Correlated Interarrival time Poisson Process (CIPP) for Modeling Broadband Teletraffic

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Abstract

In this paper, we propose a new traffic model (with correlations in interarrivals) for both composite arrival process of packets in broadband networks and individual source modeling. The traffic model involves a generalization of Poisson process which is referred to as Correlated Interarrival time Poisson Process (CIPP). The CIPP, a stationary counting process, is parameterized by a correlation parameter 'ρ' which represents the degree of correlation in adjacent interarrivals in addition to 'λ' the intensity of the process. It is shown by simulations that this arrival process models well the real time traffic source, in particular the video source. The burstiness measure [1] corresponding to this process is evaluated. We explore the self-similar behavior of CIPP by various methods including wavelet analysis. It turns out that although CIPP is *not* strictly a self-similar model, it *does* exhibit self-similar behaviour over a range of time scales. We study various properties of CIPP relevant for teletraffic modeling.

Keywords CIPP, interarrivals, burstiness measure, self-similarity, teletraffic modeling.

1 Introduction

The integral part of broadband networks is the statistical multiplexer. Statistical multiplexing involves switching any of the N input traffic streams (which are asynchronous and statistical) to any of the N output streams. Now, each of the N output streams can be modeled as a single server queue in which the arrival stream being the superposition of the N input streams in general. Accordingly, this single server (with such arrival streams) has been of interest to telecommunication researchers over the years. Hence, understanding the nature of traffic in such heterogeneous multiservice networks is essential for engineering, operations and performance evaluation of these networks. In literature, the following models for cell[1] level traffic have been reported: Poisson, Bernoulli, and D-MAP [2]. Researchers

[1] Fixed size (53 byte) ATM packet.

have used the following analytical models for traffic at burst level: on-off processes, Non-Renewal Markov Chain (MC) [3], Interrupted Poisson Process (IPP) [4], Markov Modulated Poisson Process (MMPP) [1, 5], and Gaussian models [6]. The above models are essentially, short range dependent (SRD) and they rely heavily on the modulating process for their correlation in increments.

A significant contribution in the area of teletraffic modeling is the discovery of long-range dependence (LRD) in Ethernet LAN traffic in Bellcore [7], metropolitan area network (MAN) traffic [8], wide area network (WAN) traffic and variable bit rate (VBR) video traffic [9]. Moreover, teletraffic researchers have observed that there is a *consistent positive correlation* in the packet interarrivals of Ethernet traces (p. 229 of [10]) and in the cell interarrivals from video codecs [11]. All these led Poisson process obsolete for modeling the broadband teletraffic (despite its analytical simplicity) and over and above, forced researchers to discard the conventional short-range dependent (SRD) models and look for models based on LRD and self-similarity [12, 10, 13, 14]. But, it is to be noted that models based on self-similarity or LRD are analytically *intractable* (except that, they have some bounds for their queueing performance [13, 14]). Hence, researchers were looking for models, which exhibit self-similarity to a range of time scales on one-end and on the other hand are analytically tractable. But, this pseudo-self-similarity could be brought in by introducing strong correlation in increments in the candidate models. The correlation in increments could be brought in a number of ways: (i) by considering correlations in the counts sequence, (ii) by considering correlations in the rate process, (iii) by introducing correlations by modulating the underlying arrival process through a Markov chain, or (iv) by considering correlations in the *interarrival sequence* itself. In literature, researchers have considered models which capture correlation in the counts sequence (packets per unit time) as in discrete autoregressive (DAR(1) model [15]), or in the (traffic) rate process as in autoregressive (AR) model [16] or self-similar models [7, 14], or the correlation introduced by the modulating Markov chain as in Markov modulated models [17]. Hence, it is of interest to come up with a counting process whose interarrivals are correlated, develop the corresponding queueing theory and apply it for performance modeling of real world queues.

In light of the above, we have defined such a process (herein called Correlated Interarrival time Poisson Process (CIPP)), derived its distribution and developed the corresponding queueing theory [18]. The main idea here is to approximate the LRD behavior by extremely large and slowly decaying SRD (by setting a high correlation between adjacent interarrival times). In this paper, we will restrict our discussions to modeling broadband teletraffic by CIPP, leaving the performance modeling of real world traffic by CIPP queues to a separate paper. The paper is organized as follows: In Section 2, we give the mathematical derivation of the CIPP process. Section 3 gives the statistical properties of the CIPP and its properties which are relevant for broadband teletraffic modeling. In Section 4, validation of CIPP for modeling broadband teletraffic is presented. Section 5 concludes the paper.

2 Correlated Interarrival Time Poisson Process (CIPP)

Numerous researchers have *compounded, modified and generalized* the Poisson process [19]. Their attempts have been to manipulate λ (say, expressing it as a linear function of 't') and to get a model which suits a specific application. While the motivation for the Generalized Poisson Distribution(GPD) as introduced by Consul and Jain [20], is to explain the extraordinary value of frequency or unequal mean and variance in the numerical data in different fields, the motivation for correlated Poisson Distribution (CPD) introduced by Drezner [21] is to explain the correlations. They do not consider explicit assumption of correlation in interarrival times. But this has been attempted by Wold [22], Cox [23], and Gaver and Lewis [24]. Wold [22] has made an attempt to generalize the Poisson process by obtaining dependent, but marginally exponentially distributed intervals between events. Similarly, Cox [23] has defined a sequence of random variables with conditionally exponential distributions. Here, we assume correlation in interarrivals explicitly, and arrive at a distribution.

Axiomatic Derivation of CIPP: We wish to define a counting process $N(t)$ which retains as far as possible all the nice properties of the Poisson process, but has correlated interarrivals. Typically, we would like to have interarrivals being exponentially distributed apart from being correlated. Motivated by the analytical simplicity of the method of generating stationary correlated exponential sequence (first order exponential autoregressive process (EAR(1))) developed by Gaver and Lewis [24], we present the following formalism for defining CIPP. Consider arrivals occurring in time on the interval $(0, \infty]$. For $t > 0$, let $N(t)$ be the number of arrivals that have occurred in the half-open interval $(0, t]$. Consequently, $N(t)$ and $N(t+h) - N(t) \quad (h > 0)$, assume only non-negative integer values. We now state the following axioms.

Axioms:

1. Since we begin counting arrivals at time 0, we define $N(0) = 0$.

2. **Correlation Structure for Interarrivals:** Let X_n be the interarrival time between the n^{th} arrival instant (T_n) and $(n-1)^{th}$ arrival instant (T_{n-1}). Then,

$$X_{n+1} = \rho X_n + \epsilon_{n+1} \qquad 0 \leq \rho < 1 \quad n = 1, 2, 3 \ldots \tag{1}$$

where, $\{\epsilon_n\}_{n=2}^{\infty}$ is an iid sequence, with $\epsilon_n,\ n > 1$, being a product of Bernoulli random variable[2] (B) with parameter ρ and exponential random variable (V) with parameter λ. B is statistically independent of V. $\{X_n\}$ forms a stationary sequence with the exponential distribution characterized by parameter λ.

