Power Optimal Signaling for Fading Multi-access Channel in Presence of Coding Gap

Ankit Sethi, Prasanna Chaporkar, and Abhay Karandikar

***Abstract*—In a multi-access fading channel, dynamic allocation of bandwidth, transmission power and rates is an important aspect to counter the detrimental effect of time-varying nature of the channel. Most of the existing work on dynamic resource allocation assumes *capacity achieving codes* for various signaling schemes like TDMA, FDMA, CDMA and successive decoding. For the capacity achieving codes, the rate achievable by the user is $\log(1+SNR)$, where SNR denotes the signal to noise ratio of the user at the receiver side. However, codes that are used in practice have a finite gap to capacity, i.e., the achievable rate is $\log(1+\frac{SNR}{\Gamma})$ for $\Gamma > 1$. The exact value of Γ depends on the coding strategy and the desired bit error rate. Many existing resource allocation techniques that are optimal for capacity achieving codes perform sub-optimally in presence of the coding gap. For example, successive decoding does not always minimize the sum power required for providing the desired rate to each of the users for $\Gamma > 1$. The problem of minimizing the sum power while guaranteeing the required rate to each of the users is important for both real-time and non real-time applications, and is addressed here. We obtain the resource allocation that is optimal for the above problem in presence of the coding gap.**

I. Introduction

We consider a Gaussian multi-access fading channel with perfect channel side information (CSI) at the transmitters and the receiver. This models many important practical systems including the uplink of wireless LANs and the cellular systems. In a multi-access fading channel, dynamic allocation of bandwidth, transmission power and rates is an important aspect to counter the detrimental effect of time-varying nature of the channel [1], [2], [3]. Most of the existing work on dynamic resource allocation assumes *capacity achieving codes* for various signaling schemes, such as code-division multiple access (CDMA), time-division multiple access (TDMA) and frequency-division multiple access (FDMA) [4], [5], [6]. For capacity achieving codes, the rate achievable by the user is given by $\log(1+SNR)$, where SNR denotes the signal to noise ratio of the user at the receiver side. However, codes used in practical scenario have a finite gap to capacity. For a variety of uncoded and coded modulations, this gap to capacity can be approximated by scaling SNR with a factor $(1/\Gamma)$ for $\Gamma > 1$ [7], i.e., the achievable rate is approximately $\log(1+\frac{SNR}{\Gamma})$. Moreover, this gap to capacity is constant with SNR for a number of coding techniques and depends only on the probability of error (P_e). For example, in case of PAM/QAM, $\Gamma = 9.5$ dB at $P_e = 10^{-7}$. Strictly speaking, this coding gap to capacity is a function of SNR, but it can be approximated to be constant over a large range of SNR. The modified SNR can now be used in any optimization setting in the same way as that for the capacity achieving codes ($\Gamma = 1$).

We consider the system with M users. (We use the terms "user" and "transmitter" interchangeably.) A user k requires rate R_k in each slot, where slot duration is equal to the channel coherence time. Thus, the channel gain is assumed to be constant in a slot, but it can vary from slot to slot. Let $\boldsymbol{h}(t) = [h_1(t) \cdots h_M(t)]$ denote the channel gains in slot t, i.e., if user k transmits at power P in slot t, then the received power is $h_k(t)P$. The coding strategy, and hence the coding gap Γ is specified. *Our aim is to determine a signaling strategy and the resource allocation for the given signaling strategy so as to minimize the sum transmit power while providing the desired rate to each of the users for any given $\boldsymbol{h}(t)$.* We note that for any given signaling strategy, the resource allocation has to be dynamic depending on $\boldsymbol{h}(t)$. Clearly, this problem is of interest for real time applications as they require strict delay guarantees. Next, we illustrate why this problem is of interest even for non-real time applications.

