

1. [5 points] Prove that the n -repetition code and the $(n, n-1)$ single parity check code are the dual codes of each other.
2. [5 points] Prove that $(C^\perp)^\perp = C$ when C is a linear block code. *Hint:* $\dim C + \dim C^\perp = n$ where n is codeword length.
3. [5 points] Let the generator matrix of an (n, k) binary linear block code C be of the form $\begin{bmatrix} I_k & P \end{bmatrix}$ where I_k is the $k \times k$ identity matrix and P is a $k \times n-k$ matrix. Show that $\begin{bmatrix} P^T & I_{n-k} \end{bmatrix}$ is a parity check matrix for C .
4. [5 points] Let C be a linear block code with parity check matrix \mathbf{H} . Prove that

$$\mathbf{v} \in C \iff \mathbf{v} \cdot \mathbf{H}^T = \mathbf{0}$$