Abstract—Network contingencies often cause branch overloading and voltage violation and thus leading to security problems. This paper illustrates an approach for determining the most suitable locations for installing thyristor controlled series capacitors to eliminate line overloads under single contingency. Using contingency severity index, the mostly affected branches are ranked. An optimization problem for finding the best locations among the ranked branches to install TCSC, and settings for the installed TCSCs, is formulated and solved. The proposed approach uses the new penalty parameter-less constraint handling scheme based Real coded Genetic Algorithm. Results are compared with conventional Real Coded Genetic Algorithm (RGA) and Particle Swarm Optimization (PSO) algorithm. Illustrations using the IEEE 6-bus and 30-bus test systems aim to exhibit the effectiveness of the proposed method.

I. INTRODUCTION

The security of power system can be defined as its ability to withstand a set of severe but credible contingencies and to survive transition to an acceptable new steady state condition. In present day power system, there will be an increase in number of situations where power flow equations have either no real solution or solution with violating operating limits such as voltage limit (insecure case), particularly, in contingency analysis and planning applications. Contingency screening and ranking is one of the components of on-line system security assessment. The target of contingency ranking and screening is to rapidly grade the decisive contingencies from a large list of plausible contingencies and rank them according to their severity for further rigorous analysis. Various Performance Index (PI) based methods for contingency screening and ranking have been reported in literature [1]-[3].

FACTS devices are solid state devices that have the capability of control of various electrical parameters in transmission networks. These devices, by controlling the power flows in the network, can help to reduce the flows in heavily loaded lines, resulting in an increased loadability, low system loss, improved stability of the network, and reduced cost of production [4], [5].

Thyristor Controlled Series Capacitor (TCSC) is one such device which offers smooth and flexible control for security enhancement with much faster response compared to the traditional control devices [6]. While there have been numerous studies concerning the utilization of these devices so far, most of the researchers have focused on issues such as transient stability improvement, sub-synchronous resonance (SSR) mitigation, damping of power swings, avoiding voltage collapse, enhancing power system reliability, etc [7],[8]. However, TCSC can be used effectively in maintaining system security in case of a contingency, by eliminating or reducing overloads along the selected network branches.

In this paper, utilization of the TCSC during single contingencies is investigated. In order to evaluate the suitability of a given branch for placing a TCSC, contingency severity index is calculated for each branch [9]. This index is used to rank the branches that are mostly affected during all the possible single contingencies. After having the ranked list of branches, an optimization problem is formed to find out the best locations among the ranked branches to install the TCSCs and and to determine the best settings of the installed TCSC with respect to single contingencies. The objective used in this problem is to eliminate or reduce the line overloads and increase the security margin.

Population based co-operative and competitive stochastic search algorithms are employed in the recent years in the research area of computational intelligence. Some well established search algorithms such as Genetic Algorithm [10] and Particle Swarm Optimization [11] have been successfully implemented to solve the complex problems.

In all the previous works reported in the literature for solving this problem, constraints are handled by the use of a penalty function approach, that is, the constraint violation is multiplied by a penalty parameter and added to the objective function. Recently, Deb [12] has proposed a penalty parameterless constraint-handling scheme to overcome the difficulty in choosing penalty parameters. It was demonstrated by the improved efficiency of penalty parameter-less scheme on the class of constrained test problems. Hence, in this paper, the penalty parameter-less
II. PROBLEM FORMULATION

A. Contingency Ranking

The purpose of contingency ranking is to list the branches which are more sensitive to the largest number of contingencies. TCSCs that are in series with the selected branches will provide the most efficient control of the system flows in the largest number of contingencies. This section will describe the definitions of matrices and array, and calculation of the contingency severity index.

The participation matrix (U) is a binary matrix, whose entries are 1 or 0 depending upon whether or not the corresponding branch is overloaded, where \( n \) is the total number of branches of interest, and \( m \) is the total number of considered contingencies.

The ratio matrix (W) is an \((m \times n)\) matrix of normalized excess branch flows. \( W_{ij} \) is the normalized excess power flow (with respect to the base case flow) through branch “\( j \)” during contingency “\( i \)" and is given by:

\[
W_{ij} = \frac{P_{ij,cont}}{P_{j,Base}} - 1
\] (1)

where

\( P_{ij,cont} \) - Power flow through branch “\( j \)” during contingency “\( i \)"

\( P_{j,Base} \) - Base case power flow through branch “\( j \)".