For non-real time applications, let each user k desire a long term rate r_k. Note that unlike real-time applications, this rate need not be provided in every slot. We can view this system as follows: higher layer of the protocol stack feeds r_k bits to the multiple access control (MAC) layer buffer of the k^{th} user in every slot. In slot t, MAC layer serves $R_k(t)$ bits from the buffer, where $R_k(t) \le Q_k(t)$. Here, $Q_k(t)$ is the number of back-logged bits in the buffer (queue length) of user k in slot t. Then, to provide the required rate to each user, it suffices to ensure that the expected queue length for each user is bounded (queue is stable), mathematically $\sup_t \mathbb{E}[Q_k(t)] < \infty$ for each k. Thus, providing the required long-term rate to each user is equivalent to ensuring the queue stability for each user. Recently, the problem of minimizing the average sum power required for ensuring stability of all the queues has been studied extensively [8]. It has been shown that, in each slot, transmitting R_k bits from each user k such that a function $VP(\boldsymbol{R}, \boldsymbol{h}(t)) - \sum_{k=1}^{M} Q_k(t)R_k$ is minimized achieves the required goal, where V is a sufficiently large constant. Here, $P(\boldsymbol{R}, \boldsymbol{h})$ is the sum power required to transmit $\boldsymbol{R} = [R_1 \cdots R_M]$ bits in the multi-access channel experiencing channel gains $\boldsymbol{h}(t)$. Note that the function $P(\boldsymbol{R}, \boldsymbol{h}(t))$ depends on $\boldsymbol{R}$, $\boldsymbol{h}(t)$ and coding and signaling strategy employed. The previous work assumes that the function $P(\cdot,\cdot)$ is given, i.e., it assumes that the coding and signaling strategy is specified. The optimality of the above scheme is shown for any non-negative $P(\cdot,\cdot)$. Thus, for truly minimizing the average sum power while guaranteeing stability for a given coding scheme, one needs to determine a signaling scheme

[1]This work was done while A. Sethi was a graduate student at Indian Institute of Technology Bombay (IIT Bombay), Mumbai, India. P. Chaporkar and A. Karandikar are with IIT Bombay, Mumbai, India. (e-mails: ankits@iitb.ac.in, chaporkar@ee.iitb.ac.in and karandi@ee.iitb.ac.in) This research was supported by TTSL-IIT BOMBAY, Center for Excellence in Telecom (TICET), Mumbai.

that achieves the minimum sum power while guaranteeing $\boldsymbol{R}$ when channel gains are $\boldsymbol{h}$. This shows how our problem is of interest for non-real time applications.

Next we review the related work. In the absence of coding gap, the rate region of a multi-access fading channel is a polymatroid structure as derived in [9]. In [10], the problem of minimizing the sum power of the users under the constraints of providing them with minimum defined rates is considered. Non-orthogonal signaling like *superposition coding* along with *successive decoding* (SCSD) at the receiver is shown to be the optimal strategy with appropriate power allocation. However, the analysis here considers only the capacity achieving codes. In presence of coding gap, the results in [9], [10] do not hold. Indeed, [7] shows that the SCSD is not an optimal signaling in presence of the coding gap. Specifically, [7] considers additive white Gaussian noise (AWGN) multi-access channel with two users, and shows the existence of rates R_1, R_2 for which the power optimal signaling is FDMA and not SCSD, i.e., the minimum sum power under FDMA is lesser than that under SCSD. The sum power under a given signaling scheme is minimized over all possible resource allocations for that signaling scheme. For FDMA, resources are bandwidth and power alloted to each user, while for SCSD, resource is only the power at which each user transmits. Authors in [7] also show that FDMA is not power optimal signaling for all the values of the required rates, i.e., there exists R'_1 and R'_2 such that the power optimal signaling is SCSD. Thus, the choice of optimal signaling depends on the rate requirements. We note that [7] does not consider multi-path fading, i.e., the channel gains were assumed to be time invariant. In fading channel, the optimal signaling depends not only on the rate requirements, but also on the fading state which changes in every slot. Unlike [7], which only provides an existence result, one of our key contribution is to explicitly compute power optimal signaling scheme for any given rate requirements $\boldsymbol{R}$ and the channel fading state $\boldsymbol{h}$. Our contributions are explicitly mentioned below:

• As discussed above, for any given $\boldsymbol{R}$ and $\boldsymbol{h}$ either SCSD or FDMA is power optimal. Thus, our approach is to obtain the power optimal resource allocation for FDMA and SCSD for any given $\boldsymbol{R}$ and $\boldsymbol{h}$. The optimal signaling is then obtained as the one that requires the lesser sum power between the two signaling schemes. As a first step, we obtain the optimal bandwidth and power allocation for FDMA. We note that the concept of changing the bandwidth allotted to users depending on $\boldsymbol{h}$ has not received much attention in the literature. This is because the rate region achieved by FDMA is a strict subset of the rate region achieved by SCSD when capacity achieving codes are used for any given average power constraint. Thus, primarily, the power optimal resource allocation for SCSD is widely explored in the literature [9], [10]. But, the optimality of SCSD is no longer true in the presence of coding gap [7]. Thus, unlike previous work, we need to determine optimal resource allocation for FDMA signaling.

• Next, we obtain the optimal resource allocation for SCSD signaling. Even though the optimal resource allocation for SCSD is well known for the capacity achieving codes, the case with coding gap does not follow directly from the known results. This is because the optimal resource allocation for SCSD is obtained from the key property that the rate region is a polymatroid structure [9], [10]. This key property does not hold in the presence of coding gap, and hence the optimal resource allocation for SCSD has to be obtained afresh.