3. The counting process $N(t)$ is strictly stationary, that is, for any $r = 1, 2, \ldots$ the joint distribution of $\{N(h+t_{u1}) - N(h+t_{l1}), N(h+t_{u2}) - N(h+t_{l2}), \ldots, N(h+t_{ur}) - N(h+t_{lr})\}$ is independent of h, for any $h > 0$ and $0 < t_{l1} < t_{u1} < t_{l2} < t_{u2} < \ldots < t_{lr} < t_{ur} < \infty$

Definition: A counting process $\{N(t), t > 0\}$ satisfying Axioms 1 through 3 is called Correlated Interarrival time Poisson Process (CIPP) with parameter λ and ρ.

Remark 1: The exponential distribution for interarrivals of the counting process $N(t)$

[2] The random variable B can take values 0 or 1 with probabilities ρ and $1-\rho$ respectively.

follow from Axiom 2. The following properties are the consequence of the exponential distribution for interarrivals[3].

1. For any $t > 0$, $\quad 0 < Pr\{N(t) > 0\} < 1$. In words, in any interval (no matter how small) there is a non-zero probability that an arrival will occur.

2. For any $t \geq 0$,
$$\lim_{h \to 0} \frac{Pr\{N(t+h) - N(t) \geq 2\}}{Pr\{N(t+h) - N(t) = 1\}} = 0 \tag{2}$$
In words, in sufficiently small intervals, at most one arrival can occur: i.e., it is not possible for arrivals to happen simultaneously. The process is *orderly*, in the sense of Daley *et al* [25].

Remark 2: The main difference between the axiomatic definition of a Poisson process and CIPP, is the addition of Axiom 2 and the relaxation of the independence increment assumption (in Axiom 3).

Let $P_n(t) \triangleq Pr\{N(t) = n\}$, be the probability that the number of arrivals in the interval $(0, t]$ is n.

Lemma 1 *CIPP with parameter λ and ρ has the distribution,*

$$P_n(t) = \left\{ \begin{array}{l} (1 + \rho - (1 - \rho)\alpha_n) \\ \sum_{j=0}^{n} \frac{\alpha_j \prod_{k=0}^{n} [\alpha_j - \rho\alpha_k]}{(\rho(\alpha_j + 1) - \alpha_n + 1)(\alpha_j - \alpha_n + 1)(\alpha_j - \rho\alpha_n) \prod_{k=0, k \neq j}^{n} [\alpha_j - \alpha_k]} e^{-\frac{\lambda t}{\alpha_j}} \quad 0 \leq \rho < 1 \end{array} \right. \tag{3}$$

for $\quad n = 0, 1, 2, \ldots, \; \lambda > 0, \; t \geq 0, \quad$ *and* $\quad \alpha_j = \sum_{i=0}^{j} \rho^i$

Proof: See Appendix A for proof.

Remark 3: The prime reason for assuming (1) for the interarrival sequence, is to mimic the self-similar behavior in increments for the corresponding counting process. Note that, to mimic a self-similar process with $0.5 < H < 1$ it is necessary that the increments

[3] In fact, these are the general properties of a wide class of counting processes whose interarrivals are exponentially distributed.

should have positive correlations [26]. The reason for this particular correlation is that, (1) introduces strong positive correlation than other models based on moving average structure like EMA(1) proposed by Lawrance [27] and the one proposed by Finch [28]. We will see in later sections that, as ρ increases, the estimated local Hurst parameter $(0.5 < H < 1)$ increases. The second reason is that the correlation structure (1) allows one to derive a closed form expression for $P_n(t)$ with reasonable simplicity, which is very difficult if not impossible with the other aforementioned models. The third reason is that the analytical simplicity of the correlation structure (1) allows one to develop the corresponding queueing theory. Finally, the sequence $\{X_n\}$ is obtained as an additive random linear combination of random variables, and is thus easy to simulate on the computer.

Remark 4: In (3), as the limit $\rho \rightarrow 0$, we obtain the Poisson distribution. Thus, the Poisson distribution is approached continuously from the CIPP distribution.

Remark 5: We believe that the models that capture the correlation in the interarrival sequence is more appropriate in the context of modeling broadband teletraffic than the models that capture the correlation in the counts sequence (as done in [15] for VBR modeling) or capture correlation by the modulating Markov chain as in SRD models. Correlations in interarrivals *necessarily* imply correlations in the number of counts; however, the converse of this statement is *not* necessarily true. A good example would be a renewal process. Moreover, for an interarrival sequence forming first order Markov chain, the corresponding counts sequence (the sequence of number of arrivals in the fixed length, consecutive and disjoint intervals) possesses, in general, much stronger correlation structure (than the first order Markov chain). In this context, we would like to mention that, the discrete version of (1), namely the discrete auto-regressive (DAR(1)) is used in [15] to model the *frame size* of VBR traffic. Here, the counts rather than interarrivals form a Markov chain.

Remark 6: Note that, the number of parameters is minimal in CIPP as compared to the conventional short-range dependent (SRD) models like MMPP, DBMAP, ON-OFF processes etc. For example in MMPP, the number of parameters is N^2 for N state MMPP

($N^2 - N$ for infinitesimal generator and N for the intensity matrix Λ) while in CIPP it is always 2 namely λ and ρ.

Remark 7: It has been shown in Section 3 empirically that there exists a monotonic relationship between the correlation parameter ρ of CIPP and the estimated local Hurst parameter (a measure of degree of self-similarity) of the corresponding CIPP trace. On the contrary, in SRD models addressed in literature, too many parameters decide the degree of pseudo-self-similar behavior and one does not know which of them have crucial role in exhibiting pseudo-self-similar behavior. The above feature of CIPP makes it more suited for practical purposes, when one needs to fit CIPP to a real world data sequence (say a video sequence) of given H parameter.

Remark 8: For many reasons, using an excessive number of parameters is undesirable, especially because it increases the uncertainty of the statistical inference and the parameters are difficult to interpret. In our case, CIPP has only two parameters ρ and λ and as we mentioned, both of them have a nice physical interpretation. λ denotes the arrival rate and ρ gives the degree of self-similarity. While λ can be estimated from the mean of the process, ρ can be estimated by matching the higher order statistics of the CIPP model with the data.

Remark 9: CIPP and the Neuts Process Here, we argue that the CIPP is *not* a special case of the Neuts process. In the Neuts process [29], the memory of the process is limited to the latest phase renewal instant and the process completely *forgets* the past after that point. Note, to exhibit self-similarity or long range dependence, the process should have infinite memory. In CIPP, there is *no* renewal instant. The memory of the CIPP process at time t, is limited to the arrival instant just prior to the latest arrival instant (and the latest interarrival is again dependent on its previous interarrival and so on.) Quantitatively, this extent of memory is controlled by the correlation parameter ρ.

Within a phase renewal interval, if we allow an arrival *only* during (i, j) transitions, (the process so constructed is called Markov arrival process) one may find that interarrivals are correlated. But, this is due to the difference in marginal distribution of *holding times* of the modulating Markov chain irrespective of probability transition matrix. When there is no difference in the marginal distribution of *holding times*, it reduces to a renewal process (within the phase renewal interval). But, in CIPP, the interarrivals form a stationary sequence, but, still are correlated.

3 Properties of CIPP

For fitting the CIPP model to real-world arrival processes and the performance modeling of real-world queueing situations using CIPP queues, it would be advantageous to have explicit expressions for the statistical moments of CIPP in terms of the parameters of CIPP. To that end, we investigate some statistical properties of CIPP and also its properties relevant for teletraffic modeling in this section. Moreover, we discuss about the correlational property of the CIPP process and how it leads to large scale correlations.

