Multi-Objective Approach for Voltage Stability using Interior Point Method

P.K. Modi, S.P. Singh, and J. Sharma

Abstract—A multi-objective approach for voltage stability evaluation is presented for the power system with FACTS devices. Primal dual interior point method for nonlinear programming is proposed to solve the multi-objective problem. The problem is formulated by combining conflicting objectives from voltage stability perspective. The objectives are to maximize loading and minimize real power losses. SVC is considered to be connected to the system under study. The method is robust and efficient with reduced solution times. Effects of different types of load models such as, constant power, constant current, constant impedance and ZIP model, on the solution are also investigated. The efficient set of solution formed is quite useful for the utilities in operation and planning. The proposed method is applied to IEEE-30 bus system.

I. INTRODUCTION

Modern power systems operate under much stressed conditions, closer to operating limits due to increased demand. In some cases, changes in operating conditions could cause a progressively and uncontrolled fall of voltages leading to voltage collapse [1-2]. With the development of FACTS devices it has been possible for the power systems to operate much nearer to its stability limits [3]. SVC is one of such thyristor controlled FACTS devices, most suitable for voltage control. Several methods have been proposed, to assess the voltage stability of the power systems. These include continuation power flow, sensitivity analysis, singular value decomposition, and modal analysis [2]. Optimization methods and techniques have also been used in solving voltage stability problems [4].

Interior point methods in mathematical programming have been the largest and most dramatic area of research in optimization. Since its introduction by Frisch, this method has permanently changed the landscape of mathematical programming theory practice and computation [5]. Karmarkar’s algorithm for linear programming has led a series of remarkable advances in these methods. Today, interior point methods for linear programming as well as non-linear programming are quite promising both in theory and practice. Most of the present interior point methods are based on primal-dual approach [5]. Recently, Vanderbei described an interior point algorithm for non-convex nonlinear problems [6], which is a direct modification of his earlier method for linear and quadratic programming [7]. Major modifications include a merit function and an altered search direction to ensure that a descent direction for the merit function is obtained. This algorithm is robust and efficient with greatly reduced solution times. This work has used this algorithm.

The interior point method has been increasingly used in solving the variety of power systems problems [8]. In the area of voltage stability different variants of interior point method have been used; such as primal dual interior point method [9], predictor corrector primal dual interior point method [10] and direct interior point method [11,12]. The various problems dealt in these works are to determine maximum loadability [9, 12], reactive power margin [13], voltage collapse reliability index [14], preventive and corrective control for voltage stability [15]. From the literatures survey it is observed that interior point methods has not been used for determining the voltage stability of power systems using FACTS devices.

In practice power system planners and operators are confronted with multiple objective functions and these objective functions are generally in conflict with each other. These objectives must be dealt with carefully. Multi-objective problems frequently arise in transmission network applications, commonly due to the trade-off required between security and economy. Power systems problems have been solved by, defining as multi-objective problems in some of the work, related to optimal power flow [16] and optimal scheduling of hydro stations [17]. From voltage stability point of view a optimal reactive power scheduling method using successive multi-objective fuzzy LP technique have been presented in [18]. But so far no significant work has been done to take care of conflicting objectives while improving the voltage stability of the power systems.

In this paper, multi-objectives methodology is presented from voltage stability point of view. The objectives are to maximize loading and minimize real power losses. The primal dual non-linear interior point method is used to find the optimal solution. The weighting objectives method is used to form a Pareto optimum set. The effect of different load model has also been investigated on the problem solution. The proposed methodology is applied to IEEE-30 bus systems [19].
II. LOAD MODEL

A load model is a mathematical representation of the relationship between a bus voltage (magnitude or frequency) and the power (active or reactive) or current flowing into the bus load [20]. The accurate modeling of loads continues to be a difficult task due to several factors, such as; large numbers of diverse load components, ownership and location of load devices in customer facilities, changing load composition with time, lack of precise information on the composition of load and uncertainties regarding the characteristics of many load components, particularly for large voltage or frequency variations. IEEE task force has described the nature of loads and various approaches to model it and recommended standard load models for power flow and dynamic performance simulation [21]. Voltage sensitivity of the loads can provide some system relief following a voltage disturbance. In this paper load model as described below has been considered.