• After the power optimal resource allocation is obtained for FDMA and SCSD for the given $\boldsymbol{R}$ and $\boldsymbol{h}$, the optimal signaling is obtained as the one that requires a lower sum transmit power. Here, this strategy is referred as *adaptive strategy*. To obtain insights into when a certain signaling scheme would perform better than the other, we investigate how the optimal resource allocation depends on Γ and $\boldsymbol{h}$.

The paper is organized as follows. In Section II, we present the system model. In Sections III and IV, we obtain the optimal resource allocation for FDMA and SCSD, respectively. In Section V, we quantify the dependence of the optimal resource allocation on Γ and $\boldsymbol{h}$. In Section VI, we conclude.

II. System Model

We consider a multi-access channel fading with M users. Time is slotted. The channel is time-varying with $h_k(t)$ being the fading state of k^{th} user in slot t. The fading is assumed to be flat. We assume AWGN with spectral density σ^2. All the users use the same codes and hence have the same coding gap Γ to capacity. We consider a discrete time channel

$$Y(t) = \sum_{k=1}^{M} \sqrt{h_k(t)} X_k(t) + Z(t),$$

where $Y(t)$ is the received signal in t^{th} time slot, $X_k(t)$ is the transmitted signal of k^{th} user in t^{th} time slot, and $Z(t)$ is the noise. Let $\boldsymbol{R} = [R_1 \cdots R_M]$ denote the rate requirements.

The objective is to minimize the average sum power constrained to providing a minimum rate R_k to each user k in every channel state. Next, we obtain the optimal resource allocation for FDMA and SCSD for any given $\boldsymbol{h}$.

III. Optimal Resource Allocation for FDMA

Here, we determine the optimal bandwidth allocation and power allocation scheme that minimizes the sum power of the users constrained to providing the minimum defined rate to each k for FDMA signaling. The problem can be mathematically formulated as follows. Let the current channel state be denoted by $\boldsymbol{h}$. Let a power allocation policy be $\boldsymbol{P(h)} = [P_1(\boldsymbol{h}) \cdots P_M(\boldsymbol{h})]$, and bandwidth allocation policy $\boldsymbol{\alpha(h)} = [\alpha_1(\boldsymbol{h}) \cdots \alpha_M(\boldsymbol{h})]$. Here, $P_k(\boldsymbol{h})$ is the power allotted to k^{th} user in channel state $\boldsymbol{h}$, and $\alpha_k(\boldsymbol{h})$ is the fraction of bandwidth allotted to k^{th} user in channel state $\boldsymbol{h}$. Thus, for every k we have

$$R_k \leq \alpha_k(\boldsymbol{h}) \log \left(1 + \frac{P_k(\boldsymbol{h}) h_k}{\Gamma \sigma^2 \alpha_k(\boldsymbol{h})}\right). \tag{1}$$

Without loss of generality, we assume that the total bandwidth is 1. Clearly, the sum power is minimized when (1) is satisfied with equality for each k. Thus, from (1) we have

$$P_k(\boldsymbol{h}) = \frac{(e^{\frac{R_k}{\alpha_k(\boldsymbol{h})}} - 1)\Gamma \sigma^2 \alpha_k(\boldsymbol{h})}{h_k}.$$

Using the above relation, we get the following optimization

$$\min_{\boldsymbol{\alpha}} \sum_{k=1}^{M} \frac{(e^{\frac{R_k}{\alpha_k}}-1)\Gamma\sigma^2\alpha_k}{h_k}$$
$$\text{Subjected to: } \sum_{k=1}^{M} \alpha_k = 1, \text{and } \boldsymbol{\alpha} \in [0,1]^M. \quad (2)$$

We first note that the function $\frac{(e^{R_k/\alpha_k}-1)\Gamma\sigma^2\alpha_k}{h_k}$ is strictly convex for $\alpha_k \in [0,1]$. This is because the second derivative of the function ($= \frac{\Gamma\sigma^2}{h_k\alpha_k^3}e^{R/\alpha_k}$) is positive for $\alpha_k > 0$. Thus, the objective function is the sum of convex functions, and hence it is also a convex function. Clearly, the set of feasible solutions is convex. Thus, the above problem is an instance of the convex optimization problem [11]. For convex optimization, polynomial complexity algorithms using interior point method have been proposed [12]. These algorithms can be used to obtain the optimal resource allocation for FDMA signaling.