Mean: By virtue of (3.2) in [30], the mean of the CIPP process is λt. Mathematically, $\mathbf{E}\{N(t)\} = \lambda t$.

Correlation in Interarrivals: The correlation coefficient for interarrival time sequence is defined by,

$$\eta(k) \triangleq \frac{E\{X_i X_{i+k}\} - E^2\{X_i\}}{\sigma_{X_i}\sigma_{X_{i+k}}}$$

where, σ_{X_i} is the standard deviation of the ith interarrival time. Exploiting the stationarity of the random sequence $\{X_i\}$, one can show that the correlation coefficient of the interarrival time sequence $\{X_i\}$ in CIPP is given by $\eta(k) = \rho^k$ (see [18] for derivation).

Burstiness Measure: We consider the burstiness measure as defined in [1]. We repeat it here for convenience,

$$C_p(z) = \sum_{k=0}^{\infty} C_k z^k/(\lambda^{-1})^2, \qquad |z| < 1 \tag{4}$$

where λ^{-1} is the mean interarrival time. $C_k = \mathbf{E}\{(t_0 - \lambda^{-1})(t_k - \lambda^{-1})\}$ denotes the correlation with lag k for interarrival times and $t_0, t_1, \ldots$ are a sequence of interarrival times. $C_p(z)$ includes the complete information for the second moment of interarrival times. For Poisson process, $C_p(z) = 1$ for all z. For a renewal arrival process, $C_p(z) =$ (squared coefficient of variation in the renewal process) for all z. For a MMPP, burstiness measure has been discussed in [1]. For CIPP, it can be shown that, the correlation with lag 'k' of interarrival times (C_k) is given by $C_k = \frac{\rho^k}{\lambda^2}, \quad \rho < 1$. As a result, $C_p(z) = \frac{1}{1-\rho z}$. Figure 1 shows $C_p(z)$ versus $|z|$. One can see that CIPP does display burstiness for large ρ.

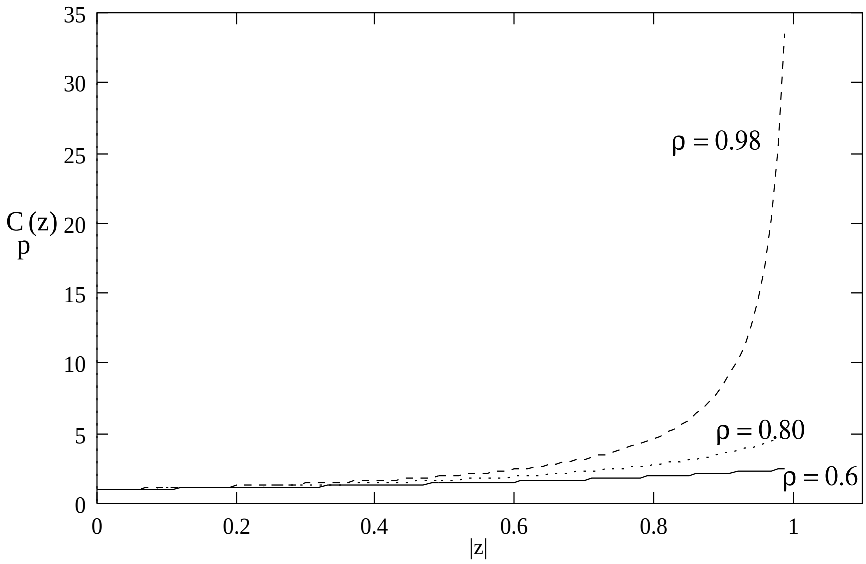


Figure 1: $C_p(z)$ versus $|z|$ for CIPP

Self-similar Property of CIPP: To compare the behavior of CIPP with the Bellcore dataset, we generate samples which are equal in length and with the same mean value as the various traces analyzed in [7] and [31]. For the synthetic traces we generate, we use $\rho = 0.976$ and $\lambda = 318.18$ arrivals per sec. The λ selected here is equal to the mean packet arrival rate in August Bellcore dataset (`pAug.TL`) and ρ is obtained by matching the correlation coefficient of the CIPP and the data (see algorithm 1 in next section). To get an idea of self-similarity of CIPP, we resort to a pictorial representation analogous to those given in [7] and [31]. We select, at random, sections of the simulated traffic and the August Bellcore dataset and plot them side by side at the same resolution level. This is shown in Figure 2. The simulated CIPP process exhibits a similar bursty behavior as the Bellcore dataset over all plotted scales. Note that the length of the Bellcore August dataset limits the number of points we can plot at the coarsest scale.

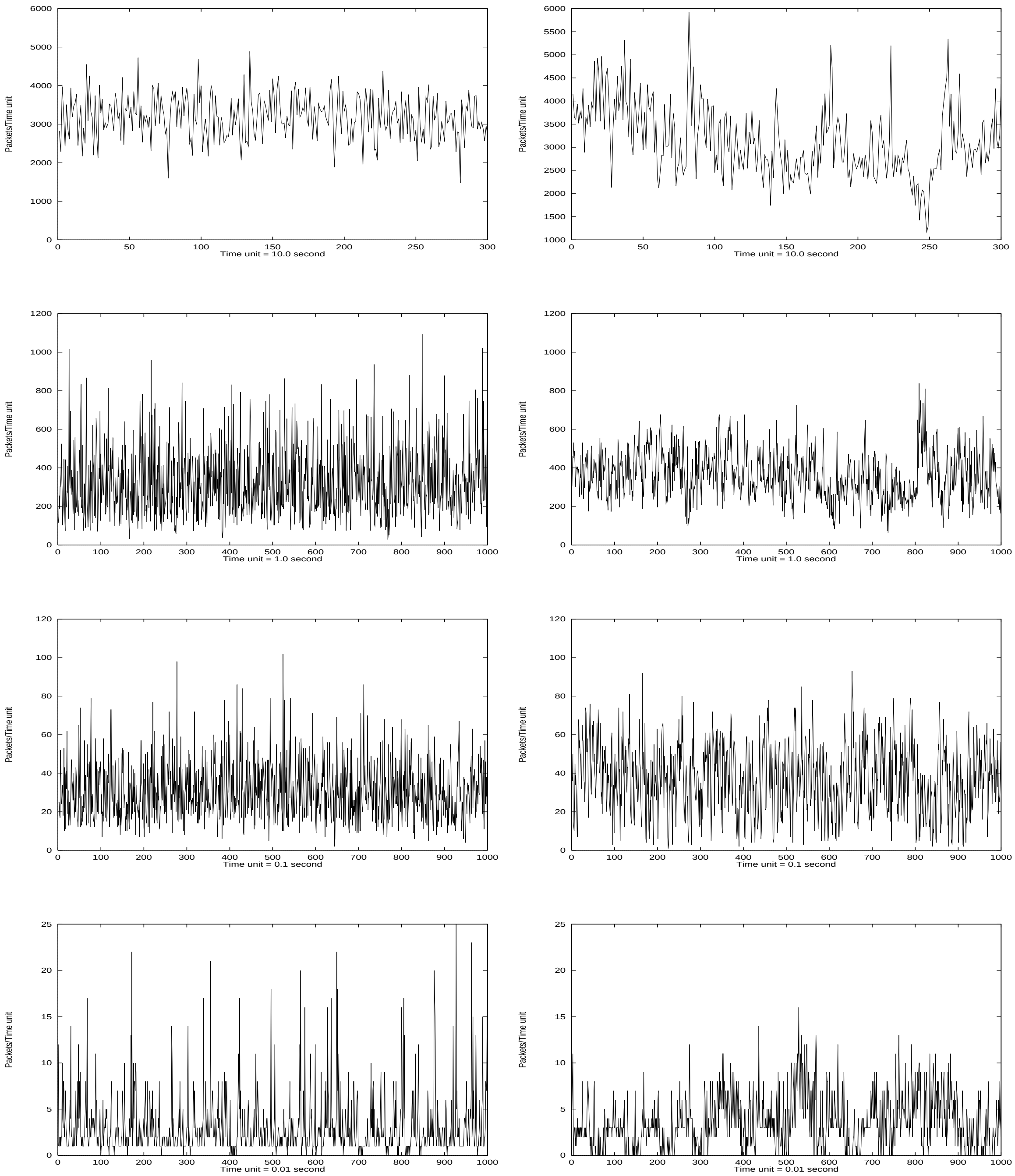
Next, we generate CIPP with different ρ and estimate the Hurst (H) parameter of the process, by the wavelet estimator proposed by Abry *et al* [32]. In Figure 3, we plot the estimated Hurst parameter for CIPP versus the correlation parameter. Note that the estimated Hurst parameter H is 0.5, corresponding to $\rho = 0$, which is true for the Poisson process. The plot shows that we can model teletraffic with higher H by using a CIPP with a larger ρ.

