ROBUST FEEDBACK SYNTHESIS FOR NARMAX MODELS USING GENERALIZED DESCRIBING FUNCTIONS

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Abstract: In this paper, a technique is proposed for synthesizing robust sampled-data feedback systems for plants described by polynomial NARMAX models. The technique is based on the concept of generalized frequency response functions, and exploits recent results that enable derivation of these directly from the NARX description of the plant. A nonlinear chemical reactor example is solved using the procedure and found to yield satisfactory results.

Keywords: NARMAX model, Nonlinear System Identification, Nonlinear Control, Robust Control, Quantitative Feedback Theory (QFT)

1. INTRODUCTION

Sampling of real finitely realizable continuous-time nonlinear systems naturally produces NARMAX (Nonlinear AutoRegressive Moving Average with eXogenous inputs) models, as demonstrated by Chen and Billings (1989). The NARMAX model provides a unified representation for a wide class of discrete-time nonlinear stochastic systems, and includes several known nonlinear input-output models, such as Hammerstein, Weiner bilinear, and state -affine, as special cases. The chief advantages of the NARMAX model over functional series representations such as the Volterra series, are that for identification the former requires a reduced number of parameters, smaller data sets, and there is no need for special input signals. With the identification results also being easier to analyze for NARMAX models, these are certainly more convenient to use than the Volterra series.

Of the various forms of NARMAX models the polynomial NARMAX model is perhaps the most suitable in practical applications, because it is linear in the parameters. Many linear identification results have been extended to the polynomial NARMAX model, and several combined routines of intelligent structure determination and parameter estimation are available, see Korenberg, *et al.* (1988). Indeed, practical identification of sev-

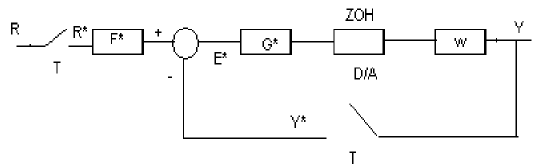

Fig. 1. The nonlinear sampled-data system.

eral industrial systems has established that most practical systems can be satisfactorily modeled by polynomial NARMAX models.

Quantitative Feedback Theory (QFT) of Horowitz (1993) is a well-established body of robust control synthesis techniques. A QFT technique for sampled-data systems comprising of a nonlinear continuous time plant modeled by differential equations has been outlined by Horowitz and Liao (1986). However there are no QFT techniques to handle systems described by polynomial NARMAX models. In view of the popularity and widespread use of these models, it is very desirable to have such a QFT technique. In this work, we propose a QFT technique for systems described by polynomial NARMAX models.

Further, in the nonlinear QFT procedures of Horowitz (1976) and Ioinovici (1987), a major computational difficulty arises while generating templates of the so-called 'LTIE plant' set. Our proposed technique is also based on the LTIE plant approach. However, in our technique a new method of computing the LTIE plant templates based on generalized describing functions is introduced. In the new method, the LTIE plant templates are generated easily and efficiently, *directly* from the coefficients of the nonlinear model. Thereby, the computational difficulties inherent in the existing methods of LTIE plant template generation are solved.

To implement the proposed procedure on nonlinear systems, an integrated package has been developed (cf. sec. 4.3). Initial experience with this package in the area of chemical process control has been encouraging (a simulated chemical reactor examples is given in sec. 5). It is hoped that the developments and software tools reported here make it possible to use QFT *on-line* in nonlinear robust control, in the not too distant future.

It is assumed in this paper that the reader is familiar with the ideas and results concerning nonlinear frequency response functions as given by Jones and Billings (1991), and with those of QFT methods as given by Horowitz (1976; 1993) and Horowitz and Liao (1986).

