CIPP BASED CHARACTERIZATION OF VBR TRAFFIC AND RELATED STATISTICAL MULTIPLEXER PERFORMANCE

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ABSTRACT

In this paper, we propose a new traffic model (with correlations in interarrivals), herein referred to as Correlated Interarrival time Poisson Process (CIPP), for modeling the VBR video traffic. The CIPP - a generalization of the Poisson process with stationary increments - is parameterized by a correlation parameter 'ρ' which represents the degree of correlation in adjacent interarrivals in addition to 'λ' the intensity of the process. We present the theory for the CIPP/M/1 queue and derive the corresponding expressions for queueing performance measures of interest. We undertake the performance modeling of statistical multiplexer with VBR video traffic input using the CIPP/M/1 queue. The simulation results show that the CIPP/M/1 queue approximates well the queueing situation as encountered by the VBR traffic in broadband networks.

1. INTRODUCTION

Two classes of traffic models have been investigated for modeling the bit rate of a VBR video source, namely, Short Range Dependent (SRD) models [1, 2, 3, 4] and Long Range Dependent (LRD) (or self-similar) models [5, 6]. The above models are intended to capture the correlation in the count sequence (packets per frame) either by an ARMA model as in [3] or by the modulating Markov chain as in SRD models. In models based on self-similarity or LRD, correlations are in the rate process (equivalently *packets per unit time*). We believe that it is more appropriate to have a model which captures the correlation in the interarrival sequence rather than the correlation in count sequence as is the case with the above models.

We know that correlations in interarrivals *necessarily* imply correlations in the number of counts; however converse of this statement is *not* necessarily true. A good example would be the renewal process. Hence, we contend that, a model capable of capturing the correlation in the interarrival sequence is a more appropriate candidate in the context of modeling teletraffic data. Moreover, the interarrival correlation plays a major role in exhibiting the self-similar behaviour in increments of the corresponding counting process. In the light of above, we are motivated to define a process, whose interarrivals are correlated. In [7], we have proposed Correlated Interarrival time Poisson Process (CIPP) for modeling VBR video traffic. The CIPP, a stationary counting process, is parameterized by a correlation parameter 'ρ' which represents the degree of correlation in adjacent interarrivals in addition to 'λ' the intensity of the process. It has been further shown that the CIPP does exhibit self-similarity over a range of time scales of interest.

In this paper, we study the CIPP/M/1 queue and discuss the implication of correlation in interarrivals on the queueing performance. We undertake the performance modeling of statistical multiplexer with *real world* VBR video traffic, using CIPP/M/1 queue. The rest of the paper is organized as follows: In Section 2, we give a brief account of the CIPP process. Section 3, gives analytical results for CIPP/M/1 queue and discusses the implications of these results. In Section 4, the performance modeling of statistical multiplexer is presented. Section 5 concludes the paper.

2. CORRELATED INTERARRIVAL TIME POISSON PROCESS (CIPP)

We wish to define a counting process $N(t)$ which retains as far as possible all the nice properties of the Poisson process, but has correlated interarrivals. Typically, we would like to have interarrivals being exponentially distributed apart from being correlated. Motivated by the analytical simplicity of the method of generating stationary correlated exponential sequence (first order exponential autoregressive process (EAR(1))) developed by Gaver and Lewis [8], we present the following formalism for defining CIPP.

Consider arrivals occurring in time on the interval $(0, \infty]$. For $t > 0$, let $N(t)$ be the number of arrivals that have occurred in the half-open interval $(0, t]$. Consequently, $N(t)$ and $N(t+h) - N(t) \quad (h > 0)$, assume only non-negative integer values. We now state the following axioms.

