EE 605: Error Correcting Codes (Autumn 2014)
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Assignment 5: 20 points — Due Date: September 25, 2014

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1. [10 points] For an (n, k) binary cyclic code, show the following.
   (a) The fraction of undetectable bursts of length $n-k+1$ is $2^{-(n-k-1)}$.
   (b) For $m > n-k+1$, the fraction of undetectable bursts of length m is $2^{-(n-k)}$.

2. [10 points] Let $\mathbf{g}(X)$ be the generator polynomial of an (n, k) binary cyclic code C. The code polynomials $\mathbf{v}(X)$ are multiples of $\mathbf{g}(X)$ of degree $n-1$ or less

   $$\mathbf{v}(X) = \mathbf{u}(X)\mathbf{g}(X)$$

   where $\mathbf{u}(X) = u_0 + u_1X + u_2X^2 + \cdots + u_{k-1}X^{k-1}$ is the message polynomial.

   Consider the code polynomials generated by message polynomials of degree $k-l-1$ or less where $l < k$, i.e. $u_{k-l} = u_{k-l+1} = \cdots = u_{k-1} = 0$. There are 2^{k-l} such code polynomials which have degree $n-l-1$ or less. They form a $(n-l, k-l)$ linear block code called the shortened cyclic code and it is not a cyclic code.

   If $\mathbf{g}(X) = (X+1)\mathbf{p}(X)$ where $\mathbf{p}(X)$ is a primitive polynomial of degree m where $n = 2^m - 1$, then show the following.
   (a) The shortened cyclic code can detect all error patterns of odd weight in the codeword of length $n-l$.
   (b) The shortened cyclic code can detect all double-bit error patterns in the codeword of length $n-l$.
   (c) The shortened cyclic code can detect all non-end-around burst errors of length $n-k$ or less in the codeword of length $n-l$.

   Note that the shortening operation destroys the cyclic property of the code. The shortened code loses the ability to detect end-around bursts of length $n-k$ or less. But we gain the ability to have an arbitrary length and still detect double-bit errors.