EE 325: Probability and Random Processes (Spring 2014)
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1. Consider a probability space $(\Omega, \mathcal{F}, P)$. For any $A, B \in \mathcal{F}$, show that $P(B \cap A^c) = P(B) - P(A \cap B)$. Use this result to deduce the following.
   - $P(A^c) = 1 - P(A)$
   - If $A \subseteq B$, then $P(A) \leq P(B)$
   - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. For events $A$ and $B$, find formulas for the probabilities of the following events in terms of $P(A)$, $P(B)$, and $P(A \cap B)$.
   - either $A$ or $B$ or both occur
   - either $A$ or $B$ but not both occur
   - at least one of $A$ or $B$ occur
   - at most one of $A$ or $B$ occur
3. If $P(A) = \frac{1}{3}$ and $P(B^c) = \frac{1}{4}$, can $A$ and $B$ be disjoint?
4. Use induction to prove that

$$P\left(\bigcap_{i=1}^{n} A_i\right) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cup A_j) + \sum_{i<j<k} P(A_i \cup A_j \cup A_k) - \cdots + (-1)^{n+1} P(A_1 \cup A_2 \cup \cdots A_n)$$

5. Prove the following:
   - If $P(B) = 1$, then $P(A|B) = P(A)$ for any $A$.
   - If $A \subset B$, then $P(B|A) = 1$ and $P(A|B) = \frac{P(A)}{P(B)}$.
   - If $A$ and $B$ are mutually exclusive, then $P(A|A \cup B) = \frac{P(A)}{P(A)+P(B)}$.
   - $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$.
6. Prove that if $P(A) > 0$ and $P(B) > 0$, then:
   - If $A$ and $B$ are mutually exclusive, they cannot be independent.
   - If $A$ and $B$ are independent, they cannot be mutually exclusive.