A Note on Information Utility of a Token Bucket Regulator

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Abstract

We consider a traffic flow constrained by a Token Bucket Regulator and analyze it from an information theoretic point of view. Specifically, we determine the maximum average information (or the entropy) it may convey in a given time; when we take into account the side information present in the form of variable packet lengths even as the flow conforms to the imposed traffic constraints. This may have an impact on a pricing policy that is based on regulator parameters.

Index Terms

Quality of Service (QoS) in Internet, Flow Control and Token Bucket Regulation, Entropy and Pricing

I. Introduction

Networks that offer Quality of Service (QoS) guarantees regulate the traffic sources using Traffic Regulators so that the resulting flows adhere to certain traffic constraints. A simple Traffic Regulator that enforces a linearly bounded flow is the Token Bucket (or Leaky Bucket) Regulator which has two parameters - the token generation rate r and the size of the token bucket B [1].

In this letter, we derive the maximum amount of information that a traffic flow can convey on an average (or the entropy) during a finite transmission interval while still conforming to the Token Bucket Regulator. We take into account the side information, that is present in the form of variable lengths of packets. The idea of using indirect means to convey information or of ‘side information channel’ has been investigated earlier, by Gallager in his pioneering paper [2]. More recently, [3] gives a detailed exposition to this idea which considers information that can be conveyed through means other than the packet contents themselves, for example, by encoding it into the timing of packets. The network, though, could mask or distort this covert channel by randomly delaying the packets. For the case considered in this paper, however, the channel becomes distortion free as long as the flow conforms to the regulator.

II. Information Utility

We consider a discrete time model, where the source transmits packets of variable lengths $x_1, x_2, ..., x_N$ at discrete times $1, 2, ..., N$ respectively; conforming to the negotiated Token Bucket Regulator, with bucket depth B and token refill rate r. We also take as a regulator parameter,

B_0, the initial token count for the bucket and denote this augmented Token Bucket Regulator by $TBR(r, B, B_0)$. Let t_j denote the number of tokens in the bucket just after the j^{th} packet transmission. Note that $t_0 = B_0$. The constraint imposed by $TBR(r, B, B_0)$ is

$$x_j \leq t_{j-1} + r \ \ ; \forall j : 1 \leq j \leq N \tag{1}$$

If all the packet lengths x_js are conforming, i.e., satisfy (1) then the number of residual tokens will evolve as

$$\begin{aligned} t_j &= \min(t_{j-1} + r - x_j, B) \ \ ; \\ t_0 &= B_0 \end{aligned} \tag{2}$$

We seek to maximize the average information that the source may convey in N transmissions or the entropy of a flow of duration N. We denote the flow entropy for a particular source by $\mathcal{E}(B_0, N)$ and the maximum achievable flow entropy by $\mathcal{E}^*(B_0, N)$. The maximum flow entropy is defined to be the information utility of the regulator. As will be shown later, a source may achieve the maximum entropy by following an optimal schedule. The dependence on the Token Bucket Regulator parameters - r and B is to be understood and will not be stated explicitly. We argue that the source has two ways of conveying information to the receiver.

1) At time j, the source transmits a packet of length x_j. It can thus contain x_j bits or $x_j \times ln2$ nates of information.
2) An indirect way of conveying information is the length x_j of the packet selected, which can form an independent alphabet. (We assume that the network does not fragment packets.) This arises because the source can transmit packets of any length so long as it conforms to the token bucket constraint, i.e., $0 \leq x_j \leq t_{j-1} + r$.