A model that expresses the active and reactive powers at any instant of time as functions of the bus voltage magnitude or frequency at the same instant is known as static load model [20]. The general form of load characteristic is given as:

\[
P_d = P_d(V) \quad Q_d = Q_d(V) \tag{1}
\]

Since in the voltage instability phenomena, frequency excursions are not of primary concern, the frequency dependence of loads is neglected. The exponential load characteristic has been used widely and given by [20]:

\[
P_d = P_{d0} \left(\frac{V}{V_0}\right)^{a_1} \tag{2a}
\]

\[
Q_d = Q_{d0} \left(\frac{V}{V_0}\right)^{a_2} \tag{2b}
\]

Where, \(V_0\) is the reference voltage, \(P_{d0}\) and \(Q_{d0}\) are the active and reactive power consumed at the reference voltage. The exponent \(a_1\) and \(a_2\) depend upon type of load and determine the sensitivity of load power to voltage.

For, constant power load, \(a_1 = a_2 = 0\), constant current load, \(a_1 = a_2 = 1\), and constant impedance load, \(a_1 = a_2 = 2\).

As the different load components exhibit different voltage characteristics, an alternative load representation is given which is based on summing up load components, which has the same or almost same exponent. Thus a polynomial load model known as ZIP model can be represented by [20]:

\[
P_d = P_{d0} \left[b_1 \left(\frac{V}{V_0}\right)^2 + b_2 \left(\frac{V}{V_0}\right) + b_3\right] \tag{3a}
\]

\[
Q_d = Q_{d0} \left[c_1 \left(\frac{V}{V_0}\right)^2 + c_2 \left(\frac{V}{V_0}\right) + c_3\right] \tag{3b}
\]

Where, \(b_1 + b_2 + b_3 = c_1 + c_2 + c_3 = 1\). These polynomial load parameters are obtained from measurements. Only the static load model given by (2) and (3) are considered, as steady state voltage instability phenomena is studied.

III. PROBLEM FORMULATION

In modern power systems, utilities are more concerned about security of the systems as well as to operate the systems to its full utilization. When security of power systems is vital to provide uninterrupted power supply to consumers, economic operation of power systems is also required for the utilities. From voltage stability point of view, the objective can be set to maximize the loadability margin. In power systems real power losses, voltage deviations from base case, total cost of real power generations and cost of var support increase, as the load increases. Therefore other objectives which can be considered are to minimize real power losses, voltage deviations from base case, total cost of real power generations and cost of var support. These objectives are in conflict with the objective to maximize the loadability margin and either of these can be combined with it to formulate a multi-objective problem. The minimization of real power losses needs more attention as because utilities are more interested in reducing the overall cost of operation. Since the generators have to supply the loads and losses, reducing the losses at increased loading will reduce the cost of generations and relieve generators from supplying higher losses.

From this consideration the problem is formulated with the objective to maximize the loading factor and to minimize the total real power losses of the systems. The equality constraints are real and reactive power balance equations at all the buses, control equations governing SVC; real and reactive power flow balance equations from/to SVC and power balance equations of SVC. Upper and lower bounds are considered on real and reactive power generations, transformers taps and SVC. In case of SVC, limits on firing angle are considered. The objective functions are:

To maximize the loading factor:

maximize \( f_1 = \lambda_{gf} \) \tag{4}

To minimize the real power losses:

\[
\text{minimize} \quad f_2 = \sum_{j=1}^{nl} G_{j(k,m)} \left[V_k^2 + V_m^2 - 2V_kV_m \cos(\delta_k - \delta_m)\right] \tag{5}
\]

subject to:

Real and reactive power balance equation:

\[
P_{SL} - (1 + \lambda_{gf}) P_{d1} - P_1 = 0, \quad Q_{SL} - (1 + \lambda_{gf}) Q_{d1} - Q_1 = 0,
\]

SVC control and balance equations:

\[
V_{svc} - V_{vc} - X_{sl}I_{svc} = 0, \quad \pi X_L B_c - 2\alpha_{svc} \sin 2\alpha_{svc} + \pi(2 - X_L / X_C) = 0, \quad I_{svc} - I_{vc} = 0, \quad Q_{svc} - V_{vc}^2 B_c = 0,
\]

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Limits on real and reactive power generations and transformer taps:
\[ P_{li,\min} \leq P_{li} \leq P_{li,\max} \]
\[ Q_{li,\min} \leq Q_{li} \leq Q_{li,\max} \]
\[ \text{tap}_{j,\min} \leq \text{tap}_j \leq \text{tap}_{j,\max} \]

Limits on loading factor:
\[ \lambda_{lf} \geq 0 \quad \text{for all } i = 1 \ldots nb, \quad j = 1 \ldots nl. \]

IV. WEIGHTING OBJECTIVES METHOD

The weighting objectives method [23] has received most attention and particular models within this method have been widely applied. The basis of this method consists in adding all the objective functions together using different weighting coefficients for each objective. It means that the multi-objective problems is changed to a scalar optimization problem by creating one function of the form:

\[ F = \sum_{i=1}^{N} w_i f_i \quad (6) \]

Where, there are N numbers of objectives and \( w_i \geq 0 \) are the weighting coefficients representing the relative importance of the criteria. It is usually assumed that:

\[ \sum_{i=1}^{N} w_i = 1 \quad (7) \]

Since, the results of solving an optimization model using (6) can vary significantly as the weighting coefficients change, and since very little is usually known about how to choose these coefficients, a necessary approach is to solve the same problem for many different values of \( w_i \). The engineers must then choose among these solutions. The weighting method is designed for use interactively when in each decision phase; values of \( w_i \) are generated on the basis of a formal or an informal procedure. It is worth noting that the objective functions are expressed in different units and weights are used to compare them and to give the relative importance of each objective. Another feature of this method is that a good knowledge of the power system is required to form the Pareto set. The multi-objective problem (5) is converted to scalar problem using weighting objectives method as follows:

maximize \( (w_1 f_1 - w_2 f_2) \quad (8) \]

Where, \( w_1 \) and \( w_2 \) are the weighting coefficients on loading factor and real power losses. The weighting coefficients have been introduced to allow a tradeoff between loading factor and real power losses and it is possible to carry out the analysis by adopting the framework of multi-objective optimization [23].

A feasible solution to a multi-objective optimization is efficient (non-inferior or Pareto optimal) if it is not possible to find another feasible solution so as to improve one of the objectives without worsening at least one of the others. The collection of efficient solutions is called the efficient set. The trade-off surface represents the values of the objectives for efficient solutions. In other words, the trade-off surface is the mirror image of the efficient set into the space of objectives. If the aggregation function is convex combination of the individual objectives in multi objective problems then the optimal solution to any set of weighting parameters is efficient.

A. General Form of Problem

The problems (4) & (5) can be represented in general form as follows:

minimize \[ f(x) \]
subject to: \[ \begin{align*}
  g(x) = 0, \\
  h_l \leq h(x) \leq h_u, \\
  x_l \leq x \leq x_u.
\end{align*} \]

Where function \( f(x) \) is the negative of \( \lambda_{lf} \) the loading factor, vector \( x \) is the state or control variables, \( g(x) \) is the power balance equations and equations governing control of FACTS device, the inequality constraints \( h(x) \) are the physical or operational limits. \( x_l \) and \( x_u \) are lower and upper bounds on variable \( x \). The problem stated in (9) is solved by Primal Dual Interior Point algorithm as given in [6] and implemented in the software LOQO [6].

