EE 605: Error Correcting Codes

Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay
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Assignment 3 : **20 points** **Due date**: October 10, 2011

Each of the following exercises is worth 5 points. Every nontrivial step in a proof should be accompanied by justification.

1. Let $g(X)$ be the generator polynomial of a binary cyclic code of length n.
   (a) Show that if $g(X)$ has $X + 1$ as a factor, the code contains no codewords of odd weight.
   (b) If n is odd and $X + 1$ is not a factor of $g(X)$, show that the code contains a codeword consisting of all ones.
   (c) Show that the code has a minimum weight of at least 3 if n is the smallest integer such that $g(X)$ divides $X^n + 1$.

2. (a) For a cyclic code, if an error pattern $e(X)$ is detectable, show that its ith cyclic shift $e^{(i)}(X)$ is also detectable.
   (b) Let $v(X)$ be a code polynomial in a cyclic code of length n. Let i be the smallest integer such that $v^{(i)}(X) = v(X)$. Show that if $i \neq 0$, i is a factor of n.

3. Consider a binary (n, k) cyclic code C generated by $g(X)$. Let $g^*(X) = X^{n-k} g(X^{-1})$ be the reciprocal polynomial of $g(X)$.
   (a) Show that $g^*(X)$ also generates an (n, k) cyclic code.
   (b) Let C^* be the cyclic code generated by $g^*(X)$. Show that C and C^* have the same weight distribution. (*Hint:* If $v(X)$ is a code polynomial in C, then $X^{n-1} v(X^{-1})$ is a code polynomial in C^*).

4. Draw the Meggitt decoder circuit for the $(7, 3)$ binary cyclic code generated by $g(X) = (X + 1)(X^3 + X + 1)$