

1. (5 points) Let  $C$  be a binary linear block code given by the vectors

$$\begin{aligned}
 & [0, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 1], [0, 1, 0, 0, 1, 0, 0], [1, 1, 0, 0, 1, 0, 1], \\
 & [0, 0, 1, 0, 0, 1, 0], [1, 0, 1, 0, 0, 1, 1], [0, 1, 1, 0, 1, 1, 0], [1, 1, 1, 0, 1, 1, 1], \\
 & [0, 0, 0, 1, 0, 0, 1], [1, 0, 0, 1, 0, 0, 0], [0, 1, 0, 1, 1, 0, 1], [1, 1, 0, 1, 1, 0, 0], \\
 & [0, 0, 1, 1, 0, 1, 1], [1, 0, 1, 1, 0, 1, 0], [0, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 0]
 \end{aligned}$$

- (a) What is the dimension of  $C^\perp$ ?  
 (b) What is the minimum distance of  $C^\perp$ ?
2. (5 points) The first row of a standard array is given below where the last four entries are missing. It is known that this standard array has 8 columns.

$$000000 \quad 110001 \quad 101010 \quad 000111 \quad * \quad * \quad * \quad *$$

- (a) Complete the standard array by giving all the remaining columns and rows.  
 (b) If the code corresponding to this standard array is used over a binary symmetric channel with crossover probability  $p$ , what is the probability of decoding error?
3. (5 points) Consider a binary linear code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Suppose a codeword from this code is sent over a binary symmetric channel with crossover probability  $p$ . What is the probability that the received vector is a codeword?

4. (5 points) Let  $C_1, C_2$  be binary linear block codes of same length  $n$  and dimensions  $k_1, k_2$  respectively. Let  $d_i$  be the minimum distance of  $C_i$  for  $i = 1, 2$ . Consider the set of vectors

$$C_3 = \left\{ [\mathbf{u} \quad \mathbf{v}] \mid \mathbf{u} \in C_1, \mathbf{v} \in C_2 \right\}$$

- (a) Show that  $C_3$  is a linear block code.  
 (b) What is the dimension of  $C_3$ ?  
 (c) What is the minimum distance of  $C_3$ ?