Axioms:

1. Since we begin counting arrivals at time 0, we define $N(0) = 0$.

2. **Correlation structure for interarrivals:** Let X_n be the interarrival time between the n^{th} arrival instant (T_n) and $(n-1)^{th}$ arrival instant (T_{n-1}). Then,

$$X_{n+1} = \rho X_n + \epsilon_{n+1} \qquad 0 \leq \rho < 1 \quad n = 1, 2, 3 \ldots \quad (1)$$

where, $\{\epsilon_n\}_{n=2}^{\infty}$ is an *iid* sequence, with $\epsilon_n,\ n > 1$, being a product of Bernoulli random variable (B) with parameter ρ and exponential random variable (V) with parameter λ. B is statistically independent of V. $\{X_n\}$ forms a stationary sequence with the exponential distribution being characterized by parameter λ.

3. The counting process $N(t)$ has stationary increments, that is, for any two points $t > s \geq 0$ and any $h > 0$, the random variables $N(t) - N(s)$ and $N(t+h) - N(s+h)$ are identically distributed. Note that, $X_1 = T_1 - T_0$ where, T_0 is the 0th arrival instant.

Definition: A counting process $\{N(t), t > 0\}$ satisfying Axioms 1 through 3 is called Correlated Interarrival time Poisson Process (CIPP) with parameter λ and ρ.

Remark 1: The axiom 2, is *more specific* and the following properties of the counting process $N(t)$ directly follow from it. 1. For any $t > 0, \quad 0 < Pr\{N(t) > 0\} < 1$.
2. For any $t \geq 0$,

$$\lim_{h \to 0} \frac{Pr\{N(t+h) - N(t) \geq 2\}}{Pr\{N(t+h) - N(t) = 1\}} = 0 \qquad (2)$$

In words, in sufficiently small intervals, at most one arrival can occur: i.e., it is not possible for arrivals to happen simultaneously. The process is *orderly*, in the sense of Daley *et al* [9].

Remark 2: The main difference between the axiomatic definition of the Poisson process and the CIPP, is the addition of Axiom 2 and the relaxation of the independence assumption in Axiom 3.

Let $P_n(t) \triangleq Pr\{N(t) = n\}$, be the probability that the number of arrivals in the interval $(0, t]$ is n.

Lemma 1 *CIPP with parameter λ and ρ has the distribution,*

$$P_n(t) = \begin{cases} (1 + \rho - (1-\rho)\alpha_n) \\ \sum_{j=0}^{n} A(j,n) e^{-\frac{\lambda t}{\alpha_j}} \qquad 0 < \rho < 1 \end{cases} \qquad (3)$$

where, $A(j,n) = \frac{\alpha_j \prod_{k=0}^{n}[\alpha_j - \rho\alpha_k]}{B(j,n)\prod_{k=0,k\neq j}^{n}[\alpha_j - \alpha_k]}$ *and* $B(j,n) = (\rho(\alpha_j + 1) - \alpha_n + 1)D(j,n)$ *where again* $D(j,n) = (\alpha_j - \alpha_n + 1)(\alpha_j - \rho\alpha_n)$*, for* $n = 0, 1, 2, \ldots,\ \lambda > 0,\ t \geq 0$ *and* $\alpha_j = \sum_{i=0}^{j} \rho^i$

The proof of the lemma and other details regarding CIPP are given in [7].

Remark 3: The prime reason for assuming (1) for the interarrival sequence is to mimic the self-similar behaviour in increments for the corresponding counting process. Note that, to mimic a self-similar process with $0.5 < H < 1$, (where H is the Hurst parameter) it is necessary that the increments should have positive correlations [10]. We choose (1) as the correlation structure for interarrivals, since, it introduces strong positive correlation than other models based on moving average structure like EMA(1) (Exponential Moving Average sequence of order 1) proposed by Lawrance [11] and the one proposed by Finch [12]. Secondly, the correlation structure (1) allows one to derive a closed form expression for $P_n(t)$ with reasonable simplicity, which is not possible with the other aforementioned models. Thirdly, the correlation structure (1), is analytically simple which allows one to develop the corresponding queueing theory (See Equations (5) and (6)). Finally, the sequence $\{X_n\}$ is obtained as an additive random linear combination of random variables, and is thus easy to simulate on the computer.