The next plot (Figure 4 (a)) is the log (Variance) versus log (aggregation level) plot for CIPP and *Bellcore* dataset. The lower most curve is a reference curve with slope 1. The behaviour is strikingly similar, with the variances for both processes decaying at the same rate, slower than 1, with increasing aggregation level.

We finally present the wavelet analysis of CIPP. Wornell [33] has proved that as long as the analyzing mother wavelet has at least one vanishing moment, the *log of variance* of the wavelet coefficients for a self-similar process increases linearly with the scale with the slope being the Hurst parameter. This interpretation of wavelet transform gives a convenient way of looking at the variance-scale plots which has been given in [31] and is used here. Figure 4 (b) shows such a plot for various processes and CIPP. One may see that for pure self-similar processes, the plot results in a linear curve. For pseudo self-similar processes the curve increases linearly and saturates at a point, where the slope is zero. Note that for CIPP the curve is linear to a much wider scale than ON-OFF processes. Also, observe that the linearity increases as ρ increases.

We now compare CIPP with Bellcore August dataset, using wavelet analysis. We have analyzed the two signals using the Daubechies-4 [34] wavelet. The plots of log of variance of wavelet coefficient versus log2 (scale) for the simulated CIPP and Bellcore August dataset shows a strong resemblance in the properties of the two processes as seen in Figure 5.

Correlational Property of CIPP: Define, $L_n(t)$ is the time required for the occurrence of nth event after t to occur. It is well known that $L_n(t)$ is exponentially distributed (See Figure 6) for memoryless processes (like say, Poisson processes). Since the presence of



Simulated CIPP trace Bellcore August Dataset

Figure 2: Demonstration of self-similar property of CIPP: A "Visual" justification.

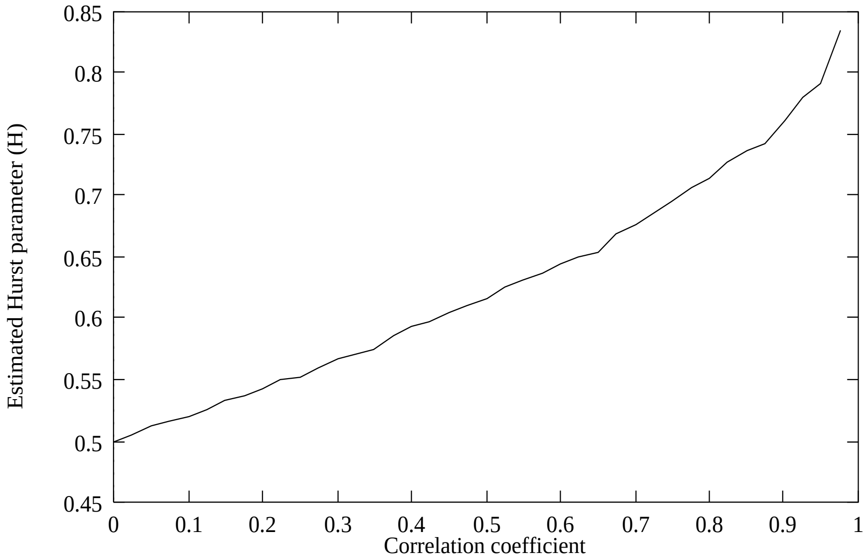
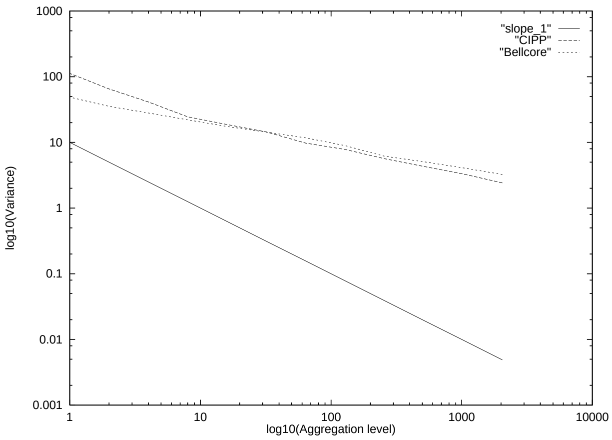


Figure 3: Estimated local Hurst parameter versus the correlation parameter

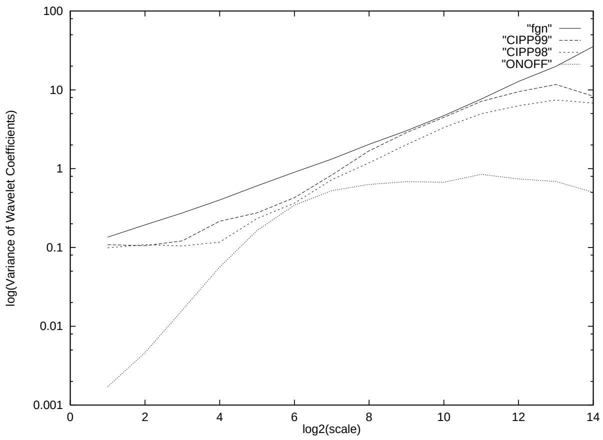
after-effects (or memory) is an important correlational property, a process can be specified by a complete pre-history up to the instant 't' under consideration. Varying degrees of non-Markovian nature may reduce the degree of indispensability of the complete pre-history. For instance, if the process is the renewal process, the pre-history need extend only up to the previous event. For this reason, Khintchine calls a renewal process, "a process with limited after-effect". This is illustrated in Figure 6. For a non-renewal process it may be necessary to specify the history even beyond the previous event. In CIPP case, the pre-history extends up to the occurrence of event previous to the latest event occurred. This feature is responsible for the self-similar behaviour to a range of time scales. Moreover, we noted in Section 4 that, the count sequence, $\{u(i)\}_1^\infty$ for CIPP, might have more complex correlation structure. This is another manifestation of the above correlation structure, which is responsible for self-similar behaviour to a range of time scales.

