An Analytical Approach for Stochastic Assessment of Phase-Angle Jumps in Large Systems

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Abstract—This paper presents an analytical method for stochastic prediction of phase-angle jump of voltage sags caused by short-circuit fault in power systems. The method represents the phase-angle jump as a continuous function of distance to the fault. A simple radial system has been used to analyze the influence of source strength and the influence of cross-section of over-head line conductor and under-ground cable. The mathematical formulation of the proposed method has been applied to the IEEE 30-bus Reliability Test System to illustrate its application.

I. INTRODUCTION

Voltage sags are short-duration (i.e. 0.5-30 cycles) reduction in the supply RMS voltage between 10-90 % of the nominal voltage; caused by short circuits, overloads and starting of large motors. The magnitude and duration are the main characteristics of voltage sag. In recent years, several stochastic methods for voltage sags estimation have been proposed [1]-[6].

The interest in voltage sags is mainly due to the problem they cause on several types of equipment like adjustable speed drives, process control equipment, and computers those are notorious for their sensitivity. Some pieces of equipment trip when RMS voltage goes below 90% for more than one or two cycles. Most equipment is only concerned with the magnitude of voltage sags. For this reason, most of the methods used for voltage sag assessment concentrate only on the analysis of voltage sag magnitude [7]-[8]. However, balanced and unbalanced faults in a power system not only cause a drop in voltage magnitude but also cause change in the phase-angle of the voltage. Therefore, power-electronics converters that use phase-angle information for their firing instants may be affected by the phase-angle jump [5], [9]-[10]. Phase angle jump manifests itself as a shift in zero-crossing of the instantaneous voltage. Phase-angle jumps during three-phase faults are due to the difference in X/R ratio between the source and the feeder and the transformation of sags to lower voltage levels. In this paper, an analytical approach for stochastic assessment of voltage sags is used to analyze the phase-angle jumps associated with voltage sag caused by balance as well as unbalance faults.

II. CALCULATION OF PHASE ANGLE JUMP

The analytical approach focuses on voltage sags caused by short-circuit faults along the lines and formulates the expression of phase-angle jump. A sample 3-bus radial system has been taken for analysis of phase-angle jump in which the influence of different source strengths and the size of overhead line conductor and under-ground cable have been studied. The influences of different impedance angles of overhead line and under-ground cable are also taken into account. The study of phase angle jumps, due to unbalanced and balanced faults, has also been extended to a meshed IEEE 30-bus Reliability Test System.

A. The method of critical distances

The method of critical distances is applicable for radial system only. To quantify sag magnitude in a radial system, the voltage divider model shown in Fig. 1 is used. The expression for voltage sag at point-of-common coupling (pcc) can be expressed as [11]

\[ \bar{V}_{se} = \frac{E - \bar{Z}_s}{\bar{Z}_f + \bar{Z}_s} \]  

Where, \( \bar{Z}_s = (R_s + jX_s) \) and \( \bar{Z}_f = R_f + jX_f \) are the feeder and source impedance respectively. In this paper the pre-event voltage is assumed 1 p.u., thus E=1. This results in the following expression for the sag magnitude as

\[ \bar{V}_{se} = \frac{\bar{Z}_s}{\bar{Z}_f + \bar{Z}_s} \]
The argument of \( V_{sag} \), thus the phase-angle jump in the voltage, is given by the following expression

\[
\Delta \phi = \arg(V_{sag}) = \tan^{-1}\left(\frac{X_S + X_f}{R_S + R_f}\right) - \tan^{-1}\left(\frac{X_S}{R_S}\right)
\]

If \( \frac{X_S}{R_S} > \frac{X_f}{R_f} \), equation (3) becomes zero and there is no phase angle jump. The phase-angle jump will thus be present if the \( X \) ratio of the source and the feeder are different.

To obtain expression for voltage sag magnitude and the associated phase-angle jump as a function of distance of the fault, we substitute \( Z_{zzL} \) in equation (2) where \( z \) is the feeder impedance per unit length and \( L \) is the distance of the fault position (on the radial feeder) from the pcc, resulting in

\[
\text{sag} = S F Z Z L V
\]

The phase angle-jump is found from

\[
\Delta \phi = \arg(V_{sag}) - \arg(Z_{zzL})
\]

The phase angle jump is thus equal to the angle in the complex plane between \( Z_S + Z_f \) and \( Z_f \) is shown in Fig. 2, where \( \phi \) is the phase angle jump.