To find the information utility for N transmissions, we consider an intermediate stage where the source has n more transmissions to make, i.e., just before $(N - n + 1)^{th}$ transmission. Let there be b tokens in the bucket, i.e., $t_{N-n} = b$. We assume that the source chooses to transmit a packet of length i with probability $p_i(b, n)$. As before $\mathcal{E}(b, n)$ denotes the entropy in nates for n slots, with b tokens to begin with (i.e., subject to the $TBR(r, B, b)$ constraint). Then the following recursive equation must hold.

$$\mathcal{E}(b,n) = \sum_{i=0}^{i=b+r} p_i(b,n)[i\ln 2 - \ln p_i(b,n)$$
$$+\mathcal{E}(\min(b+r-i,B),n-1)] \qquad (3)$$

This equation indicates that the flow entropy for duration n is a sum of the information contained in the packet length and packet contents of the first of the n transmissions, and the entropy of the remaining flow consisting of $n-1$ transmissions. To simplify notations, we define $\mathcal{E}(b,n)$ to be

$$\begin{aligned}\mathcal{E}(b,n) &= \mathcal{E}(B,n) \;\; ;\forall b > B \\ \mathcal{E}(b,n) &= \sum_{i=0}^{i=b+r} p_i(b,n)[i\ln 2 - \ln p_i(b,n) \\ &\quad +\mathcal{E}(b+r-i,n-1)] \;\; ;\forall b \leq B \end{aligned} \qquad (4)$$

We seek to find the information utility of the regulator which is the maximum possible flow entropy. We observe that the only means by which prior transmissions can constrain the rest of the flow is through the number of residual tokens left. Hence to maximize entropy in a flow of duration n, the source would follow a policy that yields maximum entropy for a flow of duration $n-1$, for each of the possible residual token states that it may reach, and this optimal policy would be independent of the probabilities of packet length selection in previous transmissions, i.e., $p_i(b,n)$. This gives that for maximum entropy,

$$\begin{aligned}\mathcal{E}^*(b,n) &= \sum_{i=0}^{i=b+r} p_i(b,n)[i\ln 2 - \ln p_i(b,n) \\ &\quad +\mathcal{E}^*(b+r-i,n-1)] \;\; ; where\ b \leq B \end{aligned} \qquad (5)$$

For maximum entropy, the probabilities $p_i(b,n)$ are chosen to be those that maximize (5) subject to

$$\sum_{i=0}^{i=b+r} p_i(b,n) = 1 \qquad (6)$$

The optimal values for probabilities, i.e., $p_i^*(b,n)$ may be determined by Lagrange-multiplier optimization. This optimization yields

$$p_i^*(b,n) = e^{i\ln 2 - 1 + \mathcal{E}^*(b+r-i,n-1)+\lambda} \;\; , i.e.,$$

$$p_i^*(b,n) \quad \propto \quad e^{i \ln 2 + \mathcal{E}^*(b+r-i,n-1)} \tag{7}$$

The constant of proportionality may be evaluated using (6). Also,

$$\begin{aligned} \mathcal{E}^*(b,0) &= 0; \;\; \forall b \\ p_i^*(b,1) &\propto 2^i \end{aligned} \tag{8}$$

Starting with (8), and using (5) and (7) recursively for $n = 1$ to $n = N$, one can compute the values of the optimal probability schedule and the corresponding entropy for a flow of duration N subject to $TBR(r, B, B_0)$. Numerical computations reveal that entropy increases almost linearly with r while logarithmically with B.

III. Conclusion and A Remark on Pricing based on Token Bucket Parameters

Given assured performance guarantees, pricing of services would be a function of the regulator parameters and the flow duration. Our analysis gives the information utility offered to a consumer as a function of these parameters. It has been argued that a sustainable pricing policy must be a linear function of both the regulator parameters r and B [4]. This is because, these parameters translate linearly to the amount of bandwidth and buffer space required in the network. A function that is non-linear would allow entities to make profits by buying in bulk and selling in chunks or vice-versa. For example, if the prices were to increase sub-linearly (say, logarithmically) with buffer space then a broker may make profits by buying a large amount of buffer space and selling it in small fragments.

The information theoretic utility, as we show, however is not a linear function of B. It increases much slowly with B. This makes for an interesting case for a consumer seeking to maximize his utility for a given price and thus influence the selection of parameter values for the Token Bucket Regulator.

References

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