V. RESULTS AND DISCUSSIONS

The proposed method is applied to IEEE-30 bus systems. The system is studied with the assumption that a SVC connected to one of the system bus. In this case it is assumed to be connected at Bus-30. having ±100MVAR.

The voltage and angle at slack bus are kept fixed at initial value. The real power generations, which are zero at PV buses i.e. no generator is connected at these buses, are also kept fixed at zero. However reactive power generations are allowed to change at these buses. The system is studied with four different types of load models i.e. Constant power model, Constant current model, Constant impedance model, ZIP model. In case of ZIP model the coefficients are considered as follows:

\[ b_1 = c_1 = 0.6, \quad b_2 = c_2 = 0.1, \quad b_3 = c_3 = 0.3; \]

In all the test systems under study, 500 sets of weighting parameters are generated by randomly varying the \( w_1 \) and \( w_2 \) between 0 and 1 such that their sum is always unity. The individual objective function levels are calculated for each set using primal dual nonlinear interior point method. The efficient set is formed from the efficient solutions. This set can be used by the operator to take decisions. To limit the space only five solution for each case of load model for the test systems.
systems are shown in Table-1.

TABLE-1: LOADABILITY AND LOSSES FOR IEEE-30 BUS SYSTEM

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Load Model</th>
<th>(W_1)</th>
<th>(W_2)</th>
<th>Loading Factor</th>
<th>Losses in p.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Constant Power</td>
<td>0.9501</td>
<td>0.0499</td>
<td>0.6498</td>
<td>0.9655</td>
</tr>
<tr>
<td>2</td>
<td>Constant Power</td>
<td>0.7689</td>
<td>0.2311</td>
<td>0.6099</td>
<td>0.7306</td>
</tr>
<tr>
<td>3</td>
<td>Constant Power</td>
<td>0.6068</td>
<td>0.3932</td>
<td>0.5101</td>
<td>0.5136</td>
</tr>
<tr>
<td>4</td>
<td>Constant Power</td>
<td>0.8913</td>
<td>0.1087</td>
<td>0.6434</td>
<td>0.8919</td>
</tr>
<tr>
<td>5</td>
<td>Constant Power</td>
<td>0.7621</td>
<td>0.2379</td>
<td>0.6072</td>
<td>0.7215</td>
</tr>
<tr>
<td>6</td>
<td>Constant Power</td>
<td>0.9501</td>
<td>0.0499</td>
<td>0.9040</td>
<td>1.1666</td>
</tr>
<tr>
<td>7</td>
<td>Constant Power</td>
<td>0.7689</td>
<td>0.2311</td>
<td>0.8730</td>
<td>0.9950</td>
</tr>
<tr>
<td>8</td>
<td>ZIP</td>
<td>0.6068</td>
<td>0.3932</td>
<td>0.7394</td>
<td>0.7151</td>
</tr>
<tr>
<td>9</td>
<td>ZIP</td>
<td>0.8913</td>
<td>0.1087</td>
<td>0.9001</td>
<td>1.1219</td>
</tr>
<tr>
<td>10</td>
<td>ZIP</td>
<td>0.7621</td>
<td>0.2379</td>
<td>0.7004</td>
<td>0.7963</td>
</tr>
<tr>
<td>11</td>
<td>ZIP</td>
<td>0.9501</td>
<td>0.0499</td>
<td>1.5423</td>
<td>1.3074</td>
</tr>
<tr>
<td>12</td>
<td>ZIP</td>
<td>0.7689</td>
<td>0.2311</td>
<td>1.3569</td>
<td>1.2780</td>
</tr>
<tr>
<td>13</td>
<td>ZIP</td>
<td>0.7621</td>
<td>0.2379</td>
<td>1.5364</td>
<td>1.2764</td>
</tr>
<tr>
<td>14</td>
<td>ZIP</td>
<td>0.9501</td>
<td>0.0499</td>
<td>0.9474</td>
<td>1.1504</td>
</tr>
<tr>
<td>15</td>
<td>ZIP</td>
<td>0.7689</td>
<td>0.2311</td>
<td>0.9203</td>
<td>0.9991</td>
</tr>
<tr>
<td>16</td>
<td>ZIP</td>
<td>0.6068</td>
<td>0.3932</td>
<td>0.7999</td>
<td>0.7494</td>
</tr>
<tr>
<td>17</td>
<td>ZIP</td>
<td>0.8913</td>
<td>0.1087</td>
<td>0.9438</td>
<td>1.1096</td>
</tr>
<tr>
<td>18</td>
<td>ZIP</td>
<td>0.7621</td>
<td>0.2379</td>
<td>0.9181</td>
<td>0.9917</td>
</tr>
</tbody>
</table>

