

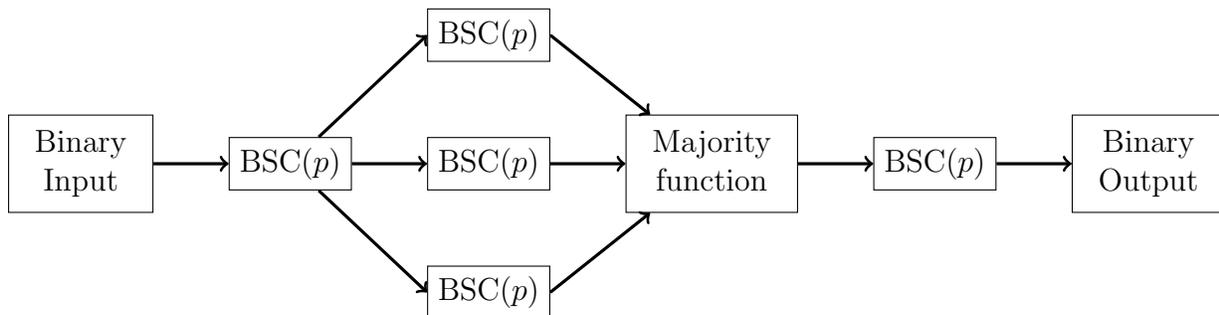
EE 605: Error Correcting Codes  
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Assignment 1 : **20 points**

**Due date:** August 4, 2010

Each of the following exercises is worth 5 points. Every nontrivial step in a proof should be accompanied by justification.

1. Prove that the Hamming distance satisfies the triangle inequality, i.e.  $d(\mathbf{u}, \mathbf{v}) \leq d(\mathbf{u}, \mathbf{w}) + d(\mathbf{w}, \mathbf{v})$  for all  $n$ -tuples  $\mathbf{u}, \mathbf{v}, \mathbf{w}$ .
2. Calculate the crossover probability of the binary symmetric channel which is equivalent to the system below.



3. Derive the optimal decoding rule for a  $2n$ -repetition code for use over a binary symmetric channel with crossover probability  $p$ . Is the optimal decoding rule unique? Calculate the average probability of error for this code when the optimal decoding rule is used.
4. Consider a binary block code  $C$  of length  $n$  having minimum distance  $d_{min}$  where  $d_{min}$  is an odd integer. Show that when a overall parity bit is added to all the codewords in  $C$  we obtain a code of length  $n + 1$  and minimum distance  $d_{min} + 1$ .