The probability array (P) is an \((m \times 1)\) array of branch outage probabilities. The probability of branch outage is calculated based on the historical data about the faults occurring along that particular branch in a specified duration of time. It will have the following form:

\[
P = [p_1, p_2, p_3, \ldots, p_m]^T
\] (2)

where

\( p_i \) - Probability of occurrence for contingency “\( i \)"

and is taken as 0.02.

\( m \) - The number of contingencies

Thus the Contingency Severity Index (CSI) for branch “\( j \)” is defined as the sum of the sensitivities of branch “\( j \)” to all the considered single contingencies, and is expressed as

\[
CSI_j = \sum_{i=1}^{m} p_i u_j w_{ij}
\] (3)

Branches are then ranked by their corresponding index values. In general, the larger the index value a branch has, the more sensitive it will be.

B. Allocation of FACTS devices

Previous works on this topic investigate the generation rescheduling and load shedding as the primary corrective strategies for alleviating overloads on transmission lines [13]. In the newly emerging deregulated operation, the design of system control devices and their associated performance will have to be based on economic incentives. The rescheduling of generation and load shedding may not be acceptable by both power providers and customers, due to their significant effect on the existing power transaction contracts. In addition, they may be harmful for system security due to the discontinuous action of load shedding and slow adjustment of generation rescheduling.

An alternative solution can be devised through the use of FACTS technology. TCSC is one of the most effective FACTS devices in use today. Proper use of TCSCs can reduce or eliminate the unwanted loop flows, and hence increase the system security margin [14].

For a large-scale power system, more than one TCSC may have to be installed in order to achieve the desired performance. Once the number of devices are decided, the solution for the optimal allocation problem includes the optimal location for the devices to be installed and the optimal settings of those installed devices, so that the number of overloads are eliminated or reduced and the security margin is maximized.

C. Optimal Location of TCSC

In previous placement works, the locations of the TCSCs are chosen starting from the top of this ranked list and proceeding downward with as many branches as the number of available TCSCs. In this work, all branches having the contingency index values are considered, and the best locations to install TCSCs are determined by the proposed algorithm.

For example, if four branches have the index value and two devices are to be installed, then it is not necessary to install the devices in the first two top branches from the rank list. It is suggested that all the four branches are taken into the algorithm which takes care of the best two locations. This approach yields improved performance of the power system considered, in security point of view.

D. Optimal setting of TCSCs

In this paper TCSC is modeled as a reactance in series with the line. To determine the best possible settings of the devices for all the possible single contingencies, the optimization problem is solved using new penalty parameterless constraint handling scheme based real coded genetic algorithm. The objective function for this work is,

\[
Obj= \text{minimize} \ (F_1 + U_n) \quad (4)
\]

Where,

\( U_n \) - total number of overloads in all the lines after all the possible single contingencies

\( F_1 \) = function representing severity of overloading

\[
F_i = \sum_{c=1}^{m} \sum_{k=1}^{n} a_k \left[ \frac{P_k}{P_k^{\max}} \right]^4
\] (5)

where
m - Number of single contingency considered
n - Number of lines
ak - weight factor=1.
P_k - real power transfer on branch k.
\( P_k^{max} \) - maximum real power transfer on branch k.

The following are the constraints associated with the formulated problem.

1. TCSC constraint
The reactance of TCSC is limited from +50% to -50% of the line reactance to which it is added. If more than +50%, then the reactance of the line will be so high that it will affect the power flow under normal conditions, inspite of doing good job under contingency conditions. If added more than -50%, then the line reactance will be so less that will allow more power beyond thermal limit, even under normal conditions.

\[-0.5X_L < X_{TCSC} < 0.5X_L \]  \hspace{1cm} (6)

\( X_L \) - original line reactance in per unit
\( X_{TCSC} \) - reactance offered by TCSC

2. Voltage Stability Constraints
The bus voltage \( V_b \) must lie within the following limits and \( V_S \) represents the voltage violation.

\[
V_S = \begin{cases} 
0 & \text{if } 0.9 < V_b < 1.1 \\
0.9 - V_b & \text{if } V_b < 0.9 \\
V_b - 1.1 & \text{if } V_b > 1.1 
\end{cases}
\]  \hspace{1cm} (7)

