GOSPAL: An Efficient Strategy-Proof Mechanism for Constrained Resource Allocation

Indu Yadav
Dept. of Electrical Engineering
IIT Bombay, India
indu@ee.iitb.ac.in

Prasanna Chaporkar
Dept. of Electrical Engineering
IIT Bombay, India
chaporkar@ee.iitb.ac.in

Abhay Karandikar
Dept. of Electrical Engineering,
IIT Bombay, India
Director and Professor, IIT Kanpur
karandi@ee.iitb.ac.in

ABSTRACT

We consider allocation of a resource to multiple interested users with a constraint that if the resource is allocated to user i, then it cannot be allocated simultaneously to a predefined set of users $\mathscr{S}_i$. This scenario arises in many practical systems that include wireless networks and constrained queuing systems. It is known that the socially optimal strategy-proof mechanism is not only NP-hard, but it is also hard to approximate. This renders optimal mechanism computationally infeasible to use in practice. Here, we propose a computationally efficient mechanism and prove it to be strategy-proof. Using Monte Carlo simulations, we show that the social utility of the proposed scheme is close to that of the optimal. Further, we demonstrate how the proposed mechanism can be used for fair and efficient short-term spectrum allocation in resource-constrained large wireless networks.

CCS CONCEPTS

• **Theory of computation → Algorithmic game theory and mechanism design**;

KEYWORDS

Strategy-Proof Auctions, Resource Allocation, Wireless networks

ACM Reference Format:
Indu Yadav, Prasanna Chaporkar, and Abhay Karandikar. 2019. GOSPAL: An Efficient Strategy-Proof Mechanism for Constrained Resource Allocation. In *12th EAI International Conference on Performance Evaluation Methodologies and Tools (VALUETOOLS 2019), March 12–15, 2019, Palma, Spain.* ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3306309.3306340

1 INTRODUCTION

In today's world with increasing demand, efficient use of the limited and scarce resource has become a challenge. Generally, resource allocation mechanisms are designed as per the goals of the system, which may include maximization of social utility, efficient and fair utilization of limited resource and maximization of revenue. Auction-based mechanism [5] is a popular way of distributing the available resource among users. A well known Vickrey-Clarke-Groves (VCG) auction provides a framework for designing a strategy-proof mechanism that maximizes social utility [2, 3, 6]. Due to the computational infeasibility of VCG in many scenarios of interest, we are required to find alternatives. One such scenario called a constrained resource allocation problem is considered here. In this problem a single resource can be allocated to multiple users who are selfish and rational.

2 SYSTEM MODEL

We consider a framework for resource allocation which comprises of an auctioneer, database and a set of users. Auctioneer, a key entity, decides resource allocation and pricing schemes. The database contains information of resource available for allocation. We assume that the time is slotted, where each slot is called allocation frame. Each user i has a constraint set $\mathscr{S}_i$, such that at most one user in $\{i\} \cup \mathscr{S}_i$ can get a resource. Note that if $\{i\} \in \mathscr{S}_j$, then $\{j\} \in \mathscr{S}_i$. Each user i communicates bid valuation $q_i \in \mathscr{R}_+$ to the auctioneer at the starting of the allocation frame, which may be different from actual valuation $r_i \in \mathscr{R}_+$. Auctioneer uses some allocation mechanism π which outputs vectors $[x_i^\pi]_{i=1,\ldots,n}$ and $[p_i^\pi]_{i=1,\ldots,n}$ based on the received bids. Here, $x_i^\pi(\boldsymbol{q}) = 1$ if user i is allocated resources under π and 0 otherwise. Moreover, $x_i^\pi(\boldsymbol{q}).p_i^\pi(\boldsymbol{q})$ is the price charged from user i under π. Utility of user i is $U_i^\pi(\boldsymbol{q}) = (r_i - p_i^\pi(\boldsymbol{q})).x_i^\pi$.