Remark 4: We would like to mention here that, the number of parameters needed to exhibit the pseudo-self-similar behaviour is minimal in CIPP as compared to the conventional SRD models like MMPP and DBMAP. For example in MMPP, the number of parameters is N^2 for N state MMPP ($N^2 - N$ for infinitesimal generator and N for the intensity matrix Λ) while in CIPP it is always two namely, λ and ρ.

Remark 5: In [7], it has been shown empirically that, the (estimated) local Hurst parameter increases monotonically with ρ. This is a convenient way of associating the parameter ρ with the local Hurst parameter (a measure of degree of self-similarity) of the resulting CIPP trace, unlike in other SRD models where too many parameters decide the degree of pseudo-self-similarity, and one does not know which of them have crucial roles in exhibiting pseudo-self-similarity. The above feature of CIPP makes it more suited for practical purposes, when one needs to fit a CIPP model to the real world data, say, a video sequence of given H parameter.

Remark 6: The distribution (3) is count stationary in the sense of Lawrance [11]. An interval stationary version (again in the sense of Lawrance) can be derived along the above lines. We give here only the result. For $0 \leq \rho < 1$

$$P_n^I(t) = \sum_{j=0}^{n} \frac{\prod_{k=0}^{n}[\alpha_j - \rho\alpha_k] e^{-\frac{\lambda t}{\alpha_j}}}{D(j,n)\prod_{k=0,k\neq j}^{n}[\alpha_j - \alpha_k]} \qquad (4)$$

where $D(j,n)$ is as above.

Remark 7: For both the forms, (3) and (4), as the limit $\rho \to 0$, we obtain the Poisson case i.e., $\lim_{\rho\to 0} P_n(t) = e^{-\lambda t}\frac{(\lambda t)^n}{(n)!}$ and $\lim_{\rho \to 0} P_n^I(t) = e^{-\lambda t}\frac{(\lambda t)^n}{(n)!}$. Thus, the Poisson distribution is approached continuously from the CIPP distribution as $\rho \to 0$.

Remark 8: Neuts process and CIPP In the Neuts process

[13], the memory of the process is limited to the latest phase renewal instant and the process completely *forgets* the past after that point. Note, to exhibit self-similarity or long range dependence, the process should have infinite memory. The memory of the CIPP process at time t, is limited to the arrival instant just prior to the latest arrival instant (and the latest interarrival is again dependent on its previous interarrival and so on). Quantitatively, this extent of memory is controlled by the correlation parameter ρ.

3. CIPP/M/1 QUEUE

We consider a single server FCFS queue with CIPP arrivals and exponentially distributed service times with parameter μ. We give here only the analytical expressions for the waiting time distribution and buffer occupancy distribution.

Stationary Queue Length:- We denote the stationary probability of j number of customers in the queue by P_j. Then,

$$P_j = \begin{cases} 1 - \frac{TM(a,1)}{M(0,1)}\left(\frac{\lambda}{\lambda+a}\right) & j = 0 \\ \frac{(1-T)T^j M(a,1)}{M(0,1)}\left(\frac{\lambda}{\lambda+a}\right) & j > 0 \end{cases} \quad (5)$$

where $T = \frac{1-\rho+\rho(\frac{\mu}{\lambda})}{\frac{\mu}{\lambda}}$, $a = \frac{\mu\rho}{1-\rho}$. $M(a,1)$ and $M(0,1)$ are given by,

$$M(0,1) = \begin{cases} \frac{(\rho+\frac{\lambda}{\mu}(1-\rho))(1-\rho+\rho\frac{\lambda}{\mu})}{(1-\rho)(\rho+\frac{\lambda}{\mu})+\rho^2\frac{\lambda}{\mu}} \\ (\lim_{n\to\infty} P_1(n,\lambda,\mu)) \end{cases}$$

