Viterbi Algorithm

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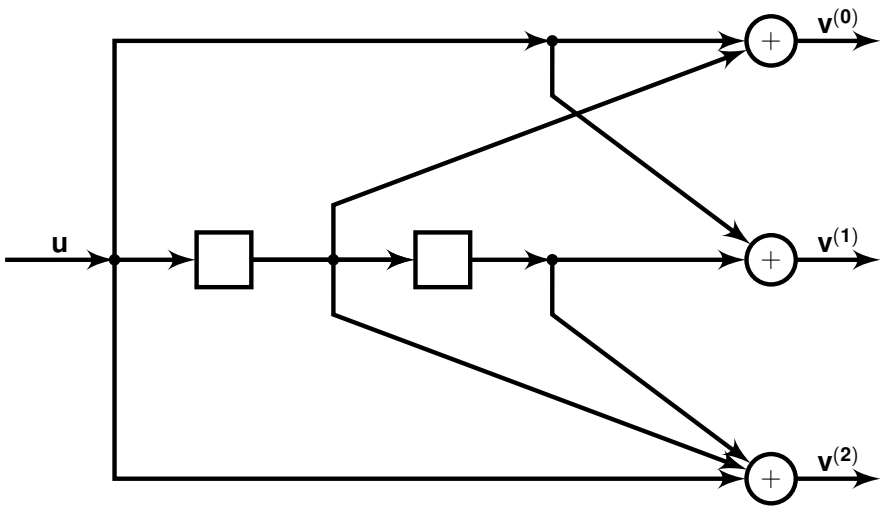
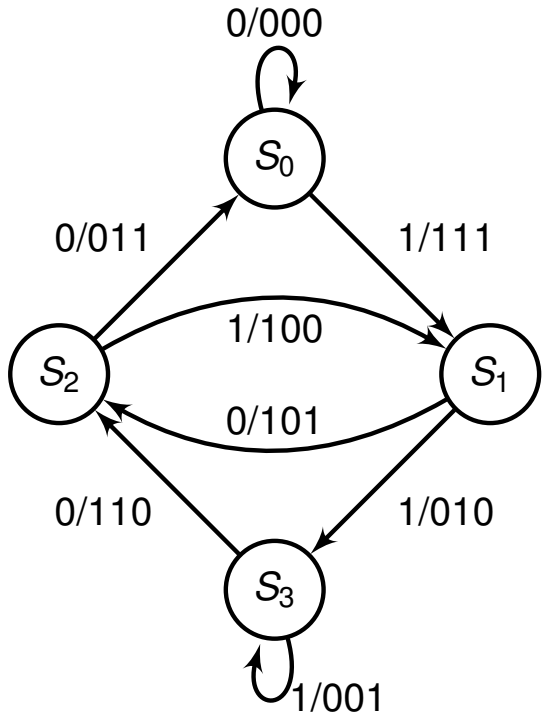
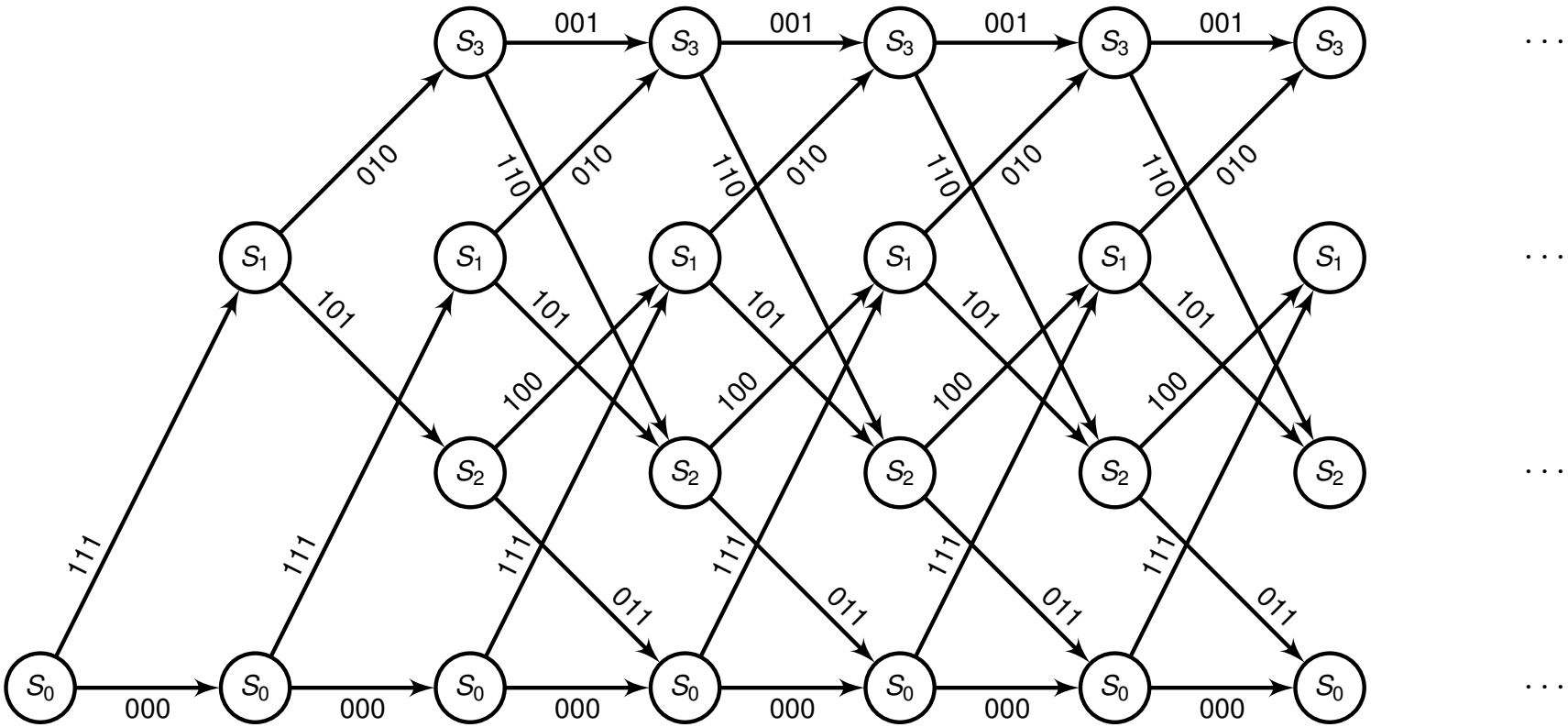
Encoder State Diagram