IV. Optimal Resource Allocation for Successive Decoding

For SCSD, we need to specify the decoding order. Let $\boldsymbol{\pi} = [\pi(1)\cdots\pi(M)]$ denote a permutation on the set of users. We say that $\boldsymbol{\pi}$ is the decoding order if $\pi(M)$ is decoded first, then $\pi(M-1)$ and so on until $\pi(1)$. Thus, for the $\pi(k)^{\text{th}}$ user, signals from the users $\pi(1)$ to $\pi(k-1)$ act as interference. Note that for a given $\boldsymbol{\pi}$, the power allocation $\boldsymbol{P}$ has to satisfy the following relations so as to provide the desired rates to each of the users. For every k,

$$R_{\pi(k)} \leq \log\left(1 + \frac{h_{\pi(k)}P_{\pi(k)}}{\Gamma(\sum_{i=1}^{k-1} h_{\pi(i)}P_{\pi(i)} + \sigma^2)}\right). \quad (3)$$

The objective is $\min_{\boldsymbol{P},\boldsymbol{\pi}} \sum_{i=1}^{M} P_i$, where $\boldsymbol{P}$ and $\boldsymbol{\pi}$ satisfy (3). Clearly, for a given $\boldsymbol{\pi}$, the minimization over $\boldsymbol{P}$ happens when (3) is satisfied with equality for every k. Thus, the problem boils down to finding the optimal decoding order.

A. Optimal Decoding Order and Minimum Sum Power

In the absence of coding gap ($\Gamma = 1$), the decoding order that minimizes the sum power while guaranteeing the desired rate to each of the users is obtained in [10]. In [10], the authors have shown that the optimal decoding order $\boldsymbol{\pi}^*$ satisfies $h_{\pi^*(1)} \leq \cdots \leq h_{\pi^*(M)}$ irrespective of the rate requirements $\boldsymbol{R}$. Thus, the optimal power allocation can be obtained using a greedy procedure. The key property utilized to prove the result is that for any given π and $\mathcal{S} \subseteq \{1,\ldots,M\}$, $\sum_{k\in\mathcal{S}} R_k = \log\left(1 + \frac{\sum_{k\in\mathcal{S}} P_k(\boldsymbol{\pi})h_k}{\sigma^2}\right)$, where $P_k(\boldsymbol{\pi})$ is the transmit power for user $\pi(k)$ under decoding order π. This property yields polymatroid structure for the rate region under SCSD (for complete details, see [10]). Now, the polymatroid structure is used to derive $\boldsymbol{\pi}^*$. We note that the aforementioned property does not hold when $\Gamma > 1$, and hence the rate region for SCSD may not be a polymatroid. Thus, the optimal decoding orders for $\Gamma = 1$ and $\Gamma > 1$ need not be the same. But, as we show in the next result, the optimal decoding order for $\Gamma > 1$ is the same as that for $\Gamma = 1$. First, we note that when (3) is satisfied with equality,

$$P_k(\boldsymbol{\pi}) = \frac{\left(e^{R_{\pi(k)}}-1\right)\Gamma\left(\sigma^2 + \sum_{i=1}^{k-1} P_i(\boldsymbol{\pi})h_{\pi(i)}\right)}{h_{\pi(k)}}. \quad (4)$$

Theorem 1: Let $\boldsymbol{\pi}^{*(\Gamma)}$ denote the optimal decoding order for a given $\Gamma > 1$. Then, $\boldsymbol{\pi}^{*(\Gamma)} = \boldsymbol{\pi}^*$ for every $\Gamma > 1$.

Proof: Suppose, for some $\Gamma > 1$, $\boldsymbol{\pi}^*(\Gamma) \neq \boldsymbol{\pi}^*$. For brevity, let $\boldsymbol{\pi} = \boldsymbol{\pi}^*(\Gamma)$. Then, there exists $m < M$ such that $h_{\pi(m)} > h_{\pi(m+1)}$. Let us construct another decoding order $\boldsymbol{\pi}'$ such that $\boldsymbol{\pi}'(k) = \boldsymbol{\pi}(k)$ for $k \notin \{m, m+1\}$, and $\boldsymbol{\pi}'(m) = \boldsymbol{\pi}(m+1)$ and $\boldsymbol{\pi}'(m+1) = \boldsymbol{\pi}(m)$. In other words, we obtain $\boldsymbol{\pi}'$ by swapping m^{th} and $(m+1)^{\text{th}}$ user in the decoding order of $\boldsymbol{\pi}$. Thus, clearly from (4), $P_k(\boldsymbol{\pi}) = P_k(\boldsymbol{\pi}')$ for every $k < m$. Now, let us consider the following:

$$\begin{aligned}\sum_{k=1}^{m+1} P_k(\boldsymbol{\pi})h_{\pi(k)} - \sum_{k=1}^{m+1} P_k(\boldsymbol{\pi}')h_{\pi'(k)} \\ = \quad & P_m(\boldsymbol{\pi})h_{\pi(m)} + P_{m+1}(\boldsymbol{\pi})h_{\pi(m+1)} \\ & -P_m(\boldsymbol{\pi}')h_{\pi'(m)} - P_{m+1}(\boldsymbol{\pi}')h_{\pi'(m+1)}.\end{aligned}$$

