EE 325: Probability and Random Processes (Spring 2025)
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Quiz 1 : 30 points February 12, 2025

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1. (5 points) Suppose $\Omega = [0, 1]$, the interval containing all non-negative real numbers less than or equal to 1. Suppose $\mathcal{F}$ is the set of subsets A of Ω such that either A or A^c is finite. Let $P : \mathcal{F} \mapsto [0, 1]$ be defined by $P(A) = 0$ if A is finite and $P(A) = 1$ if A^c is finite. Answer the following with **justification**. Each part is worth 1.25 points.
   (a) Is $\mathcal{F}$ a field?
   (b) Is $\mathcal{F}$ a σ-field?
   (c) Is P finitely additive?
   (d) Is P countably additive?

2. (5 points) Random variables X and Y are independent if the events $\{X \leq x\}$ and $\{Y \leq y\}$ are independent for all x and y in $\mathbb{R}$. Prove that if X and Y are discrete random variables that are independent, then the events $\{X = x\}$ and $\{Y = y\}$ are independent for all x and y in $\mathbb{R}$.

   Note: *You* ***cannot*** *use the following result: If X and Y are independent random variables, then the events $\{X \in A\}$ and $\{Y \in B\}$ are independent for any subsets A, B of $\mathbb{R}$.*

   The reason you cannot use it is because the fact that $\{X = x\}$ and $\{Y = y\}$ are independent for any $x, y \in \mathbb{R}$ is used to prove that the events $\{X \in A\}$ and $\{Y \in B\}$ are independent for any subsets A, B of $\mathbb{R}$. It would be a circular argument.

3. (5 points) Let $F : \mathbb{R} \mapsto [0, 1]$ be the distribution function of a random variable X. Prove that $P(X = x) = F(x) - \lim_{y \uparrow x} F(y)$.

   Hint: *Given a function $f : \mathbb{R} \mapsto \mathbb{R}$ and $c \in \mathbb{R}$, we say that $\lim_{x \uparrow c} f(x) = L$ if for all $\epsilon > 0$ there exists a δ_ϵ such that $|f(x) - L| < \epsilon$ whenever $c - \delta_\epsilon < x < c$.*

4. (5 points) A random variable X has distribution function F. What is the distribution function of $Y = aX + b$ where $a, b \in \mathbb{R}$?

5. (5 points) Consider a leap year with 366 days (including Feb 29). Suppose 180 days are drawn at random from this year without replacement. What is the probability that these 180 days have 15 days from each of the 12 months?

   Hint: *Number of days in each month: Jan, Mar, May, July, Aug, Oct, Dec have 31 days each. Feb has 29 days. Apr, June, Sept, Nov have 30 days each.*

6. (5 points) Given independent discrete random variables X_1, X_2, X_3 with probability mass functions f_1, f_2, f_3 respectively, find the probability mass functions of the following
   (a) $\max(X_1, X_2, X_3)$
   (b) $\min(X_1, X_2, X_3)$