4 Validation of CIPP as a Teletraffic Model

In this section, we justify the CIPP model for modeling broadband teletraffic. In particular, we show that CIPP models well the *real-world* MPEG-1 VBR video traces and also the composite *Bellcore* Ethernet traffic. More specifically, we carry out the "goodness of fit"



(a)



(b)

Figure 4: Self-similar property of CIPP: (a) The variance and aggregation level plot for CIPP and the *Bellcore* dataset (b) Variance scale plots for CIPP and other processes

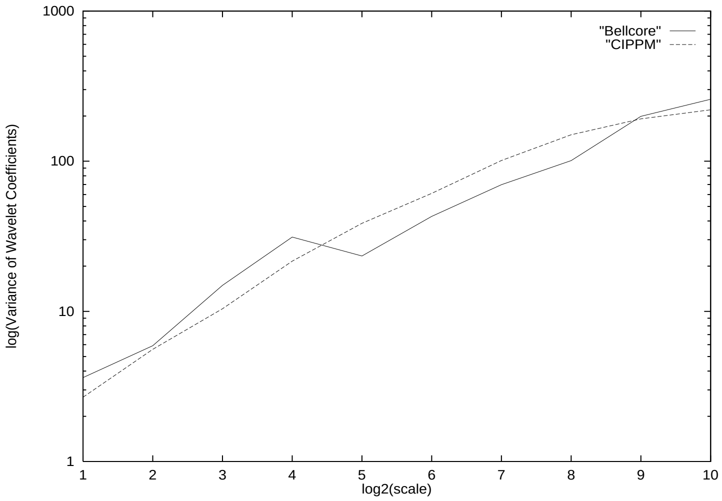


Figure 5: Variance scale plots for Bellcore dataset and the simulated CIPP process

test of CIPP to the trace data based on various second order measures and compare it favorably with the other models addressed in literature.

Fitting the CIPP Model to Real Traffic Data: In order to show that CIPP models well the data, we have to "fit" the CIPP model to the data. This requires the parameters of CIPP be estimated such that this "best fit" CIPP in some sense behaves closer to the data (with respect to some statistical measures). The estimation of λ of the 'best fit' CIPP is relatively simple due to the reason that the measured statistic, the mean of CIPP has an analytical expression involving λ ($\mathbf{E}\{N(t)\} = \lambda t$). For estimating '$\rho$', one may use

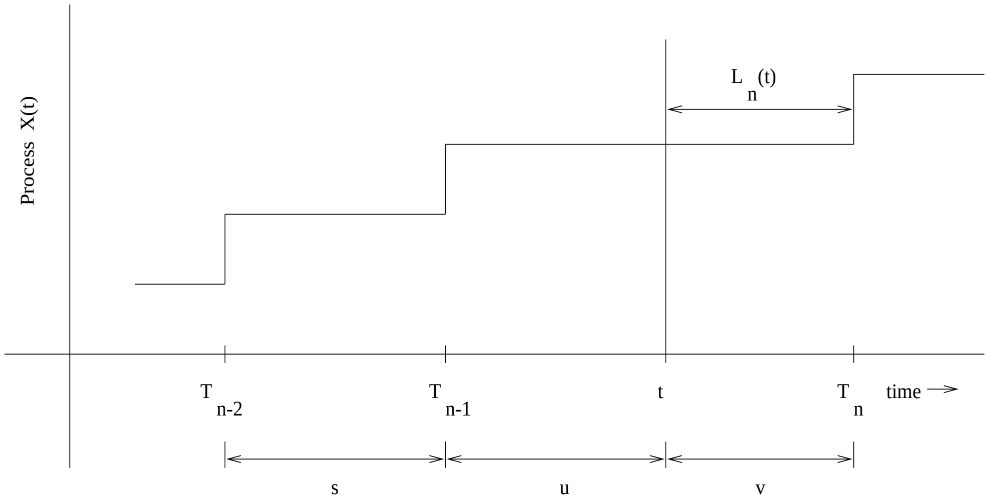


Figure 6: Correlational Property of CIPP

the elegant method of estimating ρ as given in [24]. But, since we are *more* interested in capturing the dependence structure of the real-world data, we use correlation measure (in the counts sequence or in the interarrival sequence) in some form or other for our fitting procedure. In light of the above, we choose to estimate the parameters of the CIPP model with a measure based on any of the following second order properties [35]: (1) autocorrelation of counts sequence (2) index of dispersion for intervals (IDI) and (3) index of dispersion for counts (IDC). Out of the above three, the CIPP has analytical expression only for IDI. For the other two measures, one has to estimate by simulations given λ and ρ.

Autocorrelation of Counts Sequence $\{u_T(i)\}$*:* Let $\{u_T(i)\}_{i=1}^{\infty}$ denote the number of arrivals in i^{th} slot on the positive real axis where each time slot is of equal length, T time units. The sequence $\{u_T(i)\}_{i=1}^{\infty}$ is called as *counts sequence*[4]. Then the correlation coefficient for counts sequence is defined by,

$$\zeta_T(k) \quad = \quad \frac{E\{u_T(i)u_T(i+k)\} - E^2\{u_T(i)\}}{var(u_T(i))}$$

Note that for Poisson process $\zeta_T(k) = 0$ for all $k \neq 0$.

[4]Many approaches can be used to describe packet arrival process: through the *counting process* $N(t)$ or through the sequence of arrival instants $\{T_i\}_{i=0}^{\infty}$ or through the *counts sequence* $u_T(i)$. Here, we would like to mention that the first two descriptions are well defined and the *counts sequence* $u_T(i)$ can be derived from them for a given T.

Index of Dispersion for Intervals (IDI): We use the variance of the sum of n random variables normalized by the factor $nE^2(X)$, as a measure of the variability of packet arrival processes [35]. The sequence of values

$$J_n \triangleq \frac{var(X_{i+1} + \ldots + X_{i+n})}{nE^2(X)} \tag{5}$$

with $n = 1, 2, \ldots$, is called *index of dispersion for intervals (IDI)*. It is shown in [18] that for CIPP J_n is given by,

$$J_n = \frac{1}{n(1-\rho)^2}\left[n(1-\rho^2) + 2\rho^{n+1} - 2\rho\right] \tag{6}$$

Then,

$$J \triangleq \lim_{n\to\infty} J_n = \frac{1+\rho}{1-\rho} \tag{7}$$

Since the CIPP has an analytical expression for J_n, the modeling procedure becomes simple. We have to just estimate ρ from the data, using (6).