$$M(a,1) = \begin{cases} \frac{((1-\rho)\frac{\lambda}{\mu}+\rho)(1-\rho(1-\rho)(1-\frac{\lambda}{\mu}))}{[(1-\rho)^2\frac{\lambda}{\mu}+\rho(1-\rho(1-\rho)(1-\frac{\lambda}{\mu}))]} \\ (\lim_{n\to\infty} P_2(n,\lambda,\mu)) \end{cases}$$

where, $P_1(n,\lambda,\mu) = \prod_{i=1}^{n-1} \frac{\left(\rho+(1-\rho)\frac{\lambda}{\mu}\right) \; C(i)}{(1-\rho)\frac{\lambda}{\mu}+\rho C(i)}$ and

$$P_2(n,\lambda,\mu) = \prod_{i=1}^{n-1} \frac{\left(\rho+(1-\rho)\frac{\lambda}{\mu}\right)\left[(1-\rho)C(i)+\rho^{i+1}\right]}{(1-\rho)^2\frac{\lambda}{\mu}+\rho\left[(1-\rho)C(i)+\rho^{i+1}\right]}$$

where again, $C(i) = (1-(1-\frac{\lambda}{\mu})\rho^{i+1})$

Waiting Time Distribution:- Let $Pr\{W \le x\}$ be the stationary waiting time distribution. Then,

$$Pr\{W \le x\} = 1 - \frac{TM(a,1)}{M(0,1)}\left(\frac{\lambda \; exp(-\mu x(1-T))}{\lambda+a}\right) \quad (6)$$

with $a, M(.,.)$ having the same meaning as above.

Queueing Performance Measures:- For evaluating the queueing performance of the statistical multiplexer with VBR video traffic input, we use the mean number of packets and the mean waiting time.

Mean Waiting Time: From the stationary distribution for waiting time W (6), we have,

$$\mathbf{E}\{W\} = \frac{TM(a,1)}{\mu(1-T)M(0,1)}\left(\frac{\lambda}{\lambda+a}\right) \quad (7)$$

Mean Number of Packets in the Buffer: From the stationary buffer occupancy distribution Q (5) [14] of CIPP/M/1 queue, we have

$$\mathbf{E}\{Q\} = \frac{TM(a,1)}{(1-T)M(0,1)}\left(\frac{\lambda}{\lambda+a}\right) \quad (8)$$

For the theoretical computations of queueing measures which involve the computation of $M(a,1)$ and $M(0,1)$, we take 'n' on the RHS of $M(a,1)$ and $M(0,1)$ such that the percentage error (between nth and $(n+1)$th term) is less than 0.2%. (Note that the product terms in the expression for $M(a,1)$ and $M(0,1)$ converges).

Experimental Validation of Theoretical Results: We have computed the theoretical queueing performance measures namely, the mean waiting time and the mean number of customers. We have also simulated the CIPP/M/1 queue and the results are shown in Figures 1 and 2. The figures show that the simulation results agree well with the theoretical results for all the queueing performance measures. Note also that the queueing performance measures (in our case, mean waiting times and mean number of customers) degrades as ρ increases.

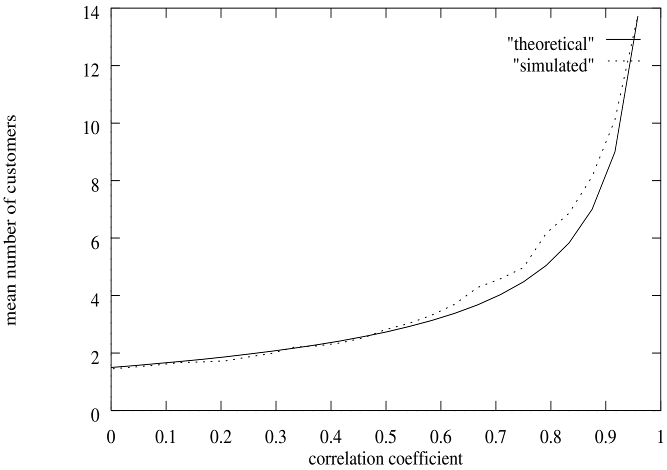

Figure 1: Mean number of packets in a CIPP/M/1 queue

3.1. Discussions

Two important issues in the context of above queueing theory analysis catches immediate attention. First one is the implications of these results and its relation to the work of Loynes [15]. The second one is the impact of correlation in interarrivals on the queueing performance measures.