3. Power balance Constraints
The equality constraints are the power balance constraints which are monitored by the load flow solution.

\[
\sum P_G = \sum P_D + P_L \]  \hspace{1cm} (8)

where

\( \sum P_G \) = Total power generation
\( \sum P_D \) = Total power demand
\( P_L \) = Losses in the network

E. GA Fitness function
The fitness function for evaluation of population individual in Conventional RGA is as follows.

Fitness = \( F_t + U_n + \lambda \cdot VS \)  \hspace{1cm} (9)

where \( \lambda \) is the penalty factor.
In the proposed penalty parameter-less method, there is no presence of \( \lambda \). The handling of voltage constraints are explained in section III B.

III. GENETIC ALGORITHM AND IMPLEMENTATION
A. Genetics Algorithm
GA starts with an initial set of random solutions called population. A population of candidate solutions, or individuals, is maintained, and individuals made to compete with each other for survival. Once evaluated, through the fitness function calculation, stronger individuals have a greater chance to contribute to the production of new individuals (the offspring) than weaker ones, which may not even contribute at all (selection procedure). Offspring are produced through recombination, whereby they inherit features from each of the parents, and through mutation, which can confer some truly innovative features as well.

In the next selection step (next generation), offspring are made to compete with each other, and possibly also with their parents. Improvement in the population arises as a consequence of the repeated selection of the best parents, which are in turn more likely to produce good offspring, and the consequent elimination of low-performers. After several generations, the algorithm converges to the best individual, which hopefully represents the optimal solution to the problem.

B. Penalty parameterless constraint handling scheme
In GA, penalty function approach is used for constraint handling. Penalty functions can be stationary or non-stationary. Stationary penalty functions add a fixed penalty when a violation occurs, as opposed to non-stationary penalty functions which add a penalty proportional to the amount which the constraint is violated, and are also a function of the iteration number. The difficulty of using penalty parameter is the selection of an appropriate penalty value for the problem.

In this paper, a constraint-handling method is employed which does not require any penalty parameter. In penalty parameter-less constraint-handling scheme, all feasible solutions have zero constraint violation and all infeasible solutions are evaluated according to their constraint violations alone. Hence, both the objective function value and constraint violation are not combined in any solution to the population. Thus there is no need to have any penalty parameter for this approach.

In penalty parameterless scheme, the fitness function is calculated using the following.

\[
F(x) = \begin{cases} 
\overline{f(x)} & \text{if } f(x) \leq f_{max} + \sum_{j=1}^{m} g_j(\overline{x}) \\
\overline{f(x)} + \sum_{j=1}^{m} g_j(\overline{x}) & \text{otherwise} 
\end{cases}
\]

where \( \overline{f(x)} \) is fitness function and \( f(x) \) is the objective function, \( g_j \) is the jth normalized absolute constraint violations and \( f_{max} \) is the objective function value of the worst feasible solution in the population. The pair-wise comparison used in tournament selection is exploited to make sure that

(i) when two feasible solutions are compared, the one with better objective function value is chosen.
(ii) when one feasible and one infeasible solutions are compared, the feasible solution is chosen.
(iii) when two infeasible solutions are compared, the one with smaller constraint violation is chosen.

In this method, penalty parameters are not needed because in any of the above three scenarios, solutions are never compared in terms of both objective function and constraint violation information. The advantages of this scheme when compared with the usual penalty parameter based scheme are:

(i) The tedious process of choosing a suitable penalty parameter can be avoided, the inappropriate choice of which will seriously affect the performance of Genetic Algorithm.

(ii) There is no need to evaluate the objective function value for individuals with constraint violation, which reduces the computation time.

C. Algorithm

The implementation of the proposed method to the device allocation problem is performed in the following steps.

Step 1. The bus data, line data, and number of FACTS devices are given as inputs.

Step 2. The location and the setting of TCSC are set as solution variables.

Step 3. The initial population of individuals are created in normalized form so as to satisfy the FACTS device constraint.

Step 4. For each individual in the population, the fitness function is evaluated in denormalized form after simulating all possible single contingencies by using AC Load flow. New Penalty parameter-less method is applied for handling the constraints.

Step 5. By applying tournament selection, Simulated Binary Crossover (SBX) and Polynomial mutation, new offspring population is created for next generation.

