*.

[1] By temporal fairness, we mean that each user has access to certain number of time slots.

[2] Fading is constant over a slot duration.

$$\text{s.t.} \liminf_{M\to\infty} \frac{1}{M}\sum_{n=1}^{M} U_i(y_i(n)q_i(n), x_i(n)) \geq C_i \ \forall i. \quad (1)$$

Due to ergodicity, we focus on the transmission policy for any slot and do not explicitly state the dependence of the channel state process on time n. Let $\mathbf{x} = (x_1, \cdots, x_N)$ denote the channel state vector. $A = (\mathbf{e_1}, \cdots \mathbf{e_N})$, where $\mathbf{e_i}$ denotes the unit vector in the i^{th} coordinate direction. Let $\mathbf{y} = (y_1, \cdots, y_N)$ be the vector of indicator random variables. Note that only one of the random variables y_i will be 1 in a given time slot. Let p be the conditional law of $(q, \mathbf{y})$ given $\mathbf{x}$, which can be decomposed as $p_1(dq|\mathbf{y},\mathbf{x})p_2(\mathbf{y}|\mathbf{x})$. Thus, we can write the optimization problem (1) as:

$$\begin{aligned} &\min \int \nu(dx_1, \cdots, dx_N) \sum_{\mathbf{y}\in A} \int_{[0,\infty)} p_1(dq|\mathbf{y},\mathbf{x})p_2(\mathbf{y}|\mathbf{x})q, \\ &\text{s.t.} \int \nu(dx_1, \cdots, dx_N) \sum_{\mathbf{y}\in \mathbf{A}} \int_{[0,\infty)} p_1(dq|\mathbf{y},\mathbf{x})p_2(\mathbf{y}|\mathbf{x}) \\ &\qquad \log(1+qy_ix_i) \quad \geq \quad C_i \ \forall i, \ \ q \geq 0. \end{aligned} \quad (2)$$

Proposition 1: The optimal policy is to select user k and transmission power q^*, where

$$\begin{aligned} k &= \arg\min_i \left\{ \left(\lambda_i - \frac{1}{x_i}\right)^+ - \lambda_i \left[\log\left(1 + \left(\lambda_i - \frac{1}{x_i}\right)^+ x_i\right) - C_i \right] \right\}, \quad (3) \\ q^* &= \left(\lambda_k - \frac{1}{x_k}\right)^+, \quad (4) \end{aligned}$$

and λ_i is the Lagrange multiplier associated with the rate constraint for user i.

Proof: The Lagrangian associated with (2) is

$$\begin{aligned} f(p_1, p_2, \boldsymbol{\lambda}) &\triangleq \int \nu(dx_1, \cdots, dx_N) \sum_{\mathbf{y}\in A} \int_{[0,\infty)} p_1(dq|\mathbf{y},\mathbf{x}) \\ &\quad p_2(\mathbf{y}|\mathbf{x}) \left(q - \sum_i \lambda_i \left[\log(1+qy_ix_i) - C_i\right] \right) \end{aligned} \quad (5)$$

where $\boldsymbol{\lambda} = (\lambda_1, \cdots, \lambda_N)$. Therefore, the optimization problem decomposes into: minimize with respect to (w.r.t.) $p_1(q|\mathbf{x},\mathbf{y})$ and then minimize w.r.t. $p_2(\mathbf{y}|\mathbf{x})$. Note that the cost function $f(p_1, p_2, \boldsymbol{\lambda})$ is linear in the joint probability distribution when the marginal distribution of $\mathbf{x}$ is fixed and the minimization is over the conditional distributions. The set of probability distributions with a fixed marginal is a closed convex set with extreme points corresponding to those distributions for which the conditional distributions are point masses [8]. Thus for each $\mathbf{x}$, we minimize over q and $\mathbf{y}$. The Lagrangian (5) is strictly convex w.r.t. q and $\mathbf{y}$ and hence the minimizer is unique. Since joint minimization over q and $\mathbf{y}$ can be done in any order, we minimize first with respect to q and then w.r.t. $\mathbf{y}$. Thus we first minimize (5) w.r.t. q for a fixed i which corresponds to $\mathbf{y} = \mathbf{e}_i$. The reduced single user min-max problem is:

$$\max_{\lambda_i} \min_q \mathcal{L}(\lambda_i, q) \quad (6)$$

where $\mathcal{L}(\lambda_i, q) = q - \lambda_i(\log(1+qx_i) - C_i)$. Denote the optimal q for $\mathbf{y} = \mathbf{e}_i$ by q_i. To minimize (6) w.r.t. q, we differentiate $\mathcal{L}(\lambda_i, q)$ w.r.t. q,

$$\frac{\partial \mathcal{L}}{\partial q} = 1 - \lambda_i \left(\frac{x_i}{1+qx_i}\right), \quad (7)$$

leading, by the Kuhn-Tucker theorem [9], to

$$q_i = \left(\lambda_i - \frac{1}{x_i}\right)^+. \quad (8)$$

Minimizing (5) w.r.t. $\mathbf{y}$ yields the optimal policy,

$$\begin{aligned} k &= \arg\min_i \left\{ q_i - \lambda_i \left[\log(1+q_ix_i) - C_i\right] \right\} \\ &= \arg\min_i \left\{ \left(\lambda_i - \frac{1}{x_i}\right)^+ - \lambda_i \left[\log\left(1 + \left(\lambda_i - \frac{1}{x_i}\right)^+ x_i\right) - C_i \right] \right\}. \quad (9) \end{aligned}$$

The optimal policy is to schedule user k which satisfies (9). The scheduled user will transmit with power $q^* = q_k$ as given in (8). ■

III. Stochastic Approximation based Online Algorithm

In this section, we propose a stochastic approximation based online optimal algorithm to estimate parameters $\boldsymbol{\lambda}$ of the optimal policy and show that the algorithm converges to the optimal $\boldsymbol{\lambda}$ with probability (w.p.) 1.

After minimizing (5) over $(q, \mathbf{y})$ as in Section II, we maximize over $\boldsymbol{\lambda}$ to obtain the optimal solution. The stochastic gradient ascent scheme for maximization over $\boldsymbol{\lambda}$ is given by,

$$\lambda_i(n+1) = \Gamma\left(\lambda_i(n) - a(n)\left[y_i(n)\log\left(1 + \left(\lambda_i - \frac{1}{x_i(n)}\right)^+ x_i(n)\right)\right] - C_i\right) \forall i \quad (10)$$

where[3]:

1) $y_i(n) = I(q_i^* - \lambda_i\left[\log(1+q_i^*x_i) - C_i\right] \leq (q_j^* - \lambda_j[\log(1+q_j^*x_j) - C_j]), j \neq i$.
2) $a(n)$ is a positive scalar sequence satisfying [10],
$$\sum_n a(n) = \infty, \quad \sum_n a(n)^2 < \infty,$$
3) $\Gamma(\cdot)$ is the projection to the set $[0, L]$ where $L \geq 0$ is a very large but finite number, i.e., $\Gamma(x) = \max(0, \min(x, L))$.

[3] $I(a \leq b) = 1$ if $a \leq b$, $= 0$ otherwise.

4) We take $a(n) = \frac{l}{n}$, where l, the initial learning rate, is a small constant.

Note that we have assumed the transmission power q to be unconstrained. However, if we impose a constraint $q \leq q_{max}$ for a prescribed $q_{max} < \infty$, then we can replace q^* by $\hat{q}^* = q^* \wedge q_{max}$[4].

In [4] [5] [6], the authors have used stochastic approximation algorithm, but the convergence proof is not discussed, also the technical proof involved in these algorithms is simple because of the differentiable functions involved in the algorithm. We now sketch the proof of convergence for the stochastic approximation scheme as outlined in (10). The details, though highly technical, are routine [11] and will be omitted here due to lack of space. We consider the channel state process to be i.i.d. across slots. The proof of convergence for the Markovian model is along similar lines.

