EE 605: Error Correcting Codes

Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay
Autumn 2010

Assignment 4 : **24 points** **Due date**: October 7, 2010

Each of the following exercises is worth 3 points. Every nontrivial step in a proof should be accompanied by justification.

1. Find the parity check matrix H for a linear binary code with generator matrix

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

2. Let C be a linear (n, k) block code with parity check matrix H. Prove that an n-tuple $\mathbf{v} \in C$ if and only if $\mathbf{v} \cdot H^T = \mathbf{0}$.

3. Let C be a linear block code and $C^\perp$ be its dual code. A code is said to be *self-dual* if $C = C^\perp$. Prove that a linear self-dual code has even length n and dimension $\frac{n}{2}$.

4. Let C be a linear block code and $C^\perp$ be its dual code. A code is said to be *self-orthogonal* if $C \subset C^\perp$. Prove that each codeword in a binary self-orthogonal code C has even weight and $C^\perp$ contains the all-ones codeword $\mathbf{1} = 111 \cdots 1$.

5. Let C be a binary linear (n, k) block code. If C contains the codeword $\mathbf{1} = 111 \cdots 1$, then prove that $A_i(C) = A_{n-i}(C)$ for $0 \le i \le n$ where $A_i(C)$ is the number of codewords in C of weight i.

6. Prove that the Hamming distance satisfies the triangle inequality, i.e. $d(\mathbf{u}, \mathbf{v}) \le d(\mathbf{u}, \mathbf{w}) + d(\mathbf{w}, \mathbf{v})$ for all n-tuples $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

7. Construct the standard array for a binary linear block code with the following generator matrix if it is to be used over a binary symmetric channel with crossover probability $p < \frac{1}{2}$.

$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   Construct the syndrome-error pattern lookup table for this code, i.e. a one-to-one mapping between the set of syndromes and set of correctable error patterns. Can this code correct all error patterns of weight 1? Weight 2?

8. Suppose a binary channel accepts codewords of lenght n and that the only error patterns which can occur are i zeros followed by j ones where $i \ge 0, j \ge 0$ and $i + j = n$. For $n = 6$, the possible error patterns are 000000, 000001, 000011, 000111, 001111, 011111, 111111. Design a binary linear (n, k) code that will correct all such error patterns while having rate as large as possible. Illustrate your construction for $n = 7$.