Index of Dispersion for Counts (IDC): For packet counts, we can define a function similar to the index of dispersion for intervals [35]. The *index of dispersion for counts (IDC)* at time t is the variance of the number of arrivals in an interval of length t divided by the mean number of arrivals in t: $I_t = \frac{\mathrm{var}(N(t))}{E(N(t))}$ where $N(t)$ indicates the number of arrivals in an interval of length t. The IDC has been defined in this manner so that for a Poisson process, the value of the IDC is 1, for all t. In estimating the IDC of the measured arrival processes, we will only consider discrete equally spaced time instants, $\tau_i \quad (i \geq 0)$. Denoting the number of arrivals in ith interval of duration $T = \tau_i - \tau_{i-1}$, by $u(i)$, we have,

$$I_n(T) = \frac{\mathrm{var}\left(\sum_{i=1}^{n} u_T(i)\right)}{\mathrm{E}\left(\sum_{i=1}^{n} u_T(i)\right)} = \frac{var(u_T(i))}{E(u_T(i))}\left[1 + 2\sum_{j=1}^{n-1}(1 - \frac{j}{n})\zeta_T(j)\right] \tag{8}$$

where $\zeta_T(j)$ is the autocorrelation coefficient of $u(i)$ at lag j. Note that, in general, $I_n(T)$ will not be a constant for renewal processes where counts in the disjoint intervals are correlated (exception is the Poisson process). Note also that the IDC is a function of the length of time unit T, over which the counts sequence is constructed from the original arrival process (characterized by the arrival instant sequence). While we can estimate $\mathrm{var}(N_t)/\mathrm{E}(N_t)$, the representation on the RHS of (8) is valid only if the data is stationary.

Experimental Results: We now attempt to fit a CIPP model to real world traffic traces by first estimating λ of the 'best fit' CIPP from measured mean statistic. Then, ρ of the 'best fit' CIPP is estimated using linear search algorithm for ρ which minimizes the mean square error between the measured statistic (one of the three measures given above) and the corresponding statistic of CIPP.

Autocorrelation of counts $\{u_T(i)\}$*:* Here, we fit a CIPP model to MPEG-1 encoded video trace using correlation coefficient as a measure. Group of Picture (GOP) sizes (sum of frame sizes of one GOP) in the frame size trace from MPEG-1 encoded video sequence, can be thought of as the number of (cell) arrivals (after segmentation of frame size trace) $\{u_{T_{GOP}}(i)\}$ in a GOP time slot (T_{GOP}). We fit a CIPP by using the following algorithm.

Algorithm 1: CIPP model fitting using correlation measure

1. Segment the bits in each frame into cells[5]. We then have the counts sequence $u_{T_f}(i)$ of cells.

2. Estimate the correlation coefficient of this counts sequence of cells. This is the correlation coefficient $\zeta^{data}_{T_{GOP}}(.)$ corresponding to data, to which we intend to match the correlation coefficient of the CIPP model.

3. Set the rate λ_{opt} of CIPP equal to $\mathbf{E}(u_{T_{GOP}}(i))/T_{GOP}$.

4. **Linear search algorithm to estimate ρ_{opt} of CIPP:**

   begin

   min = 99999999.9; step = 0.0001

   for i = 1:9999

   $\rho = step * i$

   estimate $\zeta^{CIPP}_{T_{GOP}}(.)$ for CIPP with λ_{opt} and ρ.

   compute the mean square error (P) between $\zeta^{CIPP}_{T_{GOP}}(.)$ and $\zeta^{data}_{T_{GOP}}(.)$

   if($min > P$)

[5] This is just to avoid the computational overflow while estimating the correlation coefficient of the data

min = P; $\rho_{opt} = \rho$; $\zeta^{fit}_{T_{GOP}}(.) = \zeta^{CIPP}_{T_{GOP}}(.)$;

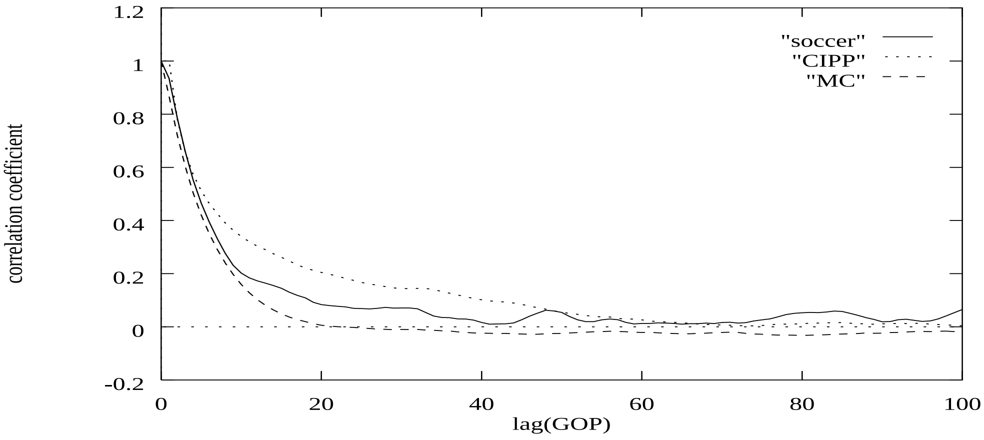
endif

end

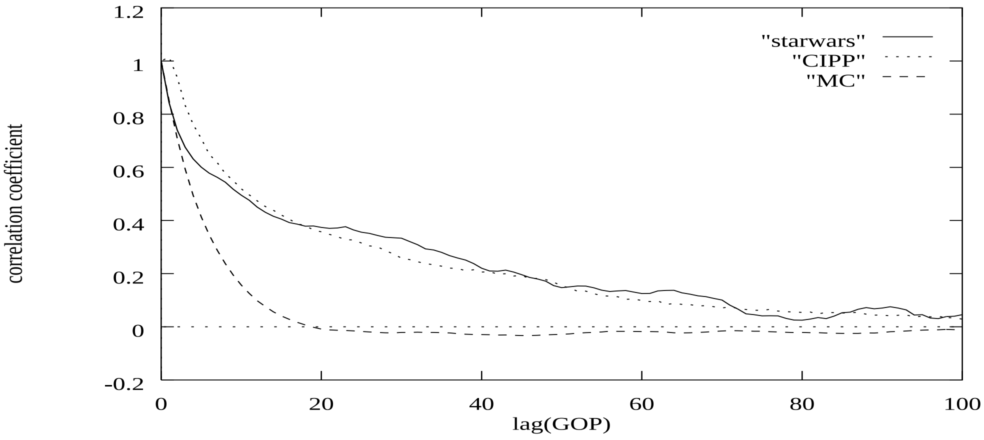
end

The correlation coefficient $\zeta^{fit}_{T_{GOP}}(.)$ is plotted along with the correlation coefficient corresponding to data $\zeta^{data}_{T_{GOP}}(.)$. For comparison, we also plot the result by fitting a Markov chain (MC) [9] to the GOP size process (Our procedure for fitting an MC to data is very similar to [9]). The number of states M in the Markov chain is taken to be G_{max}/σ_G, where G_{max} denotes the size of largest GOP and σ_G the standard deviation of the GOP sizes. Thus, the size of quantization interval is σ_G. Note that it is conventional to consider the autocorrelation function (or equivalently the correlation coefficient) of the GOP sizes since, it is difficult to get a clear picture of the long-range correlations of video traffic stream, from frame level correlations. We consider the frame size trace of MPEG-1 encoded video sequences with various Hurst (H) parameters (ranging from 0.77 to 0.96) namely, “soccer” (H = 0.77), “starwars” (H = 0.85) and “Mr.Beans” (H = 0.96) [36] for our experimental purposes. We carry out the autocorrelation fit for each of these sequences. Our results are illustrated in Figure 7. One can see that for “soccer” (H = 0.77), although both models show almost the same approximation quality, the MC model is more optimistic. However, for higher H parameter video sequences, like “starwars” and “MrBean”, the CIPP performs substantially better than the MC model. Note that the rho of the ‘best fit’ CIPPs to the sequences “soccer”, “starwars” and “MrBean” are $0.992, 0.993$ and 0.9932 respectively.