2. BACKGROUND

Consider a nonlinear SISO plant in a two degree of freedom structure. Suppose the plant is given by nonlinear continuous mapping $w : u(t) \to y(t)$, with unique continuous inverse w^{-1}. Due to the uncertainty in physical parameters, there is a denumerable set of nonlinear plants $\mathbb{W} = \{w\}$. The given finite set of deterministic inputs $\mathbf{I}$ to the system consists of the set of possible setpoint signals $\mathbb{R} = \{r\}$ and disturbances $\mathbb{D} = \{d\}$. For each $i_\alpha \in I$, there is a specified set of acceptable plant outputs $\mathbb{A}_\alpha$. The design problem is to find strictly proper LTI operators F (the prefilter) and G (the controller), such that for each $i_\alpha \in I$, the system output $y \in \mathbb{A}_\alpha, \forall\ w \in \mathbb{W}$.

For nonlinear continuous-time plants, a QFT synthesis technique to solve the above problem has been presented by Horowitz (1976). The technique, based on Schauder's fixed point theorem and valid for zero-initial conditions, is basically a two-step procedure: The first step is to find a set $\mathbb{P}_{eq}$ of what are known as LTI 'equivalent' (LTIE) plants. The second step is to solve the synthesis problem with $\mathbb{P}_{eq}$ replacing $\mathbb{W}$. It has been shown by Horowitz (1976) that for a large nonlinear problem class, the prefilter F and controller G which solve this 'equivalent' LTI problem, also solve the original nonlinear problem (i.e., for the set $\mathbb{W}$).

Next consider a nonlinear sampled-data system shown in Fig. 1. Assume that a fixed sampling period T is used, giving sampling frequency $\omega_s = \frac{2\pi}{T}$. As is customary, let $f^*(t)$ denote the impulsesampled signal, $F^*(s)$ denote the Laplace transform of $f^*(t)$, and $F(z)$ denote $[F^*(s)]_{z=e^{sT}}$.

A QFT approach similar to the nonlinear continuous-time case has been suggested by Horowitz and Liao (1986). In the sampled-data case, the actual plant input is of a staircase form (as a ZOH is used), and therefore the acceptable output set must be carefully formulated so that the $w^{-1}(a)$ indeed emerges as a staircase signal. Once this has been done, the designer can proceed by obtaining the 'equivalent' LTI set $\mathbb{P}_{eq}$, exactly as in the nonlinear continuous-time case, and apply linear sampled-data QFT techniques to the set $\mathbb{P}_{eq}$.

An improved algorithm to find the LTIE plant set has been given by Ioinovici (1987). This method does not require w^{-1} to exist, which is a constraint inherent in Howoritz's LTIE method. Further, Ioinovici demonstrated through several examples that his algorithm gives superior results to the earlier LTIE method, in terms of reduced overdesign. However, certain difficulties are found in Ioinovici's LTIE algorithm:

1. Finding *analytically* the solution of nonlinear differential equation describing the plant, for each member of I.

2. Obtaining the expressions for Laplace-transforms of each of these time-domain solutions.

3. As discussed earlier, it is very desirable in practice to have polynomial NARMAX representations for nonlinear plants. However, Ioinovici algorithm does not address plants represented as NARMAX models - in polynomial or other forms (the same is true for Horowitz's algorithm.)

A method to directly find (i.e. without solving differential equations or Laplace- transforming) the LTIE plant templates from the given Polynomial NARMAX model and the set I is described in the following section.

3. THE NARMAX MODEL AND ITS LTIE PLANT

Suppose that the nonlinear plant is represented by the model

$$\begin{aligned} y(k) = F[&y(k-1), ..., y(k-n_y), \\ &u(k-1), ..., u(k-n_u), \\ &\zeta(k-1), ..., \zeta(k-n_\zeta)] + \zeta(k) \end{aligned} \qquad (1)$$

where F is some nonlinear function of lagged input signals $u(k-n_u)$, outputs $y(k-n_y)$, and noise $\zeta(k-n_\zeta)$, with k denoting the sampling intervals and n the lags. The model in (1) is referred to as the NARMAX model. Chen and Billings (1989) rigorously proved that a nonlinear discrete-time-invariant system can always be represented by the NARMAX model in a region around an equilibrium point, subject to two sufficient conditions: (1) The response function of the system is finitely realizable (which means that distributed-parameter systems are excluded) (2) A linearized model exists if the system is operated close to the chosen equilibrium point. Further, the model can also be shown to be valid for the non-zero initial state response case.