Step 6. If maximum number of function evaluations are reached, then go to next step, else go to step 4.

Step 7. Print the best locations and corresponding settings.

IV. Simulation Results

The algorithms are implemented using MATLAB 7.0 on a PC with a Pentium IV, 1.8 GHz and 1 GB RAM. Owing to the randomness of the Evolutionary algorithms, their performance cannot be judged by the result of a single run. Many trials with independent population initializations should be made to acquire a useful conclusion of the performance of the algorithm. An algorithm is said to be robust, if it gives almost consistent result during the trials for all experiments. Hence, in this paper 10 independent trials are conducted. The best, worst, mean and standard deviation obtained in 10 trials are used to compare the performances of different algorithms. The maximum number of function evaluations is set at 10,000 for all the algorithms.

To ensure the effectiveness of the proposed algorithm, the test results are compared with the results of conventional RGA and PSO.

Parameter tuning

Optimal parameter combinations for different methods are experimentally determined by conducting a series of experiments with different parameter settings before conducting actual runs to collect the results. In RGA, crossover probability \( P_c = 0.8 \) produces the best result in terms of best and mean results.

Mutation probability \( P_m = 1/\text{number of variables} \)
Crossover index \( \eta_c = 5 \) and mutation index, \( \eta_m = 20. \)

Test system

The effectiveness of proposed approach is illustrated using the IEEE 6 bus and 30 bus test systems. It will be assumed that the impedance of all TCSCs can vary within \( \pm 50\% \) of the corresponding branch impedance.

A. IEEE 6 bus system

Table I shows the list of ranked branches after all the single contingencies are simulated. The severity index value indicates how a particular branch is affected for all single contingencies. More it is affected, higher the severity index.

<table>
<thead>
<tr>
<th>Branch</th>
<th>Rank</th>
<th>No of overloads</th>
<th>Severity index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>1</td>
<td>2</td>
<td>0.0367</td>
</tr>
<tr>
<td>1-4</td>
<td>2</td>
<td>3</td>
<td>0.0256</td>
</tr>
<tr>
<td>2-4</td>
<td>3</td>
<td>1</td>
<td>0.0213</td>
</tr>
<tr>
<td>1-5</td>
<td>4</td>
<td>2</td>
<td>0.0171</td>
</tr>
</tbody>
</table>

Four branches namely 1-2, 1-4, 2-4, and 1-5 are possessing the index values. Remaining seven lines are having zero index value. As a result, these four branches are considered for the placement of TCSC. Once the rank list is obtained, the optimal locations to install TCSC along with the settings of TCSCs are obtained using the proposed algorithm.

<table>
<thead>
<tr>
<th>Branch</th>
<th>Rank</th>
<th>No of overloads</th>
<th>Severity index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>1</td>
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<td>3</td>
<td>1</td>
<td>0.0213</td>
</tr>
<tr>
<td>1-5</td>
<td>4</td>
<td>2</td>
<td>0.0171</td>
</tr>
</tbody>
</table>

B. IEEE 30 bus system

TABLE II

<table>
<thead>
<tr>
<th>No of devices</th>
<th>PSO</th>
<th>Conventional GA</th>
<th>Penalty less GA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U</td>
<td>P</td>
<td>Fitness</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>32.665</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>32.002</td>
<td>39.002</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>31.520</td>
<td>37.520</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>30.463</td>
<td>36.463</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>29.580</td>
<td>34.580</td>
</tr>
</tbody>
</table>

Table II compares the results of three algorithms in terms of number of overloading ($U_n$), severity of overloading ($F_t$) and fitness function by placing different numbers of devices in the allotted locations. As the number of devices increases, $U_n$ and $F_t$ and thereby Fitness values decrease. When all the four allotted locations are ranked and it is given in Table IV.

The optimal location and settings of TCSC are presented in Table III.

