

1. [10 points] Let F_q be a field with p^m elements where p is a prime and m is a positive integer.
 - (a) Prove that the minimal polynomial of a primitive element in F_q has degree m .
 - (b) A degree m irreducible polynomial in $\mathbb{F}_p[x]$ is said to be primitive if the smallest value of N for which it divides $x^N - 1$ is $p^m - 1$. Show that the minimal polynomial of a primitive element in F_q is a primitive polynomial.
2. [5 points] Prove that a binary primitive BCH code with parameters m and $t = 2^{m-1} - 1$ is a repetition code.
3. [5 points] Find generator polynomials for a BCH code with length 15 and $t = 1, 2, 3$. Explain your procedure. Note that the generator polynomials may not be unique. Specify one generator polynomial for each value of t .