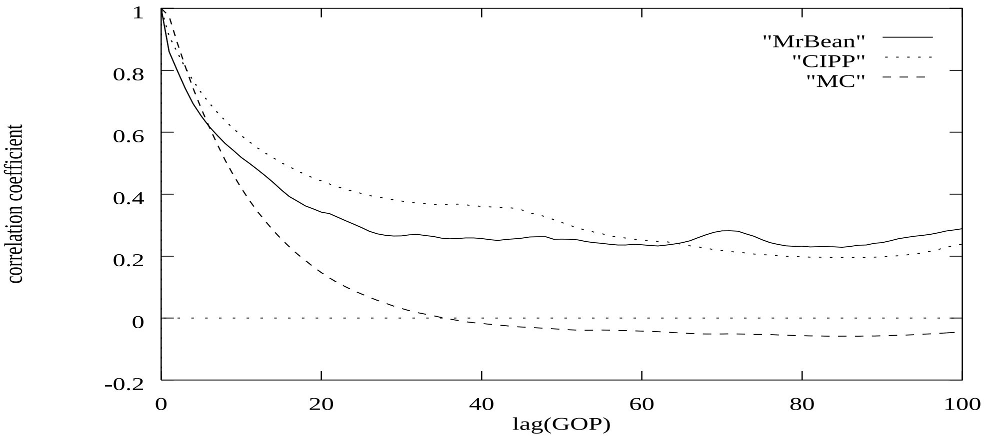
Discussions: Note that the correlation coefficient corresponding to MC model, decays very fast (with respect to lag) which is a feature of Markov models. In contrast, the counts sequence, $\{u_T(i)\}_1^\infty$ in CIPP case, may have more complex correlation structure, (may not even be a first order Markov chain) despite the fact that the interarrivals form a first order Markov chain. We strongly believe that this is the reason for non-exponential decay of correlation coefficient of counts sequence corresponding to CIPP (which accounts for the self-similar behavior to a range of time scales.)



(a)



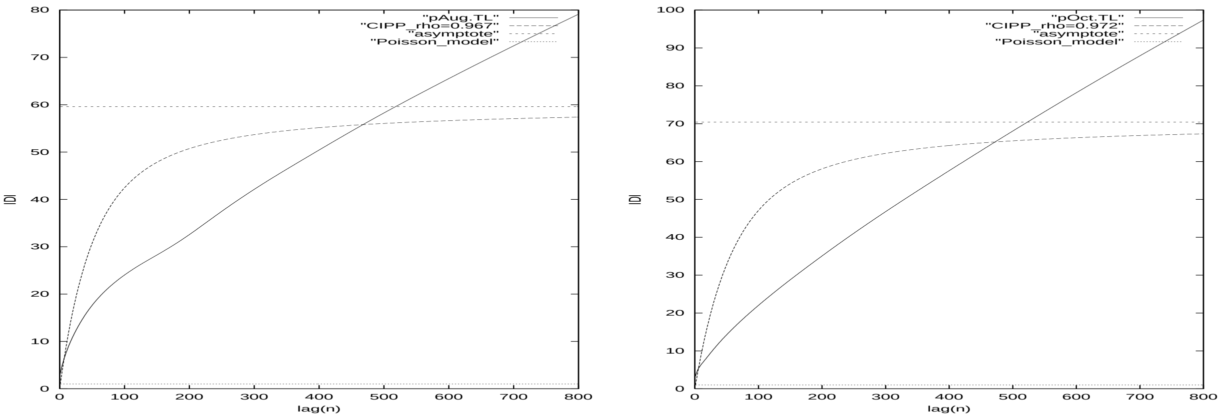
(b)



(c)

Figure 7: Experimental check on the "goodness of fit" of the CIPP as compared to MC [9] model for various video sequences [36] with autocorrelation as a measure: (a) *soccer* video sequence (b) *starwars* video sequence (c) *MrBean* video sequence

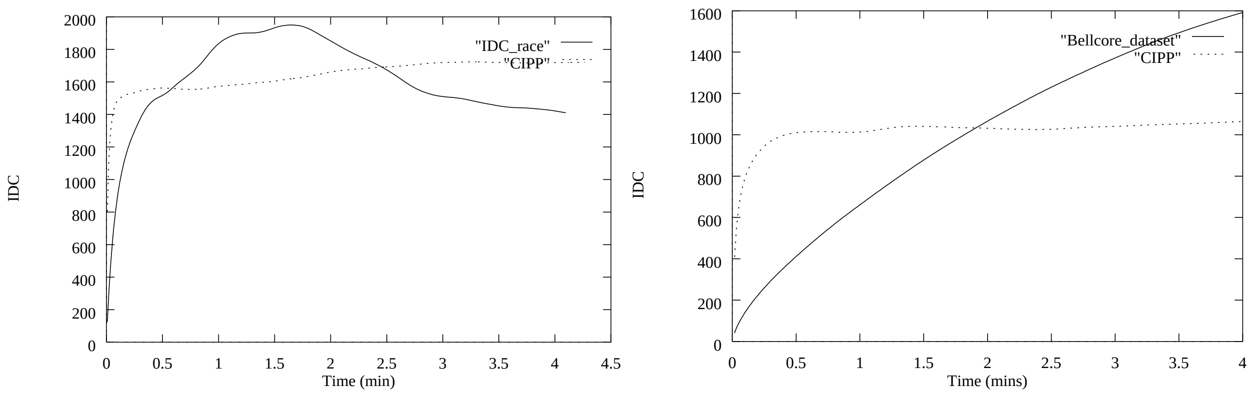
Index of Dispersion for Intervals (IDI): Here, the measure we use is IDI and the curve corresponding to data is estimated using (5). The theoretical curve corresponding to CIPP is obtained using (6). We also plot the asymptote (7) and curve corresponding to Poisson process for comparison. The algorithm which we use to fit CIPP using IDI measure is very similar to algorithm for fitting CIPP using correlation measure (see algorithm 1) except that the measure IDI for CIPP is computed *directly* using (6) in the inner loop. Figure 8 shows IDI plot for Bellcore traffic datasets [37] `pAug.TL` (Figure 8 (a)) and `pOct.TL` (Figure 8 (b)) and their corresponding (approximating) CIPP models with $\rho = 0.967$ and $\rho = 0.972$ respectively. It is seen that IDI rises sharply (initially corresponding to small lags) and eventually saturates at a point. Note this saturating point is given by (7). Note that the CIPP does substantially better than that of the Poisson process.



(a) (b)

Figure 8: IDI of Bellcore datasets and their corresponding (approximating) CIPP models (a) `pAug.TL` (b) `pOct.TL`

Index of Dispersion for Counts (IDC): Here again, the algorithm is very similar to algorithm 1 and use of (8) is made for estimation of IDC corresponding to data and CIPP. Figure 9 (a) shows the IDC plot for *race* video sequence and the corresponding CIPP match. CIPP again shows reasonable approximation quality. The corresponding plot for Bellcore dataset is shown in Figure 9 (b). Here, the approximation is not very good, possibly because CIPP is a pseudo-self-similar process, as discussed in the previous section.



