HORN ANTENNAS

(a) H-plane sectoral horn. (b) E-plane sectoral horn. (c) Pyramidal horn.

H-PLANE SECTORAL HORN

Figure - H-plane sectoral horn antenna. (a) Overall geometry. (b) Cross section through the x-z plane (H-plane).

\[ l_H^2 = R_1^2 + \left( \frac{A}{2} \right)^2 \]
\[ \phi_H = \tan^{-1} \left( \frac{A}{2R_1} \right) \]

TE10 mode in waveguide

\[ E_y = E_0 \cos \left( \frac{n \pi x}{a} \right) e^{-j \beta z} \]
\[ H_x = -E_y / Z_0, \quad Z_0 = \gamma \left( 1 - \left( \frac{A}{2} \right)^2 \right)^{-1/2} \]
Aperture phase variation
\[ = e^{-j \beta (R - R_1)} \text{ in } x \text{- direction} \]
\[ = \text{Constant} \text{ in } y \text{- direction} \]
\[ R = \sqrt{R_1^2 + x^2} \approx R_1 \left[ 1 + \frac{1}{2} \left( \frac{x}{R_1} \right)^2 \right] \]
\[ R - R_1 \approx \frac{1}{2} \frac{x^2}{R_1} \]

Aperture field distribution
\[ E_y = E_0 \cos \frac{\pi x}{\lambda} e^{-j \left( \frac{\beta}{2R_1} \right) x^2} \]

Phase error
\[ \delta = \frac{\beta}{2R_1} x^2 \]
\[ \delta_{\text{max}} = \frac{\beta}{2R_1} \left( \frac{A}{2} \right)^2 = \frac{2\pi}{\lambda} \frac{A^2}{8R_1} = 2\pi \epsilon \]
\[ \epsilon = \frac{A^2}{8\lambda R_1} = \frac{1}{8} \left( \frac{A}{\lambda} \right)^2 \frac{1}{R_1\lambda} \]
DIRECTIVITY CURVES
H-PLANE SECTORAL HORN

Figure - Universal directivity curves for an H-plane sectoral horn. For pyramidal horns the vertical axis values are $(\lambda / B)D_H$.

\[ A = \sqrt{3\lambda R_1} \]
\[ \theta_{opt} = \frac{A^2}{8\lambda R_1} \]
\[ A^2 = 3\lambda R_1 \]
\[ \delta_{max} = 2\pi \theta = 2\pi \left(\frac{3}{8}\right) = \frac{3\pi}{4} = 135^\circ \]

<table>
<thead>
<tr>
<th>( R_1 )</th>
<th>6</th>
<th>10</th>
<th>20</th>
<th>100</th>
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<tbody>
<tr>
<td>( A )</td>
<td>4.24</td>
<td>5.48</td>
<td>7.75</td>
<td>17.22</td>
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</table>
Universal Radiation Pattern

H-plane sectoral horn

Figure - Universal radiation patterns for the principal planes of an H-plane sectoral horn as shown in Fig. 8-10. The factor \((1 + \cos \theta)/2\) is not included.
E-PLANE SECTORAL HORN

SIDE VIEW

\[ L_E^2 = R_1^2 + \left( \frac{B}{2 R_2} \right)^2 \]
\[ L_E = \tan^{-1} \left( \frac{B}{2 R_2} \right) \]

Figure — E-plane sectoral horn antenna. (a) Overall geometry. (b) Cross section through the yz-plane (E-plane).

Electric Field distribution

\[ E_{ay} = E_0 \cos \frac{\pi x}{a} \ e^{-j(\beta/2 R_2) y^2} \]

\[ S = \frac{\beta^2}{2 R_2^2} y^2 \implies S_{\text{max}} = 2 \pi \left( \frac{\beta^2}{8 \lambda R_2} \right) = 2 \pi S \]

\[ S = \frac{\beta^2}{8 \lambda R_2} = \frac{1}{8} \left( \frac{\beta}{2 \lambda} \right) ^2 \frac{1}{R_2/\lambda} \]
Figure — E-plane sectoral horn antenna. (a) Overall geometry. (b) Cross section through the yz-plane (E-plane).