$$\mathbf{G}(D) = \begin{bmatrix} 1 + D & 1 + D^2 & 1 + D + D^2 \end{bmatrix}$$



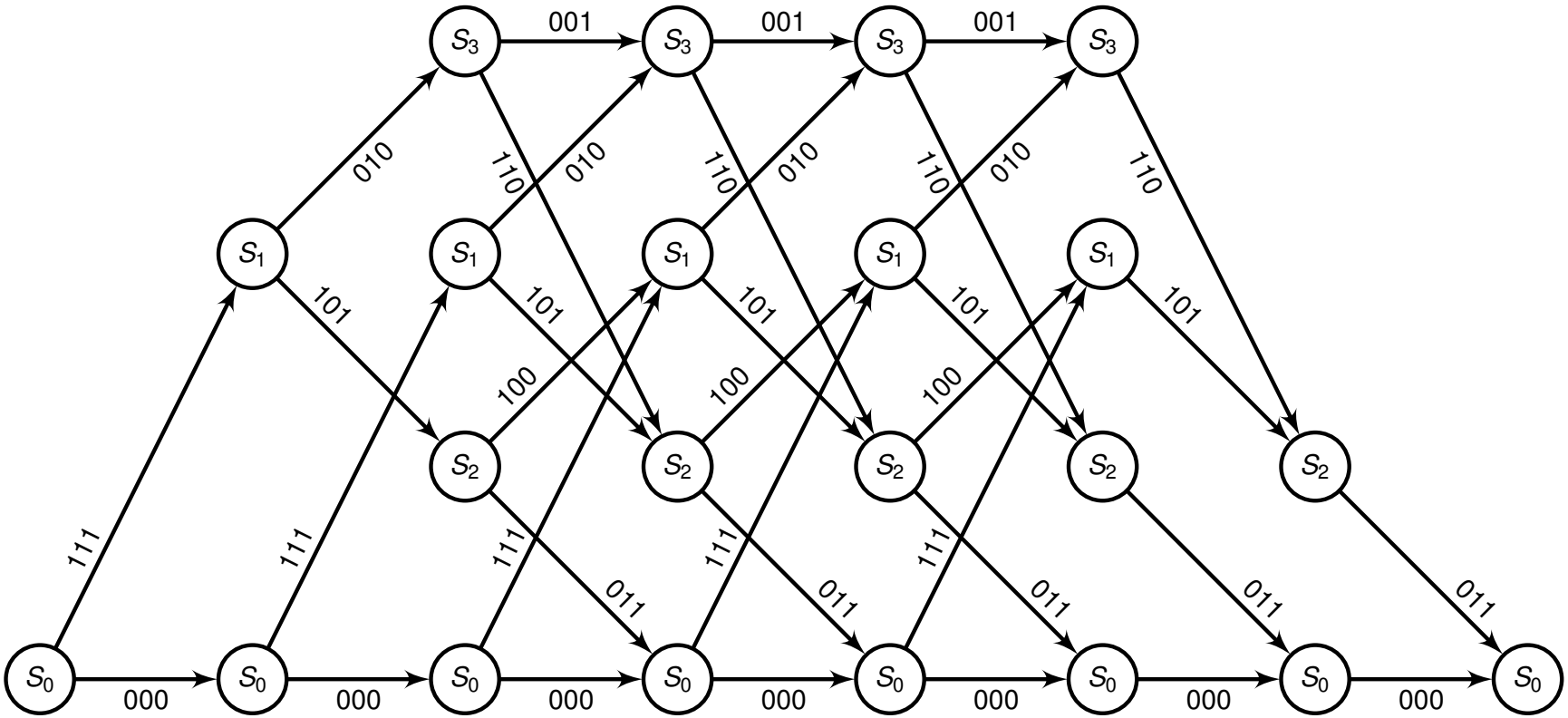


Encoder Trellis Diagram