**B. Proposed analytical method**

To determine the phase-angle jump for a meshed network, the expression for the residual phase voltages for balanced and unbalanced faults occurring along an arbitrary line are derived. For the calculation of phase-angle jump, first of all, load flow analysis is performed to get the pre-fault voltage magnitudes & phase angles. In addition, various driving-point and transfer impedances are calculated, in advance, using \( Z_{Bus} \) building algorithm.

Now let a fault position \( p \) moves along a line connecting buses \( m \) and \( n \) as shown in Fig.3, the location \( p \) at which the fault occurs is identified with the help of parameter \( \lambda \). The parameter \( \lambda \) varies from 0 to 1 as the fault position moves from bus \( m \) to \( n \). Therefore, \( \lambda \) is defined as

\[
\lambda = \frac{L_p}{L_{nm}}, 0 \leq \lambda \leq 1
\]

The driving point impedances and the transfer impedances of three sequence circuits can be expressed in terms of the sequence impedances. The sequence transfer impedances between the sensitive load bus \( i \) and fault position \( p \) can be expressed as [12]

\[
Z^{s}_{z} = (1-\lambda)Z^s_{zz} + \lambda Z^s_{nn}
\]

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Where, \( Z^{s}_{zz} \) are the sequence transfer impedance corresponding to bus \( m \) and \( i \). Similarly, \( Z^{s}_{nn} \) are the sequence transfer impedances corresponding to bus \( i \) and \( n \). The sequence driving point impedance at fault position \( p \) can be expressed as follows

\[
Z_{mp} = C \times \begin{bmatrix} Z^0_{mp} & Z^1_{mp} & Z^2_{mp} \\ Z^0_{mn} & Z^1_{mn} & Z^2_{mn} \\ Z^0_{zn} & Z^1_{zn} & Z^2_{zn} \end{bmatrix}
\]

\[
C = \begin{bmatrix} (1-2\lambda + \lambda^2) & 2(\lambda - \lambda^3) & \lambda^2 - (\lambda - \lambda^3) \\ (1-2\lambda + \lambda^2) & 2(\lambda - \lambda^3) & \lambda^2 - (\lambda - \lambda^3) \\ (1-2\lambda + \lambda^2) & 2(\lambda - \lambda^3) & \lambda^2 - (\lambda - \lambda^3) \end{bmatrix}
\]

where \( Z^{s}_{mp} \) and \( Z^{s}_{mn} \) are the driving point sequence impedances at buses \( m \) and \( n \) respectively, \( Z^{s}_{mn} \) are the driving point sequence impedances corresponding to buses \( m \) and \( n \), and \( Z^{s}_{zn} \) are the line sequence impedances between buses \( m \) and \( n \). The pre-fault voltage at fault position \( p \) is expressed as follows
The phase-angle jump at bus $i$ for SLGF can be expressed as

$$\Delta \phi_{ph,i} = \angle V_{ph,i}^f - \angle V_{ph,i}^s$$

V$^s$ and V$^f$ are the pre-fault voltages and residual phase voltages at bus $i$ due to DLGF where $ph$ represents phases A, B, and C respectively.

The phase-angle jump at bus $i$ for LLGF can be expressed as

$$\Delta \phi_{ph,i} = \angle V_{ph,i}^f - \angle V_{ph,i}^s$$

V$^s$ and V$^f$ are the pre-fault voltages and residual phase voltages at bus $i$ due to LLGF, where $ph$ represents phases A, B, and C respectively.

The phase-angle jump at bus $i$ for LLLF can be expressed as

$$\Delta \phi_{ph,i} = \angle V_{ph,i}^f - \angle V_{ph,i}^s$$

V$^s$ and V$^f$ are the pre-fault voltages and residual phase voltages at bus $i$ due to LLLF.

III. CASE STUDIES

The study was performed on two power networks: a sample 3-bus radial network and a meshed IEEE 30-Bus Reliability Test system [15]. The results are as discussed below.

A. Analysis of phase angle jump in radial network

Fig. 4 represents a simple 3 bus radial network used for the analysis of phase-angle jump.

