

The abbreviation GS X.Y.Z denotes that the problem is from Grimmett and Stirzaker's book (Chapter X, Section Y, Problem Z). The solutions to such problems can be found in the companion volume *One Thousand Exercises in Probability* by the same authors.

1. A coin shows heads with probability p and tails with probability $1 - p$. It is tossed twice where the outcome of the first toss is independent of the outcome of the second toss. Let X be the total number of heads which appear in the two tosses. What is the probability mass function of X ? What is the distribution function of X ?
2. A fair coin is tossed n times. Let Y_n denote the number of heads minus the number of tails. Find the probability mass function of Y_n .
3. Suppose independent trials of an experiment are performed until the first success. In each trial, the probability that the experiment succeeds is p . Let X denote the number of trials required to get the first success. Find the probability mass function of X . Such a random variable is called a geometric random variable.
4. (GS 2.1.2) A random variable X has distribution function F . What is the distribution function of $Y = aX + b$ where $a, b \in \mathbb{R}$?
5. Let F be a distribution function and r a positive integer. Specify a random variable which has the following distribution functions.
 - (a) $[F(x)]^r$
 - (b) $1 - [1 - F(x)]^r$
6. (GS 2.7.9) Find the distribution functions of the following as a function of the distribution function F of a random variable X .
 - (a) $X^+ = \max\{0, X\}$
 - (b) $X^- = \min\{0, X\}$
 - (c) $|X|$
 - (d) $-X$
7. (GS 3.1.3) We toss n coins, and each one shows heads with probability p , independently of the others. Each coin which shows heads is tossed again. What is the probability mass function of the number of heads resulting from the second round of tosses?