If the nonlinear function $F(\cdot)$ is continuous, it can always be arbitrarily closely approximated by a polynomial function. For practical purposes, therefore, we may choose $F(\cdot)$ as a finite polynomial function, giving us a polynomial NARMAX model.

Once a polynomial NARMAX model of the plant has been estimated, we can discard the moving average noise terms in (1) to get a polynomial NARX (Nonlinear AutoRegressive with exogenous inputs) model. This is justified, as the moving average noise terms were originally included to ensure unbiased estimation, and therefore can be dispensed with once estimation is over.

The output $y(t)$ of NARX model is expressed as

$$y(t) = \sum_{m=1}^{M} y_m(t) \qquad (2)$$

where $y_m(t)$ is $m-$th order output of system, given by

$$\begin{aligned} y_m(t) = &\sum_{p=0}^{m} \sum_{k_1,k_n=1}^{K} c_{p,q}(k_1, \ldots, k_{p+q}) \\ &\times \prod_{i=1}^{p} y(t-k_i) \prod_{i=p+1}^{p+q} u(t-k_i) \end{aligned} \qquad (3)$$

with $p+q=m$, $k=1,\ldots,K$ and $\sum_{a,b=1}^{K} = \sum_{a=1}^{K}\cdots\sum_{b=1}^{K}$.

>From a polynomial NARX model description of the plant, the n-th order GFRFs (Generalized Frequency Response Functions) can be computed using the recursive probing algorithm of Jones and Billings (1989). The algorithm yields the n-th order frequency responses to be found, without restriction on the order n. Further, this method also exposes the structure of $H_n(\cdot)$, and enables the GFRFs to be related to the structure and coefficients of the nonlinear difference equation model of the plant. These GFRFs can be subsequently used to evaluate the GDFs (Generalized Describing Function), giving us a unidimensional frequency domain representation of the nonlinear plant.

Using the recursive probing input method, the n-th order GFRFs for the NARX model (2), (3) are computed as follows:

$$\begin{aligned}
&\left(1-\sum_{k_1=1}^{K} c_{1,0}(k_1)\exp(-j(\omega_1+\cdots+\omega_n)k_1)\right)\\
&\times H_n(j\omega_1,\ldots,j\omega_n)\\
=&\sum_{k_1,k_n=1}^{K} c_{0,n}(k_1,\ldots,k_n)\\
&\times\exp(-j(\omega_1k_1+\cdots+\omega_nk_n))\\
&+\sum_{q=1}^{n-1}\sum_{p=1}^{n-q}\sum_{k_1,k_n=1}^{K} c_{p,q}(k_1,\ldots,k_{p+q})\\
&*\exp(-j(\omega_{n-q+1}k_{n-q+1}+\cdots+\omega_{p+q}k_{p+q}))\\
&\times H_{n-q.p}(j\omega_1,\ldots,j\omega_n)\\
&+\sum_{p=2}^{n}\sum_{k_1,k_p=1}^{K} c_{p,0}(k_1,\ldots,k_p)\times\\
&H_{n.p}(j\omega_1,\ldots,j\omega_n)
\end{aligned}\tag{4}$$

where $H_{n.p}(\cdot)$ is generated by the recursion

$$\begin{aligned}
H_{n.p}(\cdot)=&\sum_{i=1}^{n-p+1} H_i(j\omega_1,\ldots,j\omega_i)\times\\
&H_{n-i.p-i}(j\omega_{i+1},\ldots,j\omega_n)\times\\
&\exp(-j(\omega_1+\cdots+\omega_i)k_p)
\end{aligned}\tag{5}$$