Now, we note that

$$\begin{aligned}& P_m(\boldsymbol{\pi})h_{\pi(m)} - P_{m+1}(\boldsymbol{\pi}')h_{\pi'(m+1)} \\ & = -\Gamma\left(e^{R_{\pi(m)}}-1\right)P_m(\boldsymbol{\pi}')h_{\pi'(m)} \\ & = -\Gamma^2\left(e^{R_{\pi(m)}}-1\right)\left(e^{R_{\pi(m+1)}}-1\right)\left(\sigma^2 + \sum_{i=1}^{m-1} P_i(\boldsymbol{\pi})h_{\pi(i)}\right). \quad (5)\end{aligned}$$

Similarly,

$$\begin{aligned}& P_{m+1}(\boldsymbol{\pi})h_{\pi(m+1)} - P_m(\boldsymbol{\pi}')h_{\pi'(m)} \\ & = \Gamma\left(e^{R_{\pi(m+1)}}-1\right)P_m(\boldsymbol{\pi})h_{\pi(m)} \\ & = \Gamma^2\left(e^{R_{\pi(m)}}-1\right)\left(e^{R_{\pi(m+1)}}-1\right)\left(\sigma^2 + \sum_{i=1}^{m-1} P_i(\boldsymbol{\pi})h_{\pi(i)}\right). \quad (6)\end{aligned}$$

From (5) and (6), we conclude that

$$\sum_{k=1}^{m+1} P_k(\boldsymbol{\pi})h_{\pi(k)} = \sum_{k=1}^{m+1} P_k(\boldsymbol{\pi}')h_{\pi'(k)}. \quad (7)$$

From (4) and (7), it can be seen that $P_k(\boldsymbol{\pi}) = P_k(\boldsymbol{\pi}')$ for every $k > m+1$. Now, since $\boldsymbol{\pi}$ is the optimal decoding order, we know that

$$\begin{aligned}& \sum_{k=1}^{M} P_k(\boldsymbol{\pi}) - \sum_{k=1}^{M} P_k(\boldsymbol{\pi}') \leq 0 \\ \Rightarrow \quad & P_m(\boldsymbol{\pi}) + P_{m+1}(\boldsymbol{\pi}) - P_m(\boldsymbol{\pi}') - P_{m+1}(\boldsymbol{\pi}') \leq 0 \\ \Rightarrow \quad & P_{m+1}(\boldsymbol{\pi}) - P_m(\boldsymbol{\pi}') \leq P_{m+1}(\boldsymbol{\pi}') - P_m(\boldsymbol{\pi}) \\ \Rightarrow \quad & \frac{1}{h_{\pi(m+1)}} \leq \frac{1}{h_{\pi(m)}}. \quad (8)\end{aligned}$$

The relation (8) follows from (5) and (6) as $h_{\pi(m)} = h_{\pi'(m+1)}$ and $h_{\pi(m+1)} = h_{\pi'(m)}$ by the construction of $\boldsymbol{\pi}'$. But, note that (8) provides a contradiction as we have chosen m such that $h_{\pi(m)} > h_{\pi(m+1)}$. This proves the required. ■

Next, using examples, we demonstrate that indeed for the given $\boldsymbol{R}$ and Γ, there exist channel states $\boldsymbol{h}$ such that FDMA

Parameters	User 1	User 2	User 3	Sum
$\boldsymbol{h}$	0.7	0.6	0.5	-
$\boldsymbol{\alpha}$ in FDMA	0.2979	0.3579	0.3443	1.0000
$\boldsymbol{P}$ in FDMA	1.6×10^{-8}	2.1×10^{-8}	2.1×10^{-8}	5.8×10^{-8}
$\boldsymbol{P}$ in SCSD	10^{-6}	7×10^{-8}	4×10^{-8}	1.1×10^{-6}

TABLE I
FDMA ACHIEVES BETTER PERFORMANCE THAN THAT OF SCSD

Parameters	User 1	User 2	User 3	Sum
$\boldsymbol{h}$	0.7679	0.008	1.4386	-
$\boldsymbol{\alpha}$ in FDMA	0.1641	0.6704	0.1655	1.0000
$\boldsymbol{P}$ in FDMA	2.2×10^{-7}	4.2×10^{-6}	2.1×10^{-7}	4.63×10^{-6}
$\boldsymbol{P}$ in SCSD	4.7×10^{-8}	2.7×10^{-6}	5×10^{-7}	3.25×10^{-6}

TABLE II
SCSD ACHIEVES BETTER PERFORMANCE THAN THAT OF FDMA

gives lesser sum power than that of SCSD and vice versa. The examples are presented in Tables I and II. Here, we assume that the system has three users, $\Gamma = 7$ and $\boldsymbol{R} = [1.2\ 1.4\ 1.3]$.

