EE 325: Probability and Random Processes (Spring 2015)
Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay

Quiz 1 : 30 marks February 4, 2015

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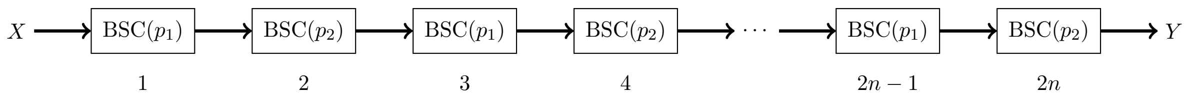
1. (5 marks) Let A, B, C be subsets of a sample space Ω. If a σ-algebra $\mathcal{F}$ contains A, B and C, what other sets must be in $\mathcal{F}$?

2. (5 marks) Let BSC(p) denote a binary symmetric channel with crossover probability p i.e. the output of the channel is not equal to its input with probability p. Consider a cascade of $2n$ binary symmetric channels where the odd numbered ones have crossover probability p_1 and the even numbered ones have crossover probability p_2 as shown in the figure. Assume that the errors introduced by each of the BSCs are independent. If X is equally likely to 0 or 1, what is the probability of $Y \neq X$?



3. (5 marks) A fair coin is tossed repeatedly until a heads appears.
   (a) Specify the sample space Ω for this experiment.
   (b) Let X be the number of tosses in an outcome of this experiment. For instance, $X = 1$ when heads appears in the first toss itself. Show that X is a random variable if we assume that the σ-field is $\mathcal{F} = 2^{\Omega}$.
   (c) Find the distribution function of X.

4. (5 marks) Find the distribution functions of the following as a function of the distribution function F of a random variable X.
   (a) $aX + b$ where $a, b \in \mathbb{R}$
   (b) $X^+ = \max\{0, X\}$
   (c) $X^- = \min\{0, X\}$
   (d) $|X|$
   (e) $-X$

5. (5 marks) Let A_r, $r \geq 1$, be events such that $P(A_r) = 1$ for all r. Show that $P\left(\bigcap_{r=1}^{\infty} A_r\right) = 1$.

6. (5 marks) Consider the equivalence relation R on the interval $[-1, 1]$ given by $x \sim y$ if $x - y$ is rational. This equivalence relation partitions $[-1, 1]$ into disjoint equivalence classes. Let $A \subset [-1, 1]$ be the set consisting of exactly one element from each of the equivalence classes. Show that A is uncountable.