Performance Modeling of Statistical Multiplexer with Composite Traffic Input by CIPP/M/1 Queue

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Abstract

Correlated Interarrival time Process (CIPP) has been proposed in [1, 2], for modeling both the composite arrival process of packets in broadband networks and the individual source modeling. The CIPP - a generalization of the Poisson process - is a stationary counting process and is parameterized by a correlation parameter 'ρ' which represents the degree of correlation in adjacent interarrivals in addition to 'λ' the intensity of the process. In this paper, we undertake the performance modeling of a statistical multiplexer with composite Ethernet traffic input using the CIPP/M/1 queue. The queueing performance measures considered are mean delay, mean number of packets, packet delay variation and overflow probability for a given threshold. Our experimental results show that the CIPP/M/1 queue models well the queueing situation as encountered by the the Ethernet traffic in a statistical multiplexer with respect to the above queueing performance measures.
Keywords CIPP, interarrivals, CIPP/M/1 queue, self-similarity.

1 Introduction

The discovery of long range dependence (LRD) in Ethernet LAN traffic in Bellcore [3], metropolitan area network (MAN) traffic [4], wide area network (WAN) traffic, and variable bit rate (VBR) video traffic [5, 6], led researchers to discard the conventional short range dependent (SRD) models [5] and look for self-similar models. One way of obtaining pseudo self-similarity was to introduce correlation. This could be achieved in a number of ways, for example, considering correlations in the count sequence by say a discrete autoregressive (DAR(1)) model, or by considering correlations in the rate process by an autoregressive (AR) model or by introducing correlations by modulating Markov chain or finally by considering *correlations in the interarrival* sequence itself. In literature, researchers have considered models which capture correlation in the counts sequence given as packets (or cells) per unit time (as in DAR(1) model [7]), or in the (traffic) rate process (as in AR model [8] or self-similar models [3]) or the correlation introduced by the modulating Markov chain as in Markov modulated models [5].

We know that correlations in interarrivals *necessarily* imply correlations in the number of counts; however converse of this statement is *not* necessarily true. A good example would be the renewal process. Hence, we contend that, a model capable of capturing the correlation in the interarrival sequence is a more appropriate candidate in the context of modeling teletraffic data. In light of the above, in [2], we proposed a new traffic model (with correlations in interarrivals) for both composite arrival process of (fixed size) packets in broadband networks and individual source modeling. The traffic model involves a generalization of Poisson process which is referred to as Correlated Interarrival time Poisson Process (CIPP). The CIPP, a stationary counting process, is parameterized by a correlation parameter 'ρ' which represents the degree of correlation in adjacent interarrivals in addition to 'λ' the intensity of the process. In [9], the corresponding queueing theory for CIPP/M/1 queue is developed. In this paper, we use the queueing theory developed for CIPP/M/1 queue for the performance modeling of statistical multiplexer with composite broadband traffic input.

In section 2, we give the stationary distributions in a CIPP/M/1 queue and the expression for the corresponding queueing performance measures. Section 3 gives the experimental results for performance modeling of statistical multiplexer with composite traffic input using the CIPP/M/1 queue. Section 4 concludes the paper.

2 The Queueing Performance Measures in a CIPP/M/1 Queue

Refer to [2] for details about the definition, distribution and properties of CIPP. In this section, we consider a single server FCFS queue with CIPP arrivals (with parameter λ and ρ) and exponential service times with parameter μ. We obtain analytical expressions for the waiting time distribution and buffer occupancy distribution. Due to space constraints, we will give here only the result. For details the reader is referred to [10].

2.1 The CIPP/M/1 Queue

Stationary queue length: We denote the stationary probability of j number of customers in the queue by P_j. Then,

$$P_j = \begin{cases} 1 - \frac{TM(a,1)}{M(0,1)}\left(\frac{\lambda}{\lambda+a}\right) & j = 0 \\ \frac{(1-T)T^j M(a,1)}{M(0,1)}\left(\frac{\lambda}{\lambda+a}\right) & j > 0 \end{cases} \tag{1}$$

where $T = \frac{1-\rho+\rho(\frac{\mu}{\lambda})}{\frac{\mu}{\lambda}}$, $a = \frac{\mu\rho}{1-\rho}$. $M(a,1)$ and $M(0,1)$ are given by,

$$M(0,1) = \begin{cases} \frac{(\rho+\frac{\lambda}{\mu}(1-\rho))(1-\rho+\rho\frac{\lambda}{\mu})}{(1-\rho)(\rho+\frac{\lambda}{\mu})+\rho^2\frac{\lambda}{\mu}} \\ (\lim_{n\to\infty} P_1(n,\lambda,\mu)) \end{cases}$$