Fig.4. Radial network

In this case, to find out the influence of source strength in the phase-angle jump, different source strengths (i.e. 750 MVA, 200 MVA, and 75 MVA) are considered, keeping the conductor size constant (i.e. 50 mm$^2$) and then, to investigate the influence of the conductor size on the phase-angle jump, different conductor sizes (i.e. 50 mm$^2$, 150 mm$^2$, and 300mm$^2$) are considered, keeping the source strength constant (i.e. 750 MVA) $Z_0=0.065$ for the 750 MVA source has been taken. For all these conductor sizes, three different fault levels (i.e. 750 MVA, 200 MVA, and 75 MVA) are considered. In addition, the feeder is considered first, as an overhead line and then, as an under-ground cable of 30 km length. The impedances of under-ground cable and overhead line are given in Table I.

1) Influence of source strength

Phase angle jump versus distance, for faults on a 50 mm$^2$ 11 kV overhead feeder line 2-3 in Fig.4, with different source strengths is presented in Fig.5. It can be observed that a stronger source makes the smaller phase-angle jump.
2) Influence of cross section

The phase-angle jump versus distance curve for 11kV overhead line of different cross sections is as shown in Fig. 6. From the overhead line impedance data shown in Table 1, X/R ratio of the feeder impedances can be calculated as: 1 for the 50 mm², 2.7 for the 150 mm², and 4.9 for the 300 mm²; it is observed from Fig. 6 that the phase-angle jump decreases with the increase in the X/R ratio of the feeder or in other words, the phase-angle jump reduces with the increase in the thickness of the conductor.

![Fig. 6. Phase-angle jump v/s distance, for overhead lines with different cross section.](image)

The influence of different cross-sections for underground cable is as shown in Fig. 7. It can be observed that cables with a smaller cross-section have a larger phase-angle jump for small distances to the fault, but the phase-angle jump also decays faster with increasing distance due to larger impedance per unit length.

B. Analysis of phase angle jump in IEEE 30-bus Reliability test system

For analysis of phase angle jump, the analytical method is also applied to the IEEE 30 bus reliability test system [15] which is a meshed network. The schematic diagram of the network as shown in Fig. 8 consists of 5 generating units, 30 buses interconnected by 41 lines, and 4 transformers.

![Fig. 7. Phase-angle jump v/s distance, for underground cables with different cross-section.](image)

Different types of faults are considered in order to show the potential of the proposed analytical method to deal with balanced and unbalanced faults. Fig. 9 shows the variation of phase angle jump at arbitrarily chosen bus 4 when single-phase-to-ground fault (SLGF), line-to-line-fault (LLF), double-line-to-ground-fault (DLGF) and three-phase-fault (LLLFI) occur at various fault positions along line 5-7.

It can be observed that highest value of phase-angle jump reached when DLGF occur. Especially, the effect of this type of fault is more noticeable when DLGF occurs closer to the far end of the line i.e. bus 7.
A similar analysis is performed considering different types of faults along line 4-6 of the system. Again, the most severe situation corresponds to DLGF. In this case, too, the phase-angle jump at bus 3 increases when the fault is closer to the end of the line. It can also be observed that all remaining fault types i.e. SLGF, LLF and LLLF have constant phase-angle jumps irrespective of the fault position.

Similarly from Fig.11, it can be observed that when faults occur along the line 23-24, the most severe situation at bus 16 correspond to DLGF when fault is closer to beginning of the line, that is closer to bus 23, and it decreases when fault is closer to the end of the line that is bus 24.

The phase-angle jump for all the buses due to SLGF, occurring along the line 4-6 is as shown in Fig.12. It is observed from Fig that for SLGF the maximum phase-angle jump is observed at bus number 30. However the maximum phase-angle jump due to SLGF is $-18^\circ$.
IV. CONCLUSIONS

The paper illustrates the methodology for calculating phase-angle jump due to balanced and unbalanced faults along the line and contributes in providing a better understanding of the behavior of the phase angle jump.

A simple 3 bus radial network has been used for analysis of phase angle jump. The influences of source strength and cross-section for over-head line conductor and under-ground cable on phase-angle jump due to different faults have been analyzed. From the analysis, it is observed that stronger source makes the phase angle jump smaller. It has been shown that cable with smaller cross-section has a larger phase-angle jump for small distance to the fault and the over-head line with thick conductor have smaller phase-angle jumps.

In case of the meshed IEEE 30-bus RTS system, phase-angle jump can exhibit a non-monotonic tendency with the distance to the fault, especially, in case of the DLGF. However, phase-angle jump does not vary much with distance for other fault types (i.e. SLGF, LLG and LLLF).

V. REFERENCES


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