Let $\tilde{y}_i(n) = y_i(n)$ with $\lambda_i(n)$ replaced by λ_i and $E_s[\,\cdot\,]$ denote the stationary expectation. Rewrite iteration (10) as,

$$\lambda_i(n+1) = \Gamma\left(\lambda_i(n) + a(n)\left[h_i\left(\boldsymbol{\lambda}(n)\right) + M_i(n+1)\right]\right),$$

where,

$$\begin{aligned} h_i(\boldsymbol{\lambda}(n)) &= E_s\Bigg[\tilde{y}_i(n)\bigg(\log\Big(1+\Big(\lambda_i - \frac{1}{x_i(n)}\Big)^+ x_i(n)\Big) \\ &\quad -C_i\bigg)\Bigg] \,|_{\lambda_i = \lambda_i(n)}, \\ M_i(n+1) &= y_i(n)\log\left(1+\left(\lambda_i(n) - \frac{1}{x_i(n)}\right)^+ x_i(n)\right) \\ &\quad -C_i - h_i\left(\boldsymbol{\lambda}(n)\right). \end{aligned}$$

This iteration will converge with probability 1 to an invariant set of the differential equation [10]

$$\dot{\boldsymbol{\lambda}}(t) = h(\boldsymbol{\lambda}(t)) + \mathbf{z}(t), \tag{11}$$

where $h(\cdot) = [h_1(\cdot), \cdots, h_N(\cdots)]$ and $\mathbf{z}(t) = [z_1(t), \cdots, z_N(t)]$ is the boundary correction term due to the projection operator Γ [10]. Note that $h_i(\boldsymbol{\lambda}) \in \partial F(\boldsymbol{\lambda})$, where,

$$\begin{aligned} F(\boldsymbol{\lambda}) &= E_s\Bigg[\min_i\bigg\{\Big(\lambda_i - \frac{1}{x_i(n)}\Big)^+ - \lambda_i\bigg(\log\Big(\\ &\quad 1+\Big(\lambda_i - \frac{1}{x_i(n)}\Big)^+ x_i(n)\Big) - C_i\bigg)\bigg\}\Bigg] \end{aligned} \tag{12}$$

is the point-wise minimum of a family of affine functions of $\boldsymbol{\lambda}$ and is a strictly concave function of $\boldsymbol{\lambda}$. ∂F denotes its superdifferential. This can be verified by invoking the recent extensions of the envelope theorem [12] [13]. Thus the ordinary differential equation (11) may be viewed as the differential inclusion

$$\dot{\boldsymbol{\lambda}}(t) \in \partial F(\boldsymbol{\lambda}(t)) + \mathbf{z}(t).$$

This is a supergradient ascent scheme for a strictly concave function and thus will converge to its unique maximum on the constraint set. If L is sufficiently large, this will be the desired vector of Lagrange multipliers by the saddle point theorem [9]. Thus, the iterates (10) converge almost surely to the Lagrange multipliers.