(a) (b)

Figure 9: IDC of *real-world* traffic traces and their respective approximating CIPP models: (a) *race* video sequence and (b) Bellcore Ethernet dataset $pAug.TL$

5 Conclusions

We have proposed a new traffic model with correlations in interarrivals. It has been shown by simulations that this arrival process models well the real time traffic source, in particular the video source. The model is motivated by the argument that a strong positive correlation in interarrivals leads to the count process which might be self-similar over a range of time scales. Simulation runs of the model also show a close match to the observed dataset as far as the "self-similarity" goes. Below we summarize some findings of the CIPP process:

1. With strong positive correlation in interarrivals one gets strong positive correlation in the counts process $u(i)$. The magnitude of the correlation depends on ρ.

2. This strong positive correlation in the interarrival sequence, results in some complex correlation structure for the counts process $u(i)$. The authors believe that this feature is responsible for the self-similar behaviour of CIPP to a range of time scales.

3. The existence of analytical results for CIPP/M/1 queue [18] coupled with the relationship between the 'ρ' of CIPP and the 'estimated local' Hurst parameter 'H' (see Section 3) will make the CIPP model very useful in network design tools or in tools supporting real-time traffic management.

The motivation for applying CIPP even when the real traffic might be long range dependent is that powerful tools have been developed for calculating performance measures for both infinite capacity queues for Markov and deterministic service times [18].

A Appendix

Construction of Stationary CIPP process

For stationarity in the counting variable, the initial point of the interval of the counting must be chosen in a particular probabilistic way[6]. These are the stationary initial conditions which we will give; they can be used to construct a count stationary point process. By doing so, we are exploiting Axiom 3.

We next use Axiom 2. Since our aim is to construct a process whose interarrivals are correlated, we first generate correlated exponential sequence (CES) $\{X_0, X_2, X_3, \ldots,\}$ as given[7] in [24] where X_0 is the length biased distribution with respect to exponential distribution with parameter λ and $X_1 = UX_0$ where again U is uniformly distributed in $[0, 1]$. Now form the partial sum,

$$T_n = X_1 + X_2 + \ldots + X_n \quad n = 1, 2, 3 \ldots \tag{9}$$

Note that, the sequence $\{T_n\}$ gives the arrival instants. Using (1), one can rewrite the above equation as[8],

$$T_n = (U + \alpha_{n-1} - 1)X_0 + \alpha_{n-2}\epsilon_2 + \ldots + \alpha_0\epsilon_n \tag{10}$$

where, X_0 is the length biased distribution with respect to the exponential distribution with parameter λ and $\alpha_j = \sum_{i=0}^{j} \rho^i$ for $0 \leq \rho < 1$, and $j = 1, 2, 3, \ldots$ We use $\Phi_W(s)$ to denote the moment generating function of the random variable W. Define, $Y = (U + \alpha_{n-1})X_0$, then,

$$\Phi_Y(s) = \frac{\lambda^2}{(\lambda + s(\alpha_{n-1} - 1))(\lambda + s\alpha_{n-1})} \tag{11}$$

[6] see for instance [38, 25, 39]

[7] Note the absence of X_1 in the sequence.

[8] Note that the sequence $\{X_0, X_2, X_3, \ldots,\}$ forms the CES and *not* the sequence $\{X_1, X_2, X_3, \ldots,\}$. Thus, each term in the RHS of (9) can be written in terms of X_0 (and not X_1).

Then exploiting independence among random variables,

$$\Phi_{T_n}(s) = \frac{\lambda}{(\lambda + s(\alpha_{n-1} - 1))(\lambda + s\alpha_{n-1})} \prod_{i=2}^{n} \Phi_\epsilon(\alpha_{n-i}s) \tag{12}$$

where,

$$\Phi_\epsilon(s) = \rho + \frac{(1-\rho)\lambda}{\lambda + s}$$

See for details [24]. Substituting this, and upon some manipulation, we get,

$$\Phi_{T_n}(s) = \frac{\lambda}{(\lambda + s\rho\alpha_{n-2})(\lambda + s\alpha_{n-1})} \prod_{i=0}^{n-2} \Phi_\epsilon(\alpha_i s) \tag{13}$$

and upon further reducing, we have,

$$\Phi_{T_n}(s) = \frac{(\lambda)^2 \prod_{i=0}^{n-3}(\lambda + s\rho\alpha_i)}{\prod_{i=0}^{n-1}(\lambda + s\alpha_i)} \tag{14}$$

Rewriting the above,

$$\Phi_{T_n}(s) = \frac{\lambda^2 \rho^{n-2} \prod_{k=0}^{n-3}(s + \frac{\lambda}{\rho\alpha_k})}{\alpha_{n-2}\alpha_{n-1} \prod_{k=0}^{n-1}(s + \frac{\lambda}{\alpha_k})} \tag{15}$$

Inverting the above expression to get the density function $f_{T_n}(t) \triangleq \mathcal{L}^{-1}(\Phi_{T_n}(s))$ and $F_{T_n}(t) \triangleq \int_0^t f_{T_n}(\tau)d\tau$ we have for distribution $F_{T_n}(t)$,

$$F_{T_n}(t) = \left\{ \begin{array}{ll} 1 - \sum_{j=0}^{n-1} \frac{\alpha_j \prod_{k=0}^{n-1}[\alpha_j - \rho\alpha_k]}{(\alpha_j - \rho\alpha_{n-2})(\alpha_j - \rho\alpha_{n-1}) \prod_{k=0,k\neq j}^{n-1}[\alpha_j - \alpha_k]} e^{-\frac{\lambda t}{\alpha_j}} & 0 \le \rho < 1 \end{array} \right. \tag{16}$$

Next we derive the distribution, (please see page 133 [40])

$$\begin{aligned} P_n(t) &= F_{T_n}(t) - F_{T_{n+1}}(t) \\ &= \left\{ \begin{array}{ll} \sum_{j=0}^{n} \frac{\alpha_j \prod_{k=0}^{n}[\alpha_j - \rho\alpha_k]}{(\alpha_j - \rho\alpha_{n-1})(\alpha_j - \rho\alpha_n) \prod_{k=0,k\neq j}^{n}[\alpha_j - \alpha_k]} e^{-\frac{\lambda t}{\alpha_j}} & \\ - \sum_{j=0}^{n-1} \frac{\alpha_j \prod_{k=0}^{n-1}[\alpha_j - \rho\alpha_k]}{(\alpha_j - \rho\alpha_{n-2})(\alpha_j - \rho\alpha_{n-1}) \prod_{k=0,k\neq j}^{n-1}[\alpha_j - \alpha_k]} e^{-\frac{\lambda t}{\alpha_j}} & 0 \le \rho < 1 \end{array} \right. \end{aligned}$$

After some manipulations, we get the required result.

$$P_n(t) = \left\{ \begin{array}{ll} (1 + \rho - (1-\rho)\alpha_n) & \\ \sum_{j=0}^{n} \frac{\alpha_j \prod_{k=0}^{n}[\alpha_j - \rho\alpha_k]}{(\rho(\alpha_j + 1) - \alpha_n + 1)(\alpha_j - \alpha_n + 1)(\alpha_j - \rho\alpha_n) \prod_{k=0,k\neq j}^{n}[\alpha_j - \alpha_k]} e^{-\frac{\lambda t}{\alpha_j}} & 0 \le \rho < 1 \end{array} \right. \tag{17}$$

for $n = 0, 1, 2, 3 \ldots$, $\lambda > 0$, $t \ge 0$, and $\alpha_j = \sum_{i=0}^{j} \rho^i$

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