V. EFFECT OF Γ AND $\boldsymbol{h}$ ON OPTIMAL RESOURCE ALLOCATION

Here, we investigate how resource allocation varies with (1) coding gap and (2) channel states.

A. Dependence on Coding Gap Γ

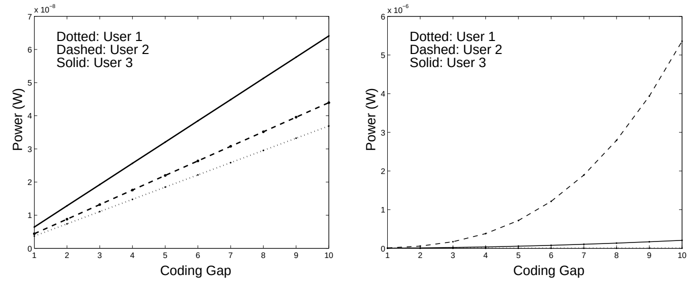
In this section, we analyze how the optimal power allocation in case of FDMA and SCSD depends on Γ under the condition that all other system variables remain unchange. We already know that when there is no coding gap ($\Gamma = 1$), the minimum sum power for all $\boldsymbol{h}$ is achieved by SCSD. But as the coding gap increases ($\Gamma > 1$) this is no longer true, i.e., for $\Gamma > 1$ there exists $\boldsymbol{h}$ such that the minimum sum power is achieved by FDMA. Here, we attempt to find a reason behind this.

1) FDMA: First, we explore the dependence of optimal power allocation under FDMA on the coding gap Γ. Let us fix $\boldsymbol{R}$ and $\boldsymbol{h}$. Now, let the optimal power allocation and bandwidth allocation for FDMA be given by $\boldsymbol{P}^{FDMA}$ and $\boldsymbol{\alpha}$ for some coding gap Γ. Now, $\boldsymbol{P}^{FDMA}$ and $\boldsymbol{\alpha}$ are the solutions to (2). From (2), it is clear that for the coding gap $\gamma\Gamma$, the optimal power and the bandwidth allocations are $\gamma\boldsymbol{P}^{FDMA}$ and $\boldsymbol{\alpha}$, respectively. Thus, the power requirement under FDMA increases in proportion to the coding gap. This can be seen in Figure 1(a).

2) Successive Decoding: Now, we explore the dependence of optimal power allocation under SCSD on the coding gap Γ. As before, let us fix $\boldsymbol{R}$ and $\boldsymbol{h}$. Let the optimal decoding order and the power allocation be $\boldsymbol{\pi}$ and $\boldsymbol{P}^{SCSD}$, respectively. Then,

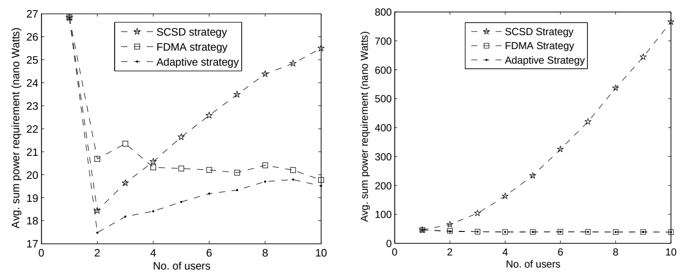
$$
\begin{aligned}
P_{\pi(k)} &= \frac{\left(e^{R_{\pi(k)}} - 1\right)\Gamma(\sigma^2 + \sum_{i=1}^{k-1} P_{\pi(i)} h_{\pi(i)})}{h_{\pi(k)}} \text{ (from (4))} \\
&\geq \frac{\left(e^{R_{\pi(k)}} - 1\right)\Gamma(\sum_{i=1}^{k-1} P_{\pi(i)} h_{\pi(i)})}{h_{\pi(k)}} \\
&\geq \frac{\Gamma^k\left(e^{R_{\pi(k)}} - 1\right)\sigma^2 \Pi_{i=1}^{k-1}\left(\left(e^{R_{\pi(i)}} - 1\right)h_{\pi(i)}\right)}{h_{\pi(k)}}
\end{aligned}
$$

Thus, the power of the user $\pi(k)$ is lower bounded by a quantity that is proportional to Γ^k. This has two implications.