[4] $a \wedge b = \min(a, b)$

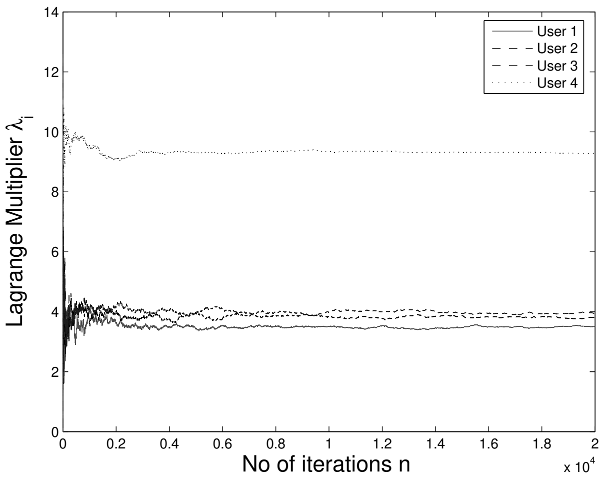

Fig. 1. Convergence for i.i.d. channel model

A. Convergence of Stochastic Approximation Algorithm

We demonstrate the convergence of $\lambda_i(n)$ via simulations. Consider a single hop network of 4 wireless users with 1 base station. We assume a Rayleigh fading channel, whose probability density function is given by $\mu(x) = \frac{1}{\gamma}e^{\frac{-x}{\gamma}}$, where $\gamma > 0$. We first show the convergence for channel model with i.i.d. Rayleigh fading. To make sense of absolute numbers, we take $\mathbf{C} = (0.6, 0.8, 0.7, 0.2)$, $\boldsymbol{\lambda}(0) = (1, 1, 1, 1)$, and $\gamma = (1, 1, 0.9, 0.3)$. Figure 1 shows the convergence of λ's for i.i.d. channel. The average power vector to achieve the desired rates over 50 independent runs is, $\mathbf{P} = (0.7557, 1.0925, 1.0546, 0.9612)$.

For simulation purposes, we next model the more general case of Markovian channel fading to demonstrate the correctness of our algorithm. Assume that the channel gain for user i obeys the auto-regressive equation,

$$x_i(n+1) = \alpha x_i(n) + (1-\alpha) g_i(n), \tag{13}$$

where noise $g_i(n)$ is Gaussian with variance σ and correlation coefficient α. We take $\alpha = 0.3$, $\mathbf{C} = (0.6, 0.8, 0.7, 0.2)$, $\boldsymbol{\lambda}(0) = (1, 1, 1, 1)$, and $\sigma = (1, 1, 0.9, 0.3)$. Figure 2 shows a particular trajectory for $\lambda_i(n)$. Our results demonstrate that the Lagrange multipliers converge for all users within 3000 iterations. The average power required is given by $(1.3677, 1.9966, 1.8971, 1.6322)$. From the absolute numbers it can be inferred that average power required for the Markovian channel is greater than for i.i.d. channel case, as expected.

Remark 1: In wireless data transfer applications, the duration of transfer is of the order of seconds, while the slot duration is of the order of microseconds. Hence, even if there is non-optimality for the initial slots, convergence will occur much before the actual completion of data transfer.

Remark 2: In practical scenarios, we may not want actual convergence to take place, or we may only like to be within the neighborhood of the optimal solution. In [14], lock-in phenomenon for stochastic approximation algorithm has been considered. If the iterate $\boldsymbol{\lambda}(n)$ is within the domain of attraction (the iterate has begun to converge), then there exists a finite