Definition 1. *A mechanism π is truthful (strategy-proof) if*

$$U_i^\pi(r_i, \boldsymbol{q}_{-i}) \geq U_i^\pi(\boldsymbol{q}), \text{ for all } \boldsymbol{q} \in \mathscr{R}_+^n.$$

3 GOSPAL MECHANISM

In this section, we describe an efficient strategy-proof mechanism for resource allocation. The mechanism is implemented in two phases: (1) Resource Allocation phase and (2) Pricing phase. Resource auctions happen at the beginning of every allocation frame and users can use the allocated resource for the frame duration.

Resource Allocation: First step is to randomly partition the set of all users $\mathscr{N}$ into at most η groups denoted as $\{G_1, \ldots, G_\eta\}$ such that $i_1, i_2 \in G_j$ then $i_1 \notin \mathscr{S}_{i_2}$, where $\eta = \max_{i \in \mathscr{N}} |\mathscr{S}_i| + 1$. Now, let Ω_g denote the set of all possible orderings of the sets $\{G_1, \ldots, G_\eta\}$. Thus, $|\Omega_g| = \eta!$. Furthermore, let $\omega_j \in \Omega_g$ denote the j^{th} ordering of the groups in the set Ω_g. We denote ω_j by a tuple $(G_{j1}, \ldots, G_{j\eta})$. A resource allocation, given group ordering ω_j is done as follows. We first assign the resource to each user in G_{j1}, then to all the users in $G_{j2} \setminus (\cup_{i \in G_{j1}} \mathscr{S}_i)$, and so on. For example, if $\eta = 3$, then there are 6 different orderings. One of the possible 6 group ordering or tuple is $\omega_j = (G_2, G_1, G_3)$. Grouping does not depend on bids. The social

VALUETOOLS 2019, March 12–15, 2019, Palma, Spain

ACM ISBN 978-1-4503-6596-3/19/03.
https://doi.org/10.1145/3306309.3306340

utility under allocation $\boldsymbol{x}(j)$ corresponding to tuple ω_j is given as

$$\tilde{U}_j(\boldsymbol{q}) = \sum_{i=1}^{n} q_i x_i(j).$$

Moreover, define $j^\star_{\boldsymbol{q}} = \arg\max_{\{j:\omega_j \in \Omega_g\}} \tilde{U}_j(\boldsymbol{q})$. Thus, $\omega_{j^\star}$ is the group permutation for which perceived utility is maximized among all possible group permutations. We propose to choose resource allocation $\boldsymbol{x}(j^\star_{\boldsymbol{q}})$. Note that even though the grouping does not depend on the bids $\boldsymbol{q}$, the chosen resource allocation does. Let $\tilde{U}^\star(\boldsymbol{q})$ denote the maximum value of the perceived social utility for the bids $\boldsymbol{q}$.

Pricing Mechanism: Let $(\epsilon, \boldsymbol{q}_{-i})$ denote the bid vector in which the bids of all the users except i are same as that in $\boldsymbol{q}$, but the bid of user i is $\epsilon > 0$. Now, the price charged from the user i is given as:

$$p_i(\boldsymbol{q}) = \left[\lim_{\epsilon \downarrow 0} \tilde{U}^\star(\epsilon, \boldsymbol{q}_{-i}) - (\tilde{U}^\star(\boldsymbol{q}) - q_i)\right] \times x_i(j^\star_{\boldsymbol{q}}). \quad (1)$$

Grouping based Optimal Strategy-Proof Allocation (GOSPAL) algorithm is strategy-proof. For proof see article [8].

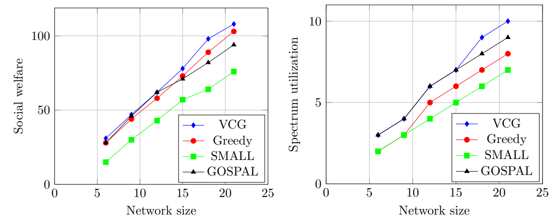
4 SIMULATION RESULTS

In this section, we evaluate the performance of GOSPAL with other existing strategy-proof spectrum allocation mechanisms using Monte Carlo Simulations. We model wireless network as a conflict graph $\mathscr{G} = (V, E)$. In the graph nodes and edges denote base stations and the interfering pair of base stations (BSs), respectively. We randomly generate graphs with given degree distribution of nodes (BSs) using configuration model [1]. We compare the performance of the network for the following parameters:

- Social Welfare: It is defined as the sum of the valuations of the base stations which are assigned channels.
- Spectrum Utilization: It is defined as the total number of base stations which are assigned channels in the allocation phase.
- Fairness across time: It quantifies disparity between the average number of times the channel is allocated to various base stations.