$$M(a,1) = \begin{cases} \frac{((1-\rho)\frac{\lambda}{\mu}+\rho)(1-\rho(1-\rho)(1-\frac{\lambda}{\mu}))}{\left[(1-\rho)^2\frac{\lambda}{\mu}+\rho(1-\rho(1-\rho)(1-\frac{\lambda}{\mu}))\right]} \\ (\lim_{n\to\infty} P_2(n,\lambda,\mu)) \end{cases}$$

where, $P_1(n,\lambda,\mu) = \prod_{i=1}^{n-1} \frac{\left(\rho+(1-\rho)\frac{\lambda}{\mu}\right)\ C(i)}{(1-\rho)\frac{\lambda}{\mu}+\rho C(i)}$ and

$$P_2(n,\lambda,\mu) = \prod_{i=1}^{n-1} \frac{\frac{\lambda T}{\mu}\left[(1-\rho)C(i)+\rho^{i+1}\right]}{(1-\rho)^2\frac{\lambda}{\mu}+\rho\left[(1-\rho)C(i)+\rho^{i+1}\right]}$$

and $C(i) = (1-(1-\frac{\lambda}{\mu})\rho^{i+1})$

Waiting time distribution: Let $Pr\{W \le x\}$ be the stationary waiting time distribution. Then,

$$Pr\{W \le x\} = 1 - \frac{TM(a,1)}{M(0,1)}\left(\frac{\lambda e^{-\mu x(1-T)}}{\lambda+a}\right) \tag{2}$$

with $a, M(.,.)$ having the same meaning as above.

2.2 Queueing Performance Measures

Mean Waiting Time: From the stationary distribution for waiting time (W) (2), we have,

$$\mathbf{E}\{W\} = \frac{TM(a,1)}{\mu(1-T)M(0,1)}\left(\frac{\lambda}{\lambda+a}\right) \tag{3}$$

Mean Number of Packets in the Buffer: From the stationary buffer occupancy distribution (the distribution of the random variable of number of customers (Q)) in a CIPP/M/1 queue (1), we have

$$\mathbf{E}\{Q\} = \frac{TM(a,1)}{(1-T)M(0,1)}\left(\frac{\lambda}{\lambda+a}\right) \tag{4}$$

Packet Delay Variation: We use the term *packet delay variation (pdv)* (denoted by τ) to denote the variance of the packet transfer delay (Y). Note that $Y \triangleq W + S$ where, W is the stationary waiting time and S is the stationary service time. It can be shown that the *pdv* has the following expression [11],

$$\tau = \frac{2}{\mu^2}\left[1-\beta+\frac{\beta}{(1-T)^2}\right] - \left[\frac{1}{\mu}\left(1+\frac{\beta T}{(1-T)}\right)\right]^2 \tag{5}$$

where $\beta \triangleq \frac{M(a,1)}{M(0,1)}\frac{\lambda}{\lambda+a}$ where again, $M(0,1)$, $M(a,1)$ and a are as given above.

Overflow Probability: Overflow probability is the probability that the number of packets in the buffer exceeds a threshold in an infinite buffer queue. For CIPP/M/1 case, it is straight forward to show from (1) that the overflow probability for the threshold of K packets[1] for $K > 0$, is given by,

$$Pr\{Q \ge K+1\} = \frac{M(a,1)}{M(0,1)}\left(\frac{\lambda}{\lambda+a}\right)T^{K+1} \tag{6}$$

where Q denotes the number of packets in the system (queue and server).

3 Performance Modeling of Statistical Multiplexer with Composite Traffic input

For our experiments, we consider Bellcore traces namely, *pAug.TL* and *pOct.TL*. We consider only the arrival instants of packets (the first column of the Ethernet *Bellcore* dataset [12]) and generate exponentially distributed packet lengths (to get exponential services) in order to use the CIPP/M/1 queueing theory results. This packet stream is then fed into an infinite buffer single server FCFS queue with exponential service. Then, the queueing measures mentioned above are estimated to get the experimental results corresponding to data. To get the *theoretical* performance results, we fit the CIPP to composite traffic trace data by autocorrelation matching [2] and compute the queueing performance measures by using the expressions given above, corresponding to this "best" fit CIPP.

Mean Waiting Time: Figure 1 shows the mean waiting time performance for *pAug.TL* and *pOct.TL* and their corresponding approximating CIPP/M/1. One can see from the figure that CIPP/M/1 queue models well both the queueing situation as encountered by the Ethernet traffic in a statistical multiplexer, with respect to the mean waiting time measure.

Mean Number of Packets in the Buffer: Here, we consider the mean number of packets as a performance measure. Our results are shown in Figure 2. It can be seen from the figure that the CIPP/M/1 queue models well the performance of statistical multiplexer with composite traffic input with respect to the measure of mean number of packets.

Packet Delay Variation (pdv) We repeat the above

[1] The unit "packet" refers to the mean packet length (in bits).