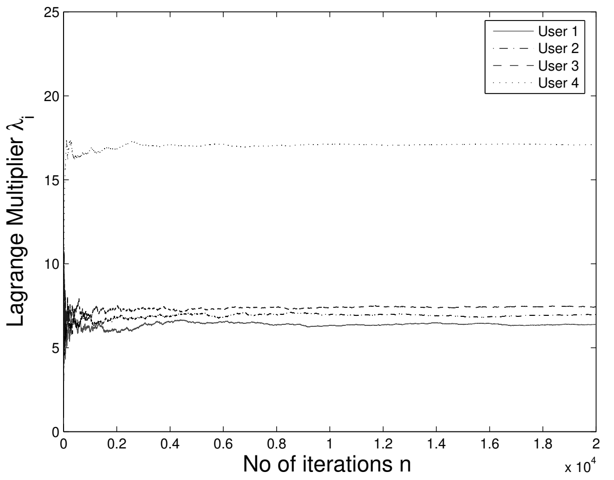

Fig. 2. Convergence for Markovian channel model

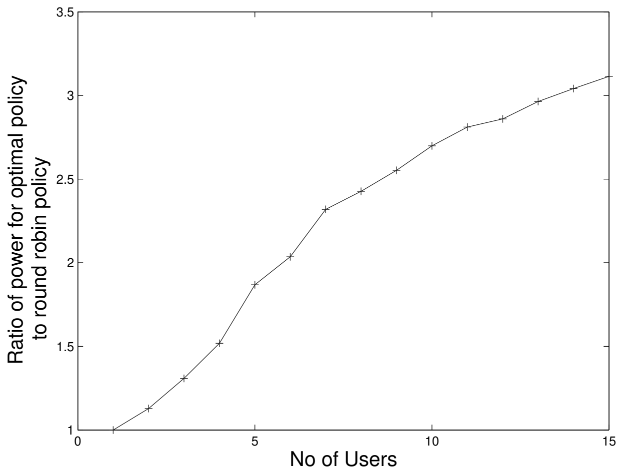

Fig. 3. Gain of the optimal policy over round robin policy

number of iterations for the iterate to be within a finite distance from the convergence point λ^*. A probabilistic lower bound is given for the occurrence of this "nearness" within finite number of iterations.

B. *Performance of Energy Optimal Opportunistic Scheduling*

We compare our scheduling policy with the round robin power scheme. Consider a symmetric system, i.e., all users have the same channel conditions and minimum rate constraints. In the round robin power scheme, we transmit with optimal power for each user in a round robin manner. The scheme reduces to power-optimal single user scheme with minimum rate guarantee dependent on N. Thus we determine the power p such that

$$\int \log(1 + p(x)x)\mu(x)\, dx = NC \tag{14}$$

is satisfied. In our simulations, we assume $C = 0.6$, $\gamma = 0.7$. Our results, shown in Figure 3, demonstrate that as the number of users increases, the ratio of average transmission power of the optimal policy to that of the round robin policy increases, but the marginal increase per user decreases. The gain obtained from variable power scheme increases with number of users, which is due to multiuser diversity.

IV. FAIRNESS

The power optimal scheduling scheme considered in Section II results in starvation of strong users in order to satisfy the rate guarantees of weak users. Hence, in this section, we develop a scheduling algorithm which minimizes power while providing short term fairness and minimum rate guarantees. We first propose a long term fair scheduler and then propose a heuristic short term fair scheduler based on the long term fair scheduler.

A. *Long term fairness*

We first formulate the problem of power optimal fair scheduling. Our method is to opportunistically schedule the user with the best channel condition such that rate guarantees and temporal fairness are achieved and average transmission power is minimized.

Let $\boldsymbol{\phi}$ be the vector of time-average fairness constraints and ϕ_i be the proportion of *temporal bandwidth* allocated to user i. Thus, ϕ_i represents the fraction of the time slots allocated to user i. Our objective is to minimize average power subject to rate and fairness constraints. Our optimization problem is the same as that of (1) with the following additional constraint

$$\liminf_{M\to\infty} \frac{1}{M} \sum_{n=1}^{M} \mathbb{E}\, y_i(n) \geq \phi_i \quad \forall i. \tag{15}$$

Using the ergodicity assumption from Section II, the Lagrangian with the fairness constraint is:

$$\mathcal{L}(p_1, p_2,\ \boldsymbol{\lambda}\,) \triangleq \int \nu(dx_1, \cdots, dx_N) \sum_{y\in A} \int_{[0,\infty)} p_1(dq|\mathbf{y}, \mathbf{x})$$
$$p_2(\mathbf{y}|\mathbf{x}) \left(q - \sum_i \lambda_i \left[\log(1 + q y_i x_i) \;\; - C_i\right] + \sum_i \lambda_i'(y_i - \phi_i) \right),$$