Loynes Results: Loynes in [15], has considered more general queues with interarrival and service times forming stationary sequences (with arbitrary dependence among themselves and arbitrary cross correlation). He has proved that, for the stability of the queue, the ratio of arrival rate to service rate should be less than 1. The correlation structure

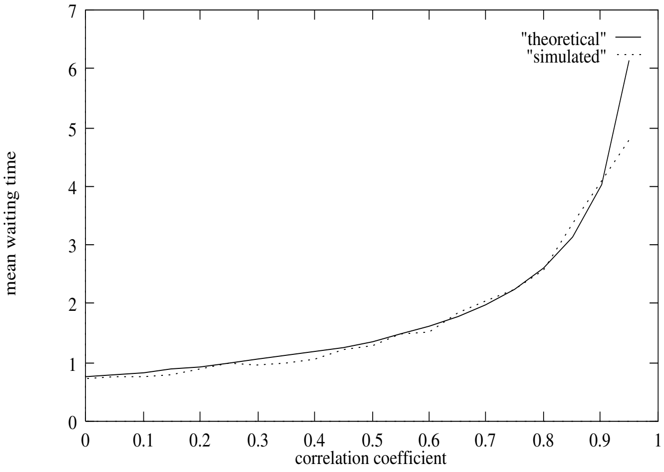

Figure 2: Mean waiting time of packets in a CIPP/M/1 queue

among the interarrival sequence has nothing to do with the stability of the queue. In the present case also, one can see from the Equations (5) and (6) that, ρ has nothing to do with the stability of the queue. It is only the ratio of λ to μ (which is implicit in T) that decides the stability of the queue.

Impact of Correlations on Queueing Performance: One of the intriguing question in the queue with correlated arrivals is about the impact of correlation in interarrivals on the queueing performance. Livny *et al* [16], after carrying out extensive simulations have concluded that positive correlation always leads to degradation in queueing performance (in their case mean waiting time). The effect of negative correlations, however, is not clear; it might either degrade or improve the queueing performance. We saw that in CIPP, the increments have positive correlation. Moreover, the analytical results (and their verification by simulations) given in Section 3, shows that the queueing performance degrades as ρ increases. Thus we have analytical results (to best of the knowledge of the authors, it is the first time in literature to have such a result) corroborating Livny's empirical observation.

4. PERFORMANCE MODELING OF STATISTICAL MULTIPLEXER WITH *REAL WORLD* VBR VIDEO TRAFFIC AS INPUT USING CIPP/M/1 QUEUE

Simulation Methodology: Given a *real world* VBR video traffic trace, the bit stream in the ith frame of size $B(i)$ is segmented into $N_f(i)$ packets, such that $N_f(i) = \overset{argmax}{k} \{k | \sum_{j=1}^{k} V_i(j) < B(i)\}$ where the packet lengths $V_i(j)$ (length of jth packet of ith frame) are geometrically distributed. Then all the packets $N_{T_f}(i)$ in the ith frame interval are accommodated with equal intervals within the frame interval. This geometric sequence (generated for the packet length) is used later as service interval sequence (divided by the link capacity), when we simulate queues with such arrival process. We use the above procedure as an approximation for exponential service[1]. (The purpose of the whole procedure is to (i) consider exponential distribution for service intervals, in order to use the theory for CIPP/M/1 queue for our performance modeling study, (ii) obtain realistic results considering the fact that the real world broadband switch serves a packet proportional to the length of the packet. This forces us to consider the variable sized packets for our study). Then the smoothed packet stream is fed into an infinite buffer single server queue. We estimate the queueing performance measures, the mean waiting time for packets and the mean number of packets from simulations. We thus get the experimental results corresponding to data. To get the theoretical results, we fit the CIPP to the video trace data by autocorrelation matching (details omitted here) and compute the above mentioned queueing performance measures corresponding to the "best fit" CIPP.