Electric Field distribution

\[ \text{Eay} = E_0 \cos \frac{\pi x}{a} e^{-j(\beta/2R_z) y^2} \]

\[ \delta = \frac{B}{2R_z} y^2 \Rightarrow \delta_{\text{max}} = 2\pi \left( \frac{B^2}{8\lambda R_z} \right) = 2\pi S \]

\[ S = \frac{B^2}{8\lambda R_z} = \frac{1}{8} \left( \frac{B}{\lambda} \right)^2 \frac{1}{R_z/\lambda} \]
**DIRECTIVITY CURVES**

*E-plane Sectoral Horn*

Figure - Universal directivity curves for an E-plane sectoral horn. For pyramidal horns the vertical axis values are \((\lambda/4)D_1\).

**Optimum Directivity**

\[
B = \sqrt{2\lambda R_2}
\]

\[
S_{\text{opt}} = \frac{B^2}{8\lambda R_2} = \frac{1}{4}
\]

\[
\theta_{\text{max}} = 2\pi S_{\text{opt}} = \frac{\pi}{2} = 90^\circ
\]
Figure: Universal radiation patterns for the principal planes of an E-plane sectoral horn antenna as shown in Fig. 5. The factor $(1 + \cos \theta)/2$ is not included.
**Pyramidal Horn Design**

\[ A = \sqrt{3\lambda R_1} \approx \sqrt{3\lambda l_H} \quad - (1) \]

\[ B = \sqrt{2\lambda R_2} \approx \sqrt{2\lambda l_E} \quad - (2) \]

**Gain**
\[ G = \zeta \mu p \frac{4\pi}{\lambda^2} A_p \quad (A_{eq} = \frac{1}{2} A_p) \]
\[ = \frac{1}{2} \frac{4\pi}{\lambda^2} AB = \frac{2\pi}{\lambda^2} AB \quad - (3) \]

For physical realization of horn
\[ R_E = R_H \]

\[ (B-b)\sqrt{\left(\frac{\lambda E}{B}\right)^2 - \frac{1}{4}} = (A-a)\sqrt{\left(\frac{l_H}{A}\right)^2 - \frac{1}{4}} \quad - (4) \]

From (1) - (4)

\[ \left[ \sqrt{2\sigma} - \frac{b}{\lambda} \right]^2 (2\sigma-1) = \left( \frac{G}{2\sqrt{\pi}} \frac{1}{\xi} - \frac{\sigma}{\lambda} \right)^2 \left( \frac{\xi^2}{18\pi^2} \frac{1}{\xi} - 1 \right) \]

\[ \sigma = \xi E/\lambda \]

Solve eqn. (5) iteratively

First trial \( \sigma_1 = \frac{G}{2\pi \sqrt{E}} \Rightarrow \text{Find } \sigma \)

\[ \xi E = \sigma \lambda, \quad B = \sqrt{2\lambda l_E} \]

\[ A = \frac{6\lambda^2}{2\pi B}, \quad l_H = \frac{A^2}{3\lambda} \]
Figure — Pyramidal horn antenna. (a) Overall geometry. (b) Cross section through the xz-plane (H-plane). (c) Cross section through the yz-plane (E-plane).

\[
R_E = R_H - \delta \beta/2 \left( \frac{x^2}{R_1} + \frac{y^2}{R_2} \right)
\]

\[
\varepsilon_{ay} = \varepsilon_0 \cos \left( \frac{\pi x}{A} \right) e
\]

\[
D_P = \frac{\pi}{32} \left( \frac{\lambda}{A} \right)^2 \left( \frac{\lambda}{B} \right)^2
\]
Figure — Principal plane patterns for the optimum pyramidal horn antenna of Example 1 at 9.3 GHz. The patterns include the \((1 + \cos \theta)/2\) factor. \(HP_E = 12.9^\circ\) and \(HP_H = 13.6^\circ\).

**Optimum Dimensions vs. Directivity**

Figure — Dimensions of rectangular (pyramidal) horns (in wavelengths) versus directivity (\(\tau\) for gain, \(\delta\) for loss). Thus, noting the dashed lines, a gain \(\geq 19\) dBi requires a horn length \(L = 4.25\) and \(H\)-plane aperture \(a_H = 3.7\) and an \(E\)-plane aperture \(a_E = 2.9\). These are inside dimensions. It is assumed that \(\delta\) \((E\ plane) = 0.25\) and \(\delta\) \((H\ plane) = 0.42\), making the dimensions close to optimum. It is also assumed that \(\tau\) = 0.6.
OPTIMUM DIMENSIONS VS. DIRECTIVITY

Figure 12. Dimensions of conical horn (in wavelengths) versus directivity (or gain, if no loss). Thus, noting the dashed lines, a gain of 20 dBi requires a horn length $L_1 = 6.0$ and a diameter $D_2 = 4.3$. These (inside) dimensions are close to optimum.