### TABLE III

<table>
<thead>
<tr>
<th>No of Devices</th>
<th>PSO</th>
<th>Conventional GA</th>
<th>Penalty less GA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X_{TCSC}$ (p.u)</td>
<td>$X_{TCSC}$ (p.u)</td>
<td>$X_{TCSC}$ (p.u)</td>
</tr>
<tr>
<td>1</td>
<td>1-4</td>
<td>0.0698</td>
<td>1-4</td>
</tr>
<tr>
<td>2</td>
<td>1-4</td>
<td>0.0639</td>
<td>2-4</td>
</tr>
<tr>
<td></td>
<td>1-5</td>
<td>0.1398</td>
<td>1-5</td>
</tr>
<tr>
<td>3</td>
<td>1-2</td>
<td>0.1198</td>
<td>1-2</td>
</tr>
<tr>
<td></td>
<td>1-4</td>
<td>0.0263</td>
<td>1-4</td>
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<tr>
<td></td>
<td>1-5</td>
<td>0.1504</td>
<td>2-4</td>
</tr>
<tr>
<td>4</td>
<td>1-2</td>
<td>0.1387</td>
<td>1-2</td>
</tr>
<tr>
<td></td>
<td>1-4</td>
<td>0.0193</td>
<td>1-4</td>
</tr>
<tr>
<td></td>
<td>2-4</td>
<td>0.0601</td>
<td>2-4</td>
</tr>
<tr>
<td></td>
<td>1-5</td>
<td>0.1496</td>
<td>1-5</td>
</tr>
</tbody>
</table>

The optimal location and settings of TCSC are presented in Table III.

### B. IEEE 30 bus system

Table IV shows the list of ranked branches for IEEE-30 bus system after all the single contingencies are simulated. There are 41 possible contingencies, leaving 3 branches (25-26, 9-11, 12-13) connected to isolated buses only 38 single contingencies are considered. The severity index is calculated for all the 41 lines considering 38 contingencies and the branches are ranked and it is given in Table IV.

### TABLE IV

<table>
<thead>
<tr>
<th>Branch</th>
<th>Rank</th>
<th>No of Overloads</th>
<th>Severity Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-28</td>
<td>1</td>
<td>1</td>
<td>0.1299</td>
</tr>
<tr>
<td>8-28</td>
<td>2</td>
<td>1</td>
<td>0.1217</td>
</tr>
<tr>
<td>15-23</td>
<td>3</td>
<td>2</td>
<td>0.0401</td>
</tr>
<tr>
<td>17-18</td>
<td>4</td>
<td>1</td>
<td>0.0295</td>
</tr>
<tr>
<td>27-29</td>
<td>5</td>
<td>1</td>
<td>0.0245</td>
</tr>
<tr>
<td>23-24</td>
<td>6</td>
<td>1</td>
<td>0.0210</td>
</tr>
<tr>
<td>27-30</td>
<td>7</td>
<td>1</td>
<td>0.0185</td>
</tr>
<tr>
<td>22-24</td>
<td>8</td>
<td>1</td>
<td>0.0176</td>
</tr>
<tr>
<td>21-22</td>
<td>9</td>
<td>2</td>
<td>0.0092</td>
</tr>
<tr>
<td>6-8</td>
<td>10</td>
<td>2</td>
<td>0.0066</td>
</tr>
</tbody>
</table>

Ten branches are having the index values. Hence, these ten branches are considered for the placement of TCSC.

Table V shows that penalty less GA gives reduced number of overload from 13 to 11, when 9 branches are installed. The severity of overloading is gradually decreased from 125.62 to 105.61 and thereby the fitness too, as the number of devices increases. Table VI presents the optimal location and settings obtained using penalty less GA approach.

### TABLE V

<table>
<thead>
<tr>
<th>Devices</th>
<th>PSO</th>
<th>Conventional GA</th>
<th>Penalty less GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>F_t</td>
<td>Fitness</td>
<td>U</td>
</tr>
<tr>
<td>0</td>
<td>13</td>
<td>125.62</td>
<td>13</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
<td>118.50</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>112.75</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>111.59</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>110.42</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>107.70</td>
<td>11</td>
</tr>
</tbody>
</table>

### TABLE VI

<table>
<thead>
<tr>
<th>No of Devices</th>
<th>Branch</th>
<th>$X_{TCSC}$ (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8-28</td>
<td>-0.1010</td>
</tr>
<tr>
<td>3</td>
<td>8-28</td>
<td>-0.1010</td>
</tr>
<tr>
<td></td>
<td>21-22</td>
<td>0.1118</td>
</tr>
<tr>
<td></td>
<td>6-8</td>
<td>0.1043</td>
</tr>
<tr>
<td>5</td>
<td>6-8</td>
<td>-0.1092</td>
</tr>
<tr>
<td></td>
<td>23-24</td>
<td>0.0279</td>
</tr>
<tr>
<td></td>
<td>22-24</td>
<td>0.0882</td>
</tr>
<tr>
<td>7</td>
<td>15-23</td>
<td>-0.0149</td>
</tr>
<tr>
<td></td>
<td>27-29</td>
<td>-0.0320</td>
</tr>
<tr>
<td></td>
<td>6-28</td>
<td>-0.0299</td>
</tr>
<tr>
<td></td>
<td>21-22</td>
<td>0.0118</td>
</tr>
<tr>
<td></td>
<td>8-28</td>
<td>-0.1021</td>
</tr>
<tr>
<td></td>
<td>23-24</td>
<td>0.0279</td>
</tr>
<tr>
<td></td>
<td>22-24</td>
<td>0.1043</td>
</tr>
<tr>
<td>9</td>
<td>6-28</td>
<td>-0.1091</td>
</tr>
<tr>
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<tr>
<td></td>
<td>27-30</td>
<td>-0.0056</td>
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<tr>
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<td>0.1099</td>
</tr>
<tr>
<td></td>
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<td>-0.0118</td>
</tr>
<tr>
<td></td>
<td>15-23</td>
<td>-0.0422</td>
</tr>
</tbody>
</table>

### V. CONCLUSION

This paper presents a procedure to place the FACTS device in suitable branches in the given system to reduce system overloads and to improve the system security margin during single contingencies. Thyristor Controlled Series Capacitor is considered in this work. The location of TCSC to be placed and their settings were taken as the optimization parameters. The penalty parameter-less constraint-handling scheme has been employed for constraint handling. For the purpose of comparison, the results of various algorithms such as PSO and conventional RGA were considered. IEEE 6 bus and IEEE 30 bus test systems were used to evaluate the performance of this approach. Simulation...
results reveal that the proposed approach can give an optimal placement and settings in security point of view more consistently than any other algorithms considered.

**APPENDIX**

**Real Coded Genetic Algorithm**

GA is inherently parallel, because it simultaneously evaluates many points in the parameter space (search space). Considering many points in the search space, GA has a reduced chance of converging to the local optimum and would be more likely to converge to the global optimum.

Real-number encoding is best used for function optimisation problems. It has been widely confirmed that real-number encoding performs better than binary or gray encoding for constrained optimisation. Owing to the adaptive capability, SBX crossover and polynomial mutation operators are employed. Tournament selection is used as selection mechanism in order to avoid premature convergence. Simulated binary crossover and non-uniform polynomial mutation are briefly explained below.

**Simulated Binary Cross over**

In SBX crossover, two offspring solutions are created from two parents as follows:

\[
β_{qi} = \begin{cases} 
(2u_i)_{\frac{1}{\eta_{qi} + 1}}, & u_i \leq 0.5 \\
\frac{1}{2(1-u_i)}_{\frac{1}{\eta_{qi} + 1}}, & \text{otherwise}
\end{cases}
\]

Then compute the offspring \(x_i^{(1,t+1)}\) and \(x_i^{(2,t+1)}\)

\[
x_i^{(1,t+1)} = 0.5[(1 + β_{qi})x_i^{(1,t)} + (1 - β_{qi})x_i^{(2,t)}]
\]
\[
x_i^{(2,t+1)} = 0.5[(1 - β_{qi})x_i^{(1,t)} + (1 + β_{qi})x_i^{(2,t)}]
\]

**Non-uniform Polynomial mutation**

Newly generated offspring undergoes polynomial mutation operation. Similar to SBX operator, the probability distribution can also be a polynomial function, instead of normal distribution. The new offspring is determined as follows:

\[
y_i^{(1,t+1)} = x_i^{(1,t+1)} + (x_i^U - x_i^L) \bar{δ}_i
\]

where the parameter \(\bar{δ}_i\) is calculated from the polynomial probability distribution.

\[
p(δ) = 0.5(\eta_m + 1)(1 - |δ|)^{\eta_m}
\]
\[
\bar{δ}_i = \begin{cases} 
(2r_i)^{\frac{1}{\eta_m}} - 1, & \text{if } r_i < 0.5 \\
1 - [2(1 - r_i)]^{\frac{1}{\eta_m}}, & \text{if } r_i \geq 0.5
\end{cases}
\]

Where \(\eta_m\) is the mutation index.

**REFERENCES**