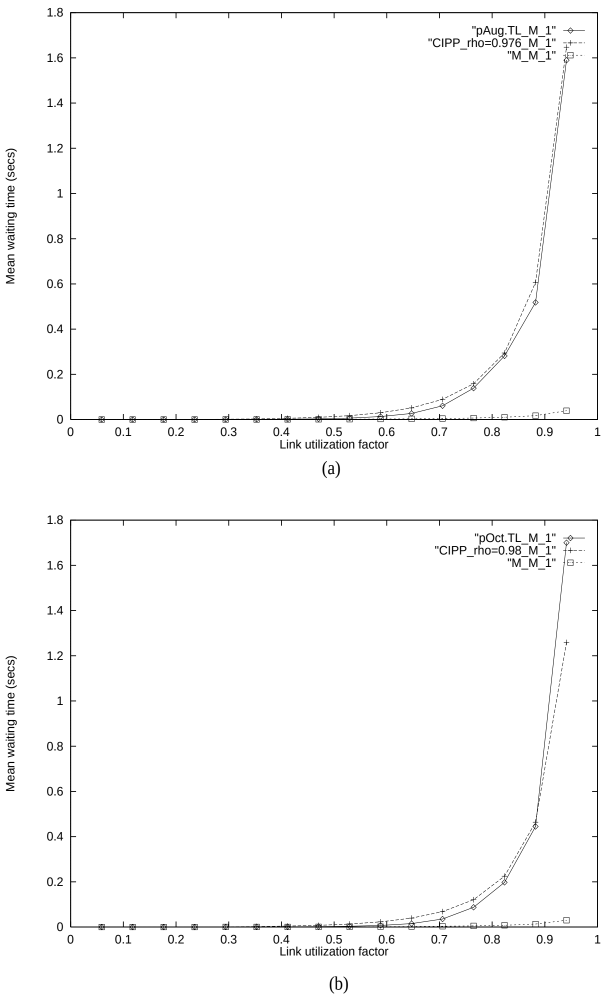

Figure 1: Mean waiting time of packets in a queue driven by Bellcore dataset and the approximating CIPP/M/1 and M/M/1 queues (a)*pAug.TL* (b)*pOct.TL*

experiment for the packet delay variation for the Bellcore dataset. Figure 3 shows the variance of *pdv* for *pAug.TL* and *pOct.TL* and their approximating CIPPs. It is seen from the figure that CIPP approximates well the queueing situation as encountered by the Ethernet traffic in a statistical multiplexer, with respect to the queueing performance measure of *pdv*.

Overflow Probability Here, we concentrate on the queueing performance measure of overflow probability for our performance modeling study. We consider a threshold of 32 packets. Figure 4 shows the overflow probability for `pAug.TL` and `pOct.TL`. From the figure, note that the CIPP again approximates well the data. Also in all the above figures, the respective performance measure for the 'best fit' M/M/1 queue is shown. Not that the results corresponding to M/M/1 queue seriously underestimates the performance measures.

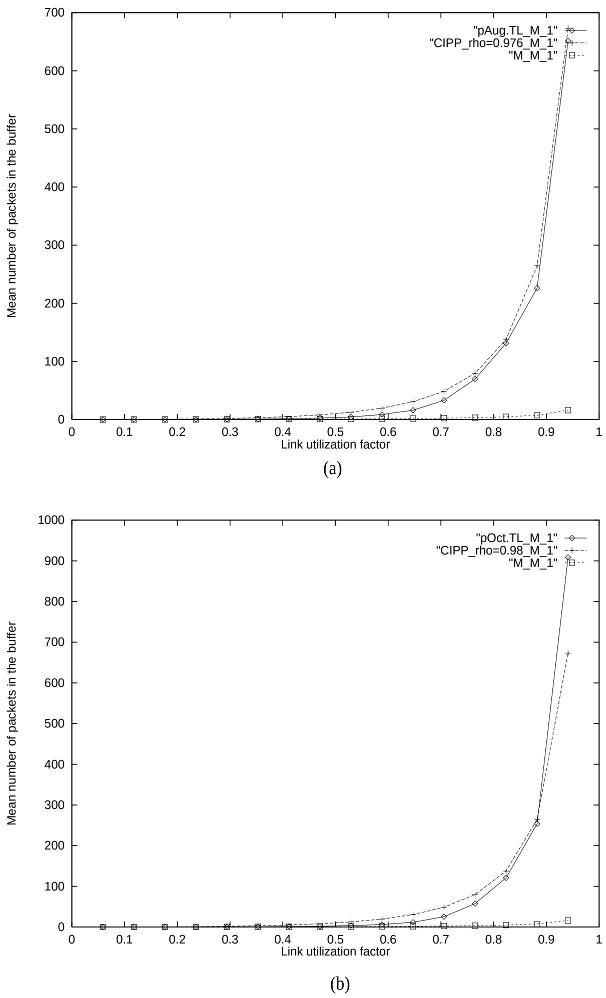

Figure 2: Mean number of packets in a queue driven by Bellcore Ethernet traffic and the approximating CIPP/M/1 queue (a)*pAug.TL* (b)*pOct.TL*

4 Conclusions

In this paper performance modeling of statistical multiplexer with Ethernet traffic using CIPP/M/1 queue has been undertaken. The experimental results show that the CIPP/M/1 approximates the queueing situation very well. Being studied the performance modeling, next immediate study would be the call admission control under CIPP traffic.