Figure 13. Experimentally observed patterns of conical horns of various dimensions.

MULTIMODE HORNS

Diagonal Horn

FIG. 14. Transformation from rectangular waveguide to diagonal horn.

$TE_{10}$ and $TE_{01}$ excited with equal amplitude and phase in a square waveguide.
**CONICAL HORN**

**FIG. 11** Calculated gain of a conical horn as a function of aperture diameter with axial length as parameter.

\[ \theta = \tan^{-1} \left( \frac{D}{2L} \right) \]

\[ S = \frac{D^2}{8AL} \]

**Sphereical wave phase error**

\[ 0.30 \text{ to } 0.375 \text{ for optimum gain} \]

**Gain (dB)**

\[ 7.0 + 20.6 \log \frac{D}{\lambda} \]
Dual Mode Conical Horn

Step of Length l

\[ z = 0 \]
\[ 2a \quad 2b \text{ (dia.)} \]
\[ d = 2a \]

Fig. 15 Dual-mode horn with generating step and its approximate aperture field distribution.

Step-less Dual Mode Horn

\[ 2a \]
\[ \theta_f \]

Fig. 17 Dual-mode pyramidal horn.

Dual Mode Pyramidal Horn

\[ d \]

\[ TE_{10} \quad TE_{12} / TM_{12} \]

Flare Angle Change

\[ \theta_f \]

\[ TE_{12} \quad TE_{10} + TE_{12} / TM_{12} \]

Fig. 18 Section through square-aperture pyramidal horn with flare-angle change.
**Conical Corrugated Horn**

**FIG.** Small-flare-angle corrugated horn at left; corrugations extended into flange at right.

**FIG.** Wide-flare scalar horn.

**FIG.** Universal patterns for small-flare-angle corrugated horns under near-balanced conditions.
(a) Cuttaged surface

(b) Noncuttaged surface

Geometry of corrugated and plane surfaces.

(a) Surface current decay on corrugated surface due to energy forced away from corrugations.

(b) Surface current decay on corrugations as a function of corrugation density.

(c) Surface current decay on corrugations as a function of corrugation shape.
Figure  Cross section of circular waveguide-fed corrugated horn with corrugated transition. Corrugations with depth of $\lambda/2$ at waveguide act like a conducting surface while corrugations with $\lambda/4$ depth in horn present a high impedance.

Figure  Cross section of circular waveguide with flange and 4 chokes for wide-beam-width high-efficacy feed of low $f/D$ parabolic reflectors.
**BROADBAND HORN**

**SHORT AXIAL LENGTH HORN**

![Sketch of a short axial length horn.](image)

**Table 1**

**Horn Dimensions**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>1.0-12.0 GHz-Horn</th>
<th>0.9-2.0 GHz-Horn</th>
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<tbody>
<tr>
<td>A</td>
<td>0.323</td>
<td>1.623</td>
</tr>
<tr>
<td>B</td>
<td>1.600</td>
<td>5.000</td>
</tr>
<tr>
<td>C</td>
<td>6.000</td>
<td>30.0000</td>
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</table>

**Cross Section Dimensions**

<table>
<thead>
<tr>
<th>Location</th>
<th>Width (in)</th>
<th>Height (in)</th>
<th>Width (in)</th>
<th>Height (in)</th>
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<td>0.872</td>
<td>7.000</td>
<td>6.200</td>
</tr>
<tr>
<td>Feed point</td>
<td>1.200</td>
<td>0.872</td>
<td>7.000</td>
<td>6.200</td>
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<tr>
<td>Launcher-horn junction</td>
<td>3.400</td>
<td>2.616</td>
<td>11.200</td>
<td>6.700</td>
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<td>Horn aperture</td>
<td>9.300</td>
<td>5.440</td>
<td>37.500</td>
<td>27.200</td>
</tr>
</tbody>
</table>

**Note**: Gain measured on 1/4 scale model.

**Fig. 19**: Sketch of a short axial length horn.

**Fig. 20**: VSWR and gain (1.0-12.0 GHz horn).

**Fig. 21**: VSWR and gain (0.9-2.0 GHz horn).
**FIG. 22:** A COMPACT, APERTURE-MATCHED ANTENNA

**FIG. 23:** MEASURED REFLECTION COEFFICIENT FOR MATCHED AND UNMATCHED ANTENNAS

**FIG. 24:** GAIN OF VARIOUS ANTENNAS