

1. (a) (1 point) Write down the multiplication operation table over \mathbb{F}_{11} .
- (b) (2 points) Enumerate all the points of the elliptic curve $Y^2 = X^3 + 4X + 5$ over \mathbb{F}_{11} .
- (c) (2 points) For each point P on the above elliptic curve, calculate the point $2P$.
2. (5 points) Answer the following questions in the context of the Bitcoin system.
 - (a) Describe the steps involved in generating a Pay-to-Public-Key-Hash (P2PKH) address from a private key.
 - (b) Alice wants to buy a book from Bob. He emails his Bitcoin P2PKH address to Alice. But a single character is missing from the address. Bob made a mistake while typing it. How can Alice find the **location and value** of the missing character in the address without contacting Bob?
 - (c) The merchant Bob wants to create a *vanity P2PKH address* to share with this customers. He wants it to start with the characters `1bob...`. How can he generate such an address?
 - (d) Under what conditions is storing bitcoins in a P2PKH output safer than storing bitcoins in a pay-to-public-key output?
 - (e) Suppose a merchant waits for six confirmations on a Bitcoin payment before transferring some goods to a customer. Describe how a 51% attacker can execute a double spend attack on such a merchant.
3. (2 points) The Merkle Patricia trie corresponding to the key-value pairs { `646f` : 'verb', `646f67` : 'puppy', `646f6765` : 'coin', `686f727365` : 'stallion' } is given below. Suppose the last key-value pair (corresponding to the value 'stallion') is deleted from the trie. Write down the modified trie.

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rootHash [ ⟨16⟩, hashA ]
hashA    [ ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, hashB, ⟨⟩, ⟨⟩, ⟨⟩, hashC, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩ ]
hashC    [ ⟨20 6f 72 73 65⟩, 'stallion' ]
hashB    [ ⟨00 6f⟩, hashD ]
hashD    [ ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, hashE, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, 'verb' ]
hashE    [ ⟨17⟩, hashF ]
hashF    [ ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, hashG, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, ⟨⟩, 'puppy' ]
hashG    [ ⟨35⟩, 'coin' ]

```

4. (4 points) Answer the following questions in the context of the Ethereum system.
 - (a) Decode the following using RLP decoding: `0xc6827a77c10401`.
 - (b) How does the **nonce** field in the transaction data structure prevent replay attacks? Describe a replay attack which becomes possible if this field is omitted.
 - (c) Explain why the **v** field in the transaction data structure is not mandatory and is included as a convenience to reduce computation. *Hint: Show how the public key can be identified from a message and signature (r, s) even when v is unknown.*
 - (d) In Ethash mining, the **mixHash** field can be calculated from the partial header hash, **nonce**, and dataset (DAG). Explain the reasoning behind including the **mixHash** field in the header by describing a DoS attack (on nodes which validate blocks) which is possible if the **mixHash** field had been omitted.