EE 605: Error Correcting Codes

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Midsemester Exam : **30 points** **Duration**: 120 minutes

Each of the following questions is worth 5 points.

1. Construct the standard array and syndrome decoding table for the $(6, 3)$ binary linear block code with generator matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

   Decode the following received vectors using the syndrome table generated.

   (a) 110110

   (b) 110111

   (c) 110001

2. Consider a binary linear code with generator matrix

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

   Suppose N codewords from this code are sent over a binary symmetric channel with crossover probability p. What is the probability that no undetected errors occur during this transmission?

3. State and prove the Singleton bound for binary block codes.

4. Let C_1 be an (n, k_1) binary linear block code with minimum distance d_1 and let C_2 be an (n, k_2) binary linear block code with minimum distance d_2. Consider the following set of $2n$-tuples

$$C = \{(\mathbf{u}, \mathbf{u} + \mathbf{v}) | \mathbf{u} \in C_1, \mathbf{v} \in C_2\}.$$

   Prove that the set C is a binary linear block code with dimension $k = k_1 + k_2$ and minimum distance $d_{min} = \min\{2d_1, d_2\}$.

5. Let C_1 and C_2 be two cyclic codes of same length n with generator polynomials $g_1(X)$ and $g_2(X)$ respectively. Show that $C_1 \subseteq C_2$ if and only if $g_2(X)$ divides $g_1(X)$.

6. Let C_1 and C_2 be two cyclic codes of same length n with generator polynomials $g_1(X)$ and $g_2(X)$ respectively. Show that $C_1 \cap C_2$ is a cyclic code. What is its generator poynomial?