where λ_i' is the Lagrange multiplier associated with the constraint (15), $\boldsymbol{\lambda}$ is the vector $(\lambda_1, \cdots, \lambda_N, \lambda_1', \cdots, \lambda_N')$. Following the approach adopted in Section II, we obtain the optimal policy as: Select user k and transmission power q^* where

$$\begin{aligned} k &= \arg\min_i \Bigg\{ \left(\lambda_i - \frac{1}{x_i}\right)^+ - \lambda_i \Big[\log \Big(\\ &\qquad 1 + \left(\lambda_i - \frac{1}{x_i}\right)^+ x_i \Big) - C_i \Big] + \lambda_i'(1 - \phi_i) \Bigg\} \quad (16) \\ q^* &= \left(\lambda_k - \frac{1}{x_k}\right)^+. \quad (17) \end{aligned}$$

Using the stochastic approximation algorithm from Section II, the Lagrange multiplier update equations can be written as

$$\begin{aligned} \lambda_i(n+1) &= \Bigg[\lambda_i(n) - a(n) \Big[y_i(n) \log \Big(\\ &\qquad 1 + \left(\lambda_i - \frac{1}{x_i(n)}\right)^+ x_i(n) \Big) \Big] - C_i \Bigg]^+ \\ \lambda_i'(n+1) &= [\lambda_i'(n) - a(n)(y_i(n) - \phi_i)]^+ \quad \forall i \quad (18) \end{aligned}$$

The optimality of the above scheme can be proved in a manner similar to that in Section II.

B. *Short term fairness*

In Section IV-A, we have considered long term fairness. Long term fairness guarantees average proportional time share. However, one of the problems associated with long term fairness is starvation or Head of Line (HOL) blocking. There exist conditions when a user may not get a chance to transmit even after being assured a minimum rate guarantee. Thus, it is important to consider a short term fair scheduler.

We consider a window of size $M \geq N$ slots. In a short term fair scheduler, we allocate time share equal to $\phi_i M$[5] to user i over this window and say that the scheduler is short term fair over the window M. The case $M \to \infty$ is same as the long term fairness. We first discuss the case when $M = N$. Let $\mathcal{A}$ be the set of users, i.e., user $k \in \mathcal{A}$. For $M = N$, we can allocate a maximum of one slot per user. We first select the user from the set $\mathcal{A}$ which is optimal for that time slot from (16)[6]. Let k be the optimal user. We remove user k from the list: $\mathcal{A} = \mathcal{A} \setminus \{k\}$. We repeat the above process on modified $\mathcal{A}$. We call this policy as *elimination policy*. A heuristic based algorithm for general M is explained below.

Algorithm 1 Temporal Short Term Fair Scheduling

```
Slot vector v= M(φ_1, φ_2, ··· , φ_N)
A = {1, 2, ··· , N}
i = 1
for i ≤ M do
  for each j ∈ A do
    Choose optimal k using (16).
    Transmit with power q*
    {v}_k = {v}_k − 1
    if {v}_k ≤ 0 then
      A = A \ {k}
    end if
  end for
end for
```

In our simulations, we assume that the channel gains are Markovian across slots, as in (13). To make sense of absolute numbers, we assume $\mathbf{C} = (0.6, 0.8, 0.7, 0.2)$ and $\boldsymbol{\phi} = (0.3, 0.4, 0.2, 0.1)$. We implement Algorithm 1 and plot the average power required with increasing window size, as shown in Figure 4. The power required is a decreasing function of window size. It may be noted that in short term fair scheduler more emphasis is given to providing temporal fairness, but in the process, the actual rate obtained may deviate from the desired rates. Thus there is trade off between window size and actual rates achieved.