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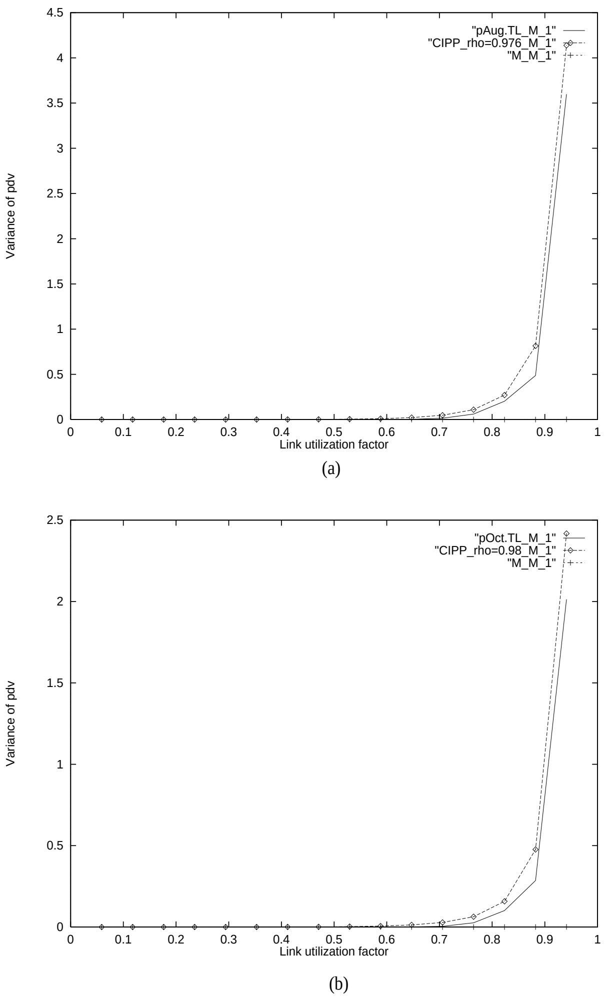

Figure 3: Variance of *pdv* in a queue driven by Bellcore and that of the approximating CIPP/M/1 queue (a)$pAug.TL$ (b)$pOct.TL$

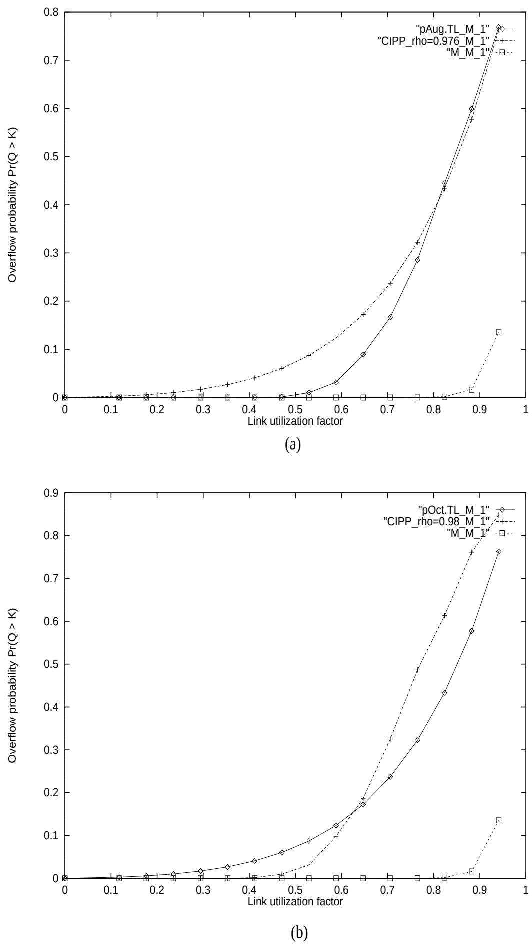

Figure 4: Overflow probability in a queue driven by Bellcore dataset and the corresponding quantity in the approximating CIPP/M/1 and M/M/1 queues (a) `pAug.TL` (b) `pOct.TL`. Here, the threshold buffer size is $K = 32$ packets

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