The recursion finishes at $p=1$ and

$$\begin{aligned}
H_1(j\omega_1,\ldots,j\omega_n)=&H_n(j\omega_1,\ldots,j\omega_n)\\
&\times\exp(-j(\omega_1+\omega_n)k_1)
\end{aligned}\tag{6}$$

Next, the n-th order multidimensional output spectrum is found:

$$\begin{aligned}
Y(j\omega_1,\ldots,j\omega_n)=&H_n(j\omega_1,\ldots,j\omega_n)\\
&\times\prod_{i=1}^{n} U(j\omega_i)
\end{aligned}\tag{7}$$

where $U(j\omega)$ represents the normalized input spectrum. Then, the unidimensional output spectrum is obtained:

$$\begin{aligned}
Y_n(j\omega)=&\frac{1}{(2\pi)^{n-1}}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\\
&Y_n(j\omega_1,j(\omega_2-\omega_1),\ldots,j(\omega_n-\omega_{n-1}))\\
&\times d\omega_1\ldots d\omega_{n-1}
\end{aligned}\tag{8}$$

The total unidimensional output spectrum is given by summation of all the n-th order unidimensional output spectrums:

$$Y(j\omega)=\sum_{n=1}^{N} Y_n(j\omega)\tag{9}$$

Finally, this output response is used to evaluate the GDF:

$$N(A,j\omega)=\frac{Y(j\omega)}{AU(j\omega)}\tag{10}$$

where A denotes the input amplitude or waveform scaling factor. Note that $N(A,j\omega)$ is considered undefined whenever $U(j\omega)=0$.

It is easily seen that the GDF characterizes precisely Ioinovici's LTIE plant that corresponds to the given polynomial NARMAX model and the input signal considered in (7).

Thus, the GDF provides a new and powerful frequency domain representation of a wide class of nonlinear systems. This characterization is subsequently used in the proposed procedure as a basis for controller synthesis using the principles of sampled-data QFT.

4. A QFT PROCEDURE FOR POLYNOMIAL NARMAX MODELS

4.1 *The Basic Idea*

Suppose that a polynomial NARMAX model description of the nonlinear plant is available. From this model, a NARX model is extracted, and the $n-$th order GFRFs generated using (4)-(6). For any particular input signal $i_\alpha \in I$, the corresponding GDF is obtained by finding the total output frequency response and then diving it by the input signal spectrum, as given in (7)-(10). Now, the GDF is unidimensional in frequency, so a single magnitude and phase (Bode) response is got at each frequency. By repeating the procedure over I, a band of (instead of a single) magnitude and phase plots is obtained at each frequency. This response band forms the template of the LTIE plant at each frequency.

These LTIE plant templates are next used for feedback synthesis using linear sampled-data QFT methods. The resulting controller and prefilter when used on the original nonlinear plant, are guaranteed to achieve the given specs. This has been shown in general for a large class of nonlinear systems using fixed-point theorems of nonlinear function analysis by Horowitz (1993).

4.2 *The Proposed Procedure*

We now give the complete synthesis procedure.

(1) For the considered plant, formulate the set I of signals for which the design is to be performed, and the set A of acceptable output responses.

(2) Generate appropriate input-output data set for experimental (or simulated) identification of the non-linear plant. Using an integrated structure determination and parameter estimation algorithm of Korenberg, *et al.* (1988), identify a parsimonious model in the polynomial NARMAX form from these data sets. Validate the model using model validation methods for nonlinear systems.

(3) From the identified NARMAX model, obtain the NARX model (2)-(3) by discarding the moving average noise terms.

(4) From the obtained NARX model, find GFRFs $H_1\left(j\omega_1\right), H_2\left(j\omega_1, j\omega_2\right), ..., H_n\left(j\omega_1, \ldots, j\omega_n\right)$ using (4)-(6) where the highest order n is consummate with the nonlinearity of the model. Evaluate these functions in the output frequency domain, and use the same domain for all further work.

(5) Pick any input signal $i_\alpha \in I$, and find its input spectrum $U(j\omega)$.