We consider small networks of size up to 21 BSs to compare the result with the optimal outcome of VCG. We assume bid at each BS is uniformly distributed in the interval $[5, 15]$ and the maximum degree of a BS is restricted to 4. We consider only one channel is available in the database for allocation. All the results are averaged over 100 iterations for 100 different topologies and bid values across base stations. In Fig. 1, it is observed that the social utility of GOSPAL and greedy [9] is close to the optimal utility obtained using VCG, whereas the performance of SMALL [7] is the worst among all the mechanisms. The poor social utility/welfare of SMALL is justified as BSs of only one group can be allocated channel at the cost of sacrificing the lowest bidder in the group. In spectrum utilization performance of GOSPAL is close to the VCG mechanism. Moreover, GOSPAL provides a significant improvement in spectrum utilization over greedy and SMALL (see Fig. 1).

Next, we perform simulations to see how various algorithms perform when the resource allocation process is repeated periodically. To understand the fairness in resource allocation across users, we generate a random network topology. For the given topology, we observe resource allocation for 100 different bid values uniformly distributed in the interval $[15, 25]$ at each BS,and determine Jain's



(a) Social Welfare **(b) Spectrum utilization**

Figure 1: Performance comparison for different algorithms in small network.

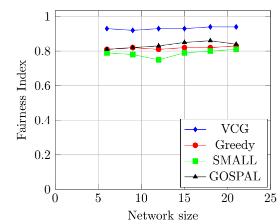


Figure 2: Fairness measure for various alogrithms.

Fairness index [4]. We consider bids are independent and identically distributed (iid) across users and time. The fairness for various schemes is shown in Fig. 2. It can be seen that GOSPAL achieves better fairness compared to the other sub-optimal mechanisms greedy and SMALL.

5 CONCLUSION

In this paper, we consider resource allocation problem among multiple users with constrained set. We propose a startegy-proof and computationally efficient mechanism GOSPAL, which is feasible to implement even in large number of users. Using simulation results, we observe that GOSPAL achieves social utility and resource utilization close to the optimal. GOSPAL also achieves better fairness index for resource allocation among the users in comparison to the other existing schemes.

Acknowledgement: This work is funded by the Ministry of Electronics and Information technology (MeitY), Government of India.

REFERENCES

[1] Fan Chung and Linyuan Lu. 2002. Connected Components in Random Graphs with given Expected Degree Sequences. *Annals of combinatorics* 6, 2 (2002), 125–145.
[2] Edward H Clarke. 1971. Multipart Pricing of Public Goods. *Public choice* 11, 1 (1971), 17–33.
[3] Theodore Groves. 1973. Incentives in Teams. *Econometrica: Journal of the Econometric Society* 41, 4 (1973), 617–631.
[4] Rajendra K Jain, Dah-Ming W Chiu, and William R Hawe. 1984. A Quantitative Measure of Fairness and Discrimination. *Eastern Research Laboratory, Digital Equipment Corporation, Hudson, MA* (1984).
[5] Vijay Krishna. 2009. *Auction Theory.* Academic press.
[6] William Vickrey. 1961. Counterspeculation, Auctions, and Competitive Sealed Tenders. *The Journal of finance* 16, 1 (1961), 8–37.
[7] Fan Wu and Nitin Vaidya. 2011. SMALL: A Strategy-proof Mechanism for Radio Spectrum Allocation. In *IEEE INFOCOM.* 81–85.
[8] Indu Yadav, Prasanna Chaporkar, and Abhay Karandikar. 2018. GOSPAL: An Efficient Strategy-Proof mechanism for Constrained Resource Allocation. *arXiv preprint arXiv:1811.11578* (2018).
[9] Xia Zhou, Sorabh Gandhi, Subhash Suri, and Haitao Zheng. 2008. eBay in the Sky: Strategy-proof Wireless Spectrum Auctions. In *ACM ICMCN.* 2–13.