(a) FDMA (b) SCSD

Fig. 1. Variation of power of users with coding gap when (a) FDMA strategy (b) SCSD strategy is used. Here $M = 3$, $[R_1, R_2, R_3] = [1.0, 1.4, 1.9]$, $[h_1, h_2, h_3] = [0.4, 0.6, 0.5]$, $\sigma = 10^{-5}$



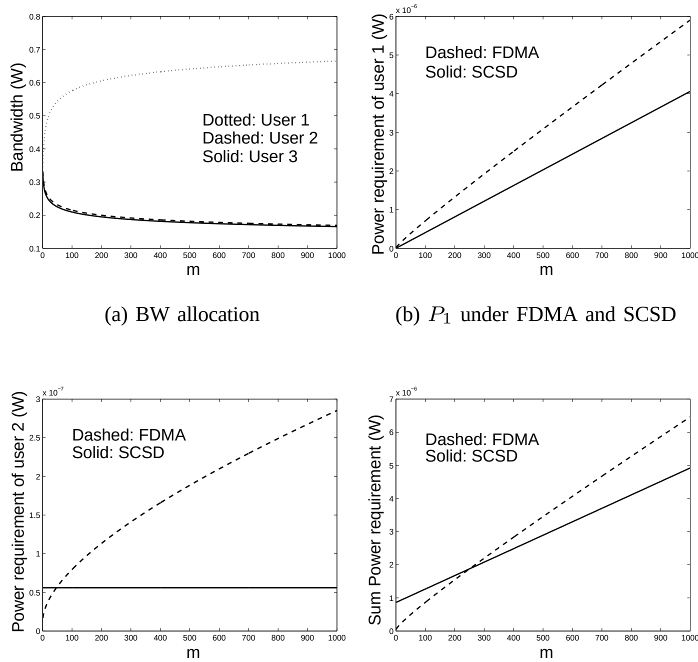
(a) $\Gamma = 1.3$ (b) $\Gamma = 2.5$

Fig. 2. Variation of the minimum average sum power of users for FDMA, SCSD and Adaptive Strategy in a Rayleigh Fading channel with number of users in the system. Here $MR^0 = 5$, $\sigma = 10^{-5}$. In (b), the plots for FDMA strategy and Adaptive strategy are overlapping.

Firstly, as Γ increases, the transmit power of the users increases exponentially, where the exponent depends on $\boldsymbol{\pi}$. This can be seen in Figure 1(b). It follows that as Γ increases the power consumption of a user under SCSD increases at a much higher rate than that of the respective user under FDMA strategy (except for the user that is decoded last). This explain why FDMA can achieve better performance than SCSD for $\Gamma > 1$. Secondly, the power consumption of the user to be decoded first is lower bounded by a quantity proportional to Γ^M where M is the number of users in the system. Thus, as the number of users in the system will increase, FDMA strategy will start outperforming SCSD and the power consumption in FDMA strategy will eventually converge to that of the 'Adaptive Strategy'. This can be seen in Figure 2(a) and Figure 2(b). These show the variation of minimum average sum power required by the users in a Rayleigh Fading channel when FDMA strategy, SCSD strategy and Adaptive strategy are used with the number of users in the system. Here, for a given number of users in the system, the minimum rate required for every user is same i.e $R_1 = R_2.. = R_M = R^0$ where $MR^0 = 5$.

B. Dependence on the Channel State $\boldsymbol{h}$

First, we note that changing $\boldsymbol{h}$ to $\gamma\boldsymbol{h}$ is equivalent to changing Γ to Γ/γ while keeping the same channel state. Hence, the observations in the previous section apply for scaler



(a) BW allocation

(b) P_1 under FDMA and SCSD

(c) P_3 under FDMA and SCSD

(d) Sum Power

Fig. 3. Here $M = 3$, $\Gamma = 7$, $[R_1, R_2, R_3] = [1.2, 1.2, 1.2]$, $[h_1, h_2, h_3] = [0.4, 0.6, 0.5]$, $\sigma = 10^{-5}$. We plot various performance measures as channel for the user 1 degrades, i.e., it changes from h_1 to h_1/m.

shift in $\boldsymbol{h}$. Now, we explore the dependence of the system performance on the channel state of an individual user.

1) FDMA: Let us consider two channel state vectors $\boldsymbol{h}^1$ and $\boldsymbol{h}^2$ such that $h_k^1 = h_k^2$ for all $k \neq m$ and $h_m^1 > h_m^2$. Thus, $\boldsymbol{h}^2$ corresponds to the channel state vector in which user m has worse channel gain than that in $\boldsymbol{h}^1$, while the channel gain of all the other users remain unchanged. Let $\boldsymbol{\alpha}^1$ and $\boldsymbol{\alpha}^2$ denote the optimal bandwidth allocation for $\boldsymbol{h}^1$ and $\boldsymbol{h}^2$, respectively. Then, we show the following.