(6) Find the n-th order output frequency response $Y_n(j\omega_1, j(\omega_2-\omega_1), \ldots, j(\omega_n-\omega_{n-1})$ using (7), the unidimensional output frequency response $Y_n(j\omega)$ using (8), the total unidimensional frequency response $Y(j\omega)$ using (9), and the GDF $N(A, j\omega)$ using (10).

(7) Repeat steps (5)-(6) over the set I (and/or over set W), to get magnitude and phase response bands at each frequency. These bands form the template of the LTIE plants at each frequency.

(8) Using the LTIE plant templates generated at the design frequencies, synthesize a controller $G(z)$ and a prefilter $F(z)$ using linear sampled-data QFT methods. The steps in sampled-data controller design using QFT are detailed by Horowitz (1993).

(9) Design verification: The performance of the closed loop system with the synthesized prefilter $F(z)$ and controller $G(z)$, and the original nonlinear plant model needs to be checked in the time domain. This can be accomplished using simulation packages such as SIMULINK (2001).

4.3 *Software Aspects*

To implement the proposed procedure, a suite of MATLAB-based program has been developed at IIT, Bombay. This suite can be categorized in terms of the following sets of programs:

1. NLID: Performs automatic structure detection, parameter estimation and model validation of multivariable nonlinear systems, see Makwana (1995).

The underlying algorithms are based on the works of Billings and co-workers (see the references).

2. NLMIMO: Finds GFRFs and GDFs for multivariable NARX models identified using NLID, with at most second order terms, see Date (1995). Program handles step command inputs for a range of amplitudes, and up to three uncertain model coefficients. The LTIE templates generated are readily usable by QFT_IIT.

3. QFT_IIT: Performs robust feedback synthesis using QFT principles, see Nataraj (1994). A recent version incorporates a fully automated controller synthesis routine based on the numerous suggestions given by Horowitz (1993) .

Using the integrated software package, the overall design cycle for the problem example given in sec. 5 took about 3 min. on a PC/Pentium-I 133 MHz.. A major portion of this time was taken up by the MATLAB-based numerical integration routine QUAD8 called upon by NLMIMO. A more efficient numerical integration routine should considerably reduce the computation time, enabling the procedure to be executed fast enough for online QFT-based control of nonlinear processes.

5. SIMULATION EXAMPLE

5.1 *Problem Description*

We test our design algorithm on a nonlinear differential equation model of an isothermal CSTR described by Eaton and Rawlings (1990). The reaction occurring in CSTR is $2A \xrightarrow{k} B$ with (reaction rate) $\propto$ (concentration of $A)^2$.Assuming that volume of liquid is constant, the mass balance equation is

$$V\frac{dC_A}{dt} = F_{in}\left[C_{A_{in}}(t) - C_A(t)\right] - KVC_A^2(t) \qquad (11)$$

where K is related to the reactor temperature by $K = K_0 e^{-E/R_gT}$. Here, C_A is the concentration of reactant A, mol/lit., $C_{A_{in}}$ is inlet concentration of A, mol/lit., F_{in} is the inlet flow, in mols/ hr., T is reactor temperature, Kelvin, V is the volume of vessel, liters, and E and R_g are physical constants. The input and output variables of CSTR are F_{in} and C_A, respectively. The reactor parameter values are $K = 0.972$ mol lit/hr., and $V = 10.0$ liters. The initial steady-state concentration of reactant A is 0.5 mol/ lit., with the inlet concentration $C_{A_{in}} = 3.6$, and $F_{in} = 0.784$.