V. Conclusions

In this paper, we have obtained a power optimal opportunistic scheme for multiuser TDMA system with minimum rate constraints for individual users. We have proposed an online optimal scheduling algorithm based on stochastic approximation

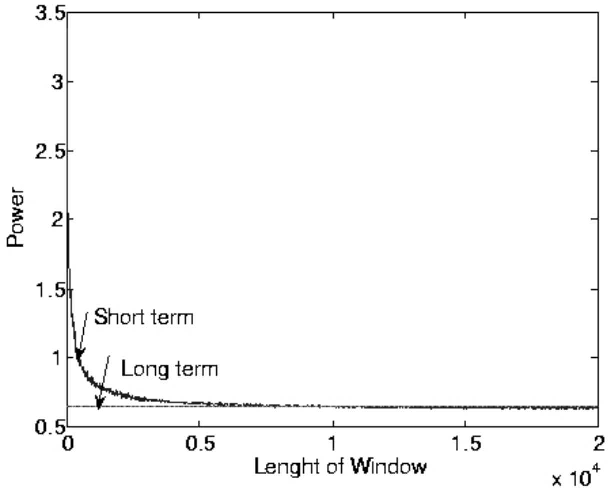

Fig. 4. Power required for the short term fairness compared to long term fairness

and argued the theoretical convergence of the policy. We have extended the approach to incorporate temporal long-term fairness. Finally we have proposed a heuristic based short term fair algorithm and compared its performance with that of long term fairness.

References

[1] R. Knopp and P. A. Humblet, "Information Capacity and Power Control in Single-cell Multiuser Communications," in *IEEE ICC*, vol. 1, pp. 331–335, June 1995.

[2] E. F. Chaponniere, P. Black, J. M. Holtzman, and D. Tse, "Transmitter directed multiple receiver system using path diversity to equitably maximize throughput." U.S. Patent No. 6449490, September 10, 2002.

[3] R. Berry and E. M. Yeh, "Cross-layer wireless resource allocation," *IEEE Signal Processing Magzine*, September 2004.

[4] X. Liu, E. Chong, and N. B. Shroff, "Opportunistic Transmission Scheduling with Resource-Sharing Constraints in Wireless Networks," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 10, pp. 2053–2065, 2001.

[5] S. Kulkarni and C. Rosenberg, "Opportunistic Scheduling Policies for Wireless Systems with Short Term Fairness Constraints," in *IEEE GLOBECOM*, vol. 1, pp. 533 – 537, December 2003.

[6] Y. Liu and E. Knightly, "Opportunistic Fair Scheduler over Multiple Wireless Channel," vol. 3, pp. 1106–1115, IEEE INFOCOM, 2003.

[7] X. Liu, E. Chong, and N. Shroff, "Joint Scheduling and Power-Allocation for Interference Management in Wireless Networks," in *Proceedings of the VTC Fall 2002*, pp. 757–766, 2002.

[8] V. S. Borkar, "On White Noise Representations in Stochastic Realization Theory," *SIAM J. Control and Optim.*, vol. 31, pp. 1093–1102, 1993.

[9] D. P. Bertsekas, *Nonlinear Programming*. Belmont, Mass.: Athena Scientific, 1999.

[10] H. Kushner and G. Yin, *Stochastic Approximation Algorithms and Applications*. New York: Springer-Verlag, 1997.

[11] A. Bhorkar, "Resource allocation in wireless networks," Master's thesis, Department of Electrical Engineering, Indian Institute of Technology, Bombay, 2006. www.ee.iitb.ac.in/uma/~bhorkar/thesis.pdf.

[12] M. Bardi and I. C. Dolcetta, *Optimal Control and Viscosity Solutions of Hamilton-Jacobi-Bellman Equations*. Boston: Birkhauser, 1997.

[13] P. Milgrom and I. Segal, "Envelope Theorems for Arbitrary Choice Sets," *Econometrica*, vol. 70, pp. 583–601, 2002.

[14] V. S. Borkar, "On the lock-in probability of stochastic approximation," *Comb. Probab. Comput.*, vol. 11, no. 1, pp. 11–20, 2002.

[5] We assume that $\phi_i M$ is an integer $\forall\, i$.

[6] In the modified algorithm, the set of users is $\mathcal{A}$.