Lemma 1: The optimal bandwidth allocation $\boldsymbol{\alpha}^1$ and $\boldsymbol{\alpha}^2$ satisfy that $\alpha_m^1 < \alpha_m^2$ and $\alpha_k^1 \geq \alpha_k^2$ for every $k \neq m$.

The proof for the lemma is omitted because of space constraints. From the above lemma it follows that change in the channel state of one user results in a change in the power allocation for all the users. Specifically, it can be shown that the power for all the users increases when the channel for any of the users degrades.

2) Successive Decoding: In this case, when the channel state for a user changes, the optimal decoding order also changes. Thus, exact impact of the change in the channel state of a user on the power allocation policy depends on the placement of the user in the decoding order before and after the change in the channel state. A special case in which the channel gain of the worst user, i.e., the user with the smallest channel gain, becomes smaller, it can be seen from (4) that the power requirement of the worst user alone increases and the power requirements of others remain the same. Note that in this case, the optimal decoding order remains the same.

3) Numerical Evaluation: The results for numerical study are presented in Fig. 3. Fig. 3(a) verifies Lemma 1. Fig. 3(b) shows the rate at which power for the user, whose channel quality worsens, increases under FDMA and SCSD. The rate is higher under FDMA than that under SCSD. Fig. 3(c) shows that the power for users whose channel remains same does not change under SCSD, while under FDMA the power requirement increases. Finally, Fig. 3(d) shows that the rate of increase of sum power under FDMA is higher than that under SCSD. Hence, even when initially FDMA was an optimal signaling, SCSD becomes the optimal signaling as the channel quality for user $k = 1$ becomes worse.

VI. Conclusions

We addressed the problem of minimizing the sum power subject to providing the desired rate to each user in multi-access fading channel in the presence of coding gap. We showed that in the presence of coding gap, SCSD is no longer an optimal strategy in all the channel states and also that there are certain channel states where FDMA outperforms SCSD. For these channel states, we determined a power optimal bandwidth allocation policy as a function of the channel state vector. This shows the benefit of the dynamic bandwidth allocation in the presence of coding gap (dynamic FDMA). Further, for the channel states where SCSD is optimal, we showed that the optimal decoding order is to decode the users in the decreasing order of their channel gains independent of their rate requirements. Finally, we developed some insights on how the minimum sum powers for SCSD and FDMA depend on the channel state vector and the coding gap.

References

[1] R. Knopp and P. Humblet, “Information capacity and power control in single-cell multiuser communications,” *IEEE International Conference on Communications, Gateway to Globalization*, vol. 1, 1995.

[2] A. Lozano, A. Tulino, and S. Verdu, “Optimum ergodic power allocation for multiuser OFDM with arbitrary signal constellations,” *IEEE Global Telecommunications Conference, GLOBECOM*, 2006.

[3] Z. Shen, J. Andrews, and B. Evans, “Optimal power allocation in multiuser OFDM systems,” *IEEE Global Telecommunications Conference, GLOBECOM*, vol. 1, 2003.

[4] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. Cambridge University Press, 2005.

[5] T. Cover and J. Thomas, *Elements of information theory*. Wiley New York, 1991.

[6] D. Yu and J. Cioffi, “Iterative water-filling for optimal resource allocation in OFDM multiple-access and broadcast channels,” *IEEE Global Telecommunications Conference, GLOBECOM*, 2006.

[7] S. Jagannathan and J. Cioffi, “Optimality of FDMA in Gaussian multiple-access channels with non-zero SNR margin and gap,” *IEEE Global Telecommunications Conference, GLOBECOM*, 2006.

[8] P. Chaporkar and S. Sarkar, “Stable scheduling policies for maximizing throughput in generalized constrained queueing,” in *IEEE INFOCOM’06*, Barcelona, Spain, Apr. 2006.

[9] D. Tse and S. Hanly, “Multiaccess fading channels. I. Polymatroid structure, optimal resource allocation and throughput capacities,” *IEEE Trans. on Info. Theory*, vol. 44, no. 7, 1998.

[10] S. Hanly and D. Tse, “Multiaccess fading channels. II. Delay-limited capacities,” *IEEE Trans. on Info. Theory*, vol. 44, no. 7, 1998.

[11] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, Mar 2004.

[12] Y. Nesterov and A. Nemirovskii, *Interior-point polynomial algorithms in convex programming*, ser. Studies in Industrial and Applied Mathematics. Society for Industrial & Applied Mathematics, Jul 1995.