(11) is rewritten in terms of deviation variables with respect to the initial steady state values:

$$\frac{dy}{dt} = \frac{C_{Ain} - C_{A,s}}{V}u - \frac{F_{in,s} + 2KVC_{A,s}}{V}y - Ky^2 - \frac{1}{V}uy \qquad (12)$$

where $y(t)$ and $u(t)$ are the deviations in C_A and F_{in} from their respective steady states $C_{A,s}$ and $F_{in,s}$. The reactor model (12) is used to generate input-output data set for identification purposes. The sampling time is taken as 0.01 hours. From this data set, a NARMAX model is first identified using program NLID described in sec. 4.3 and then a NARX model is extracted as

$$y(k) = \alpha y(k-1) + \beta u(k-1) - 0.0031u(k-1)y(k-1) \qquad (13)$$

where $\alpha = 0.8858, \beta = 0.0156$.

Next, uncertainty is introduced into the reactor parameter values, which leads to the following bounds on the estimated NARX parameter values:$\alpha \in [0.7, 0.9], \beta \in [0.012, 0.018]$.

Based on the open loop responses, for a unit step in setpoint of C_A the closed loop specs are set as follows. Steady state offset at most 2%; Maximum overshoot: 10%; Minimum and maximum 2% settling times: 0.53 and 0.65 hours, respectively. These figures of merit are translated into the frequency domain via transfer function models. The translated frequency domain specs and the original time domain ones are shown as dotted lines in Figs. 2 and 3. Moreover, a gain margin of 5 dB and a phase margin of 45^0 are sought.

5.2 *Design Execution*

The design is executed as follows.

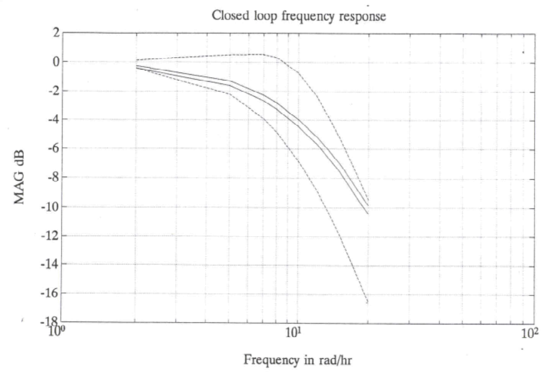

Fig. 2. The closed loop frequency responses

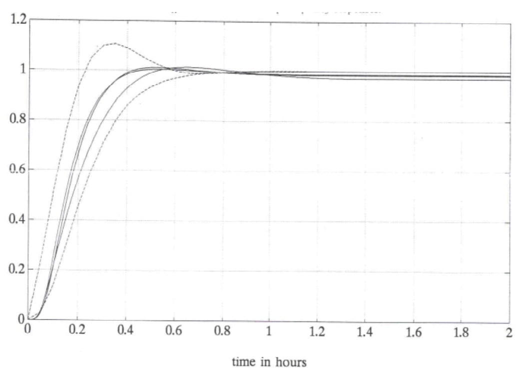

Fig. 3. The closed loop time responses obtained for the original reactor system.

1. For generating the LTIE templates corresponding to the obtained NARX model, first and second order GFRFs are used. Following step 4 of our procedure, the first two GFRFs are derived from (12) as

$$H_1\ (\exp(j\omega)) = \frac{\beta}{1 - \alpha\exp(-j\omega)}$$

$$\begin{aligned} &H_2\ (\exp(j\omega_1), \exp j(\omega_2 - \omega_1)) \\ &= \frac{0.0031\exp(-j\omega_2) H_1(\exp(j\omega_2))}{1 - \alpha\exp(-j\omega_2)} \end{aligned} \qquad (14)$$

Continuing the procedure until step 6 and using the equation

$$N(j\omega) = \frac{1}{U(j\omega)}\left[Y_1(j\omega) + Y_2(j\omega)\right]$$

the GDF is evaluated over the design frequency range. At each frequency, $N(j\omega)$ is a function of uncertain parameters α and β. Thus, by evaluating $N(j\omega)$ at different value sets of the reactor parameters, the LTIE plant template at each frequency