In the above results the losses are given in p.u. on 100 MVA base. The loading factor gives an indication that by how much amount the load can be increased on the system. It is the value over the base case solution. Thus the total load can be 1+0.6498=1.6498, where 1.0 is the base case loading (refer to the 1st result in the table) and so on. The method produces the desired solution very fast. The computation time in given in Table-2 for the systems for one particular value of weighting coefficients, i.e. \(w_1 = 0.9501\) and \(w_2 = 0.0499\). However, it has been observed that the computation time varies as the weighting coefficient varies. It is because of the convergence characteristics of the algorithm for different parameters value, as it happens in all optimization problems. However, the computation time for other values of weighting coefficients remains in the ranges of ±10% of the time given in Table-2.

TABLE-2: COMPUTATION TIME FOR ALL THE SYSTEMS

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Load Model</th>
<th>Computation time in Secs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Constant Power</td>
<td>0.036</td>
</tr>
<tr>
<td>2</td>
<td>Constant Current</td>
<td>0.031</td>
</tr>
<tr>
<td>3</td>
<td>Constant Impedance</td>
<td>0.046</td>
</tr>
<tr>
<td>4</td>
<td>ZIP</td>
<td>0.031</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper multi-objective approach for voltage stability evaluation of power system with SVC is presented. The objectives are to maximize loading and minimize real power losses. The primal dual non-linear interior point method is used to find the optimal solution. The weighting objectives method is used to form a Pareto optimum set. The effect of different load model has also been investigated on the problem solution. The primal dual interior point method can be efficiently used for voltage stability studies. The method is robust and efficient with reduced solution times. This method converges very fast. Load model has a considerable effect on the loadability margin. Of all the load models considered ZIP model combines all types of loads and can be effectively used for voltage stability evaluation purposes.

VII. REFERENCES


VIII. BIOGRAPHIES

P. K. Modi received his B.Sc (Engg.) in Electrical Engineering from Regional Engineering College, Rourkela, M.E.(Power System Engg.) from University College of Engineering, Burla, India, and Ph.D. from Indian Institute of Technology, Roorkee in 1987, 1996, and 2003 respectively. He is serving as Reader at University College of Engineering, Burla. His area of interest is power system operation and planning, FACTS devices.

S. P. Singh received his B.Sc. degree in Electrical Engineering from Aligarh Muslim University, Aligarh in 1978, and obtained M.E. and Ph.D. from University of Roorkee, India in 1980 and 1993, respectively. He is serving as Associate Professor at Indian Institute of Technology, Roorkee. His area of research includes power plant operation and control, electric machine analysis, self and line excited induction generators.

J. Sharma obtained BE in electrical engineering from Jiwaji University,Gwalior, India in 1968, ME in 1971 and Ph D in 1974 from University of Roorkee. At present he is serving as Professor in Indian Institute of Technology, Roorkee. He has published more than 150 papers in various national and international journals. His fields of interest are power system planning, operation and automation.