

1. [10 points] Random variables X and Y are independent if the events $\{X \leq x\}$ and $\{Y \leq y\}$ are independent for all x and y in \mathbb{R} . Prove that if X and Y are discrete random variables that are independent, then the events $\{X = x\}$ and $\{Y = y\}$ are independent for all x and y in \mathbb{R} . *Hint:* $\Pr[X = x] = \Pr[(X \leq x) \cap (X \geq x)] = \Pr[(X \leq x) \cap (X < x)^c] = \Pr[(X \leq x) \setminus (X < x)]$
2. [10 points] Let X and Y be independent discrete random variables taking values in the positive integers. Both of them have the same probability mass function given by

$$P[X = k] = P[Y = k] = \frac{1}{2^k} \quad \text{for } k = 1, 2, 3, \dots$$

Find the following.

- $P(\min\{X, Y\} \leq x)$
- $P[X = Y]$
- $P[X > Y]$
- $P[X \geq nY]$ for a given positive integer n
- $P[X \text{ divides } Y]$