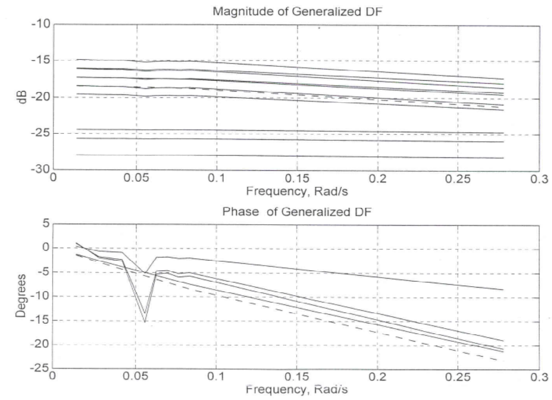

Fig. 4. Frequency responses of some LTIE plants. Dotted lines is for the linear transfer function H_1.

is got (cf. step 7). Program NLMIMO describe in sec. 4.3 is used to automate these computations. The frequency responses of the LTIE plants are plotted in Fig. 4.

An important condition to be satisfied by the plant family is that the plant templates must be topologically path connected, see Nwokah and Thompson (1989).

This condition is checked for our example as follows: From the expressions for H_1, H_2 in (14), and from (7), (8), it is seen that the domains of $Y_1(j\omega)$ and $Y_2(j\omega)$ are path connected sets, and that $Y_1(j\omega)$, $Y_2(j\omega)$ are continuous functions. Since a continuous image of a path connected set is path connected, it follows that the templates generated from (10) are indeed path connected.

For further work, the nominal plant is arbitrarily taken as the linear transfer function $H_1(\cdot)$ with $\alpha = 0.8858$, $\beta = 0.0156$.

2. The robust margin bounds and the discrete-time tracking bounds on a nominal loop transmission $L_0^*(s)$ are derived from the specs stated in sec. 5.1.

3. The synthesis of a $G(z)$ that satisfies these bounds and of an appropriate prefilter $F(z)$ is carried out using the QFT_IIT toolbox. Using the QFT_IIT tool box, a controller is obtained as

$$\begin{aligned} G_{num}(z) = &1.52z^6 + 3.29z^5 - 0.72z^4 - 5.50z^3 \\ &-1.98z^2 + 2.29z + 1.26 \end{aligned}$$

$$G_{den}(z) = z^6 - 1.49z^5 - 0.20z^4 + 1.21z^3$$
$$-0.8z^2 + 0.28z - 0.00015$$
$$G(z) = G_{num}(z) / G_{den}(z) \quad (15)$$

and a prefilter as

$$F_{num}(z) = 0.00472z^2 + 0.0094z + 0.0047$$
$$F_{den}(z) = z^2 - 1.71z + 7.25$$
$$F(z) = F_{num}(z) / F_{den}(z) \quad (16)$$

5.3 *Design Verification*

Since our proposed method is based on GDFs an error analysis for the validity of the describing function approximation is necessary, see Bergen, *et al.* (1982) and Mees and Bergen (1975). The analysis is carried out as given by Nataraj, *et al.* (1997), and verifies closed loop stability.

Before proceeding to time domain design verifications with the $G(z)$ and $F(z)$ obtained above, the closed loop frequency responses are first checked. Fig. 2 (dotted lines are the specs) shows that, over the entire range of NARX model parameters α, β, these specs are satisfactorily met in the frequency domain.

Closed loop time domain simulation studies on the nonlinear reactor model are performed using the simulation package SIMULINK. The setpoint on C_A is changed by a step of unit magnitude, and the closed loop time responses for different reactor parameter values are obtained (see Fig. 3). Over the entire range of parameter uncertainty, the reactor concentration responses (solid line figure, nearly single) are seen to be well within the time domain specs.

6. CONCLUSIONS

A new synthesis procedure for robust control of nonlinear sampled-data systems has been proposed. This procedure uses generalized describing function to characterize a given NARX model. Robust controller design is carried out using principles of nonlinear QFT. The proposed procedure enables one to apply QFT methods to the widely used polynomial NARMAX models. A simulation example of a nonlinear reactor model has been solved using the proposed procedure. The results have been found to be quite satisfactory.

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