EE 605: Error Correcting Codes

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Assignment 6 : **40 points** **Due date**:

Each of the following exercises is worth 10 points. Every nontrivial step in a proof should be accompanied by justification.

1. Determine the generator polynomials of the single error correcting and double error correcting binary primitive BCH codes of length 31. Use the primitive element of the field $\mathbb{F}_{32}$ which has minimal polynomial $p(X) = 1 + X^2 + X^5$.

2. Consider a (15,5) binary BCH code capable of correcting three or fewer errors. Let α be the primitive element in $\mathbb{F}_{16}$ with minimal polynomial $X^4 + X + 1$. The generator polynomial for the code obtained using α is $g(X) = X^{10} + X^8 + X^5 + X^4 + X^2 + X + 1$. Decode the following received polynomials using the Peterson-Gorenstein-Zierler decoder.

   (a) $X^{14} + X^7 + 1$

   (b) $X^9 + X^6 + X^3 + X^2 + X$

3. A rate $R = \frac{1}{3}$ nonsystematic convolutional encoder has the following input-output relationship,

$$\begin{aligned} v_l^{(0)} &= u_l + u_{l-1} \\ v_l^{(1)} &= u_l + u_{l-1} + u_{l-2} \\ v_l^{(2)} &= u_l + u_{l-1} + u_{l-2} + u_{l-3} \end{aligned}$$

   where $v_l^{(i)}$, $i = 0, 1, 2$ are the outputs at time l and u_l is the input at time unit l.

   (a) What is the memory order of this encoder?

   (b) What is the overall constraint length of this encoder?

   (c) What are the generator sequences (impulse responses) of this encoder?

   (d) Write down the binary generator matrix which transforms a input bit sequence of length 5 into the corresponding interleaved output sequence.

   (e) Write down the transform domain generator matrix for this encoder. Find the interleaved output sequence corresponding to the input sequence $u(D) = 1 + D + D^2$.

   (f) Draw the controller canonical form of this encoder.

   (g) Draw the observer canonical form of this encoder.

   (h) Draw the state diagram corresponding to the controller canonical form of the encoder.

4. Consider the (2,1,3) nonsystematic feedforward encoder with $\mathbf{G}(D) = [1 + D^2 \quad 1 + D + D^2 + D^3]$.

   (a) Find the GCD of its generator polynomials. Is this encoder catastrophic?

   (b) Draw the encoder state diagram.

   (c) Find a zero-output weight cycle in the state diagram other than the one around the zero state S_0.

   (d) Find an infinite weight information sequence that generates a codeword of finite weight.