Simulation Results: We consider the *race* ($H = 0.71$) MPEG-1 VBR video sequence for our experiments. Figure 3 shows our results for mean waiting time for packets in the buffer for *race* video sequence. One may see that, the CIPP/M/1 queue models well the queueing situation encountered by the *race* video sequence data, except at higher loads, where the CIPP/M/1 queue overestimates the mean waiting time. This is desirable since, for network dimensioning purpose, it is more convenient to use a model which marginally overestimates the mean waiting time. The results corresponding to mean number of packets in the buffer (Figure 4), follows the same behaviour and results are readily interpretable. The M/M/1 queue curve shown in the above plots, seriously underestimates the mean waiting time. This corroborates the work by Paxson[17].

5. CONCLUSIONS

In this paper, we have considered a counting process (herein called CIPP) with correlated interarrivals and derived its distribution. We have successfully applied the process for performance modeling of statistical multiplexer with the *real world* VBR traffic. The results show that the queueing behaviour of CIPP approximates well the queueing situation as encountered in VBR traffic in broadband networks for infinite capacity cases. We believe that, the theoretical approach proposed in this paper, will be a useful tool in supporting real-time ATM network traffic management. Future work involves considering the priority queues with CIPP arrivals.

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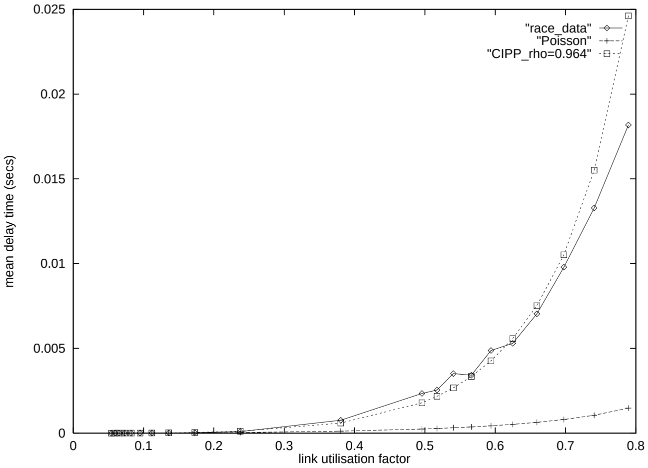

Figure 3: Mean waiting of packets in a queue with *race* video sequence input and that of approximating CIPP

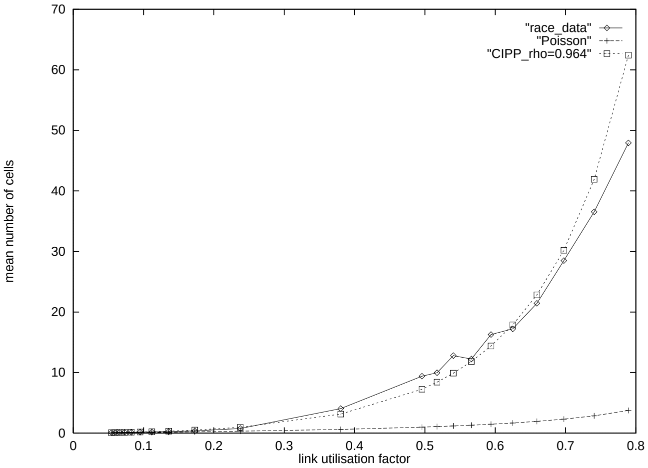

Figure 4: Mean number of packets in a queue with *race* video sequence input and that of approximating CIPP

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