Independence

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Independence

Definition

Events A and B are called independent if

$$P(A \cap B) = P(A)P(B).$$

More generally, a family $\{A_i : i \in I\}$ is called independent if

$$P\left(\bigcap_{i \in J} A_i\right) = \prod_{i \in J} P(A_i)$$

for all finite subsets J of I.

Examples

- A fair coin is tossed twice. The first toss is independent of the second toss.
- Pick a card at random from a pack of 52 cards. The suit of the card is independent of its rank.
- Two fair dice are rolled. Is the the sum of the faces independent of the number shown by the first die?

Questions

- Can an event be independent of itself?
- What is the relation between independence and mutual exclusivity?
- What is the relation between independence and conditional probability?
- Does pairwise independence imply independence?

  $\Omega = \{abc, acb, cab, cba, bca, bac, aaa, bbb, ccc\}$ with each outcome being equally likely.

  Let A_k be the event that the kth letter is a.

$$\begin{aligned} P(A_i) &= \frac{1}{3} \\ P(A_i \cap A_j) &= \frac{1}{9}, \quad i \neq j \\ P(A_1 \cap A_2 \cap A_3) &= \frac{1}{9} \end{aligned}$$

  $\{A_1, A_2, A_3\}$ are pairwise independent but not independent.

Conditional Independence

Definition

Let C be an event with $P(C) > 0$. Two events A and B are called conditionally independent given C if

$$P(A \cap B|C) = P(A|C)P(B|C).$$

Example

- We have two coins; the first is fair and the second has heads on both sides. A coin is picked at random and tossed twice. Are the results of the two tosses independent? Are they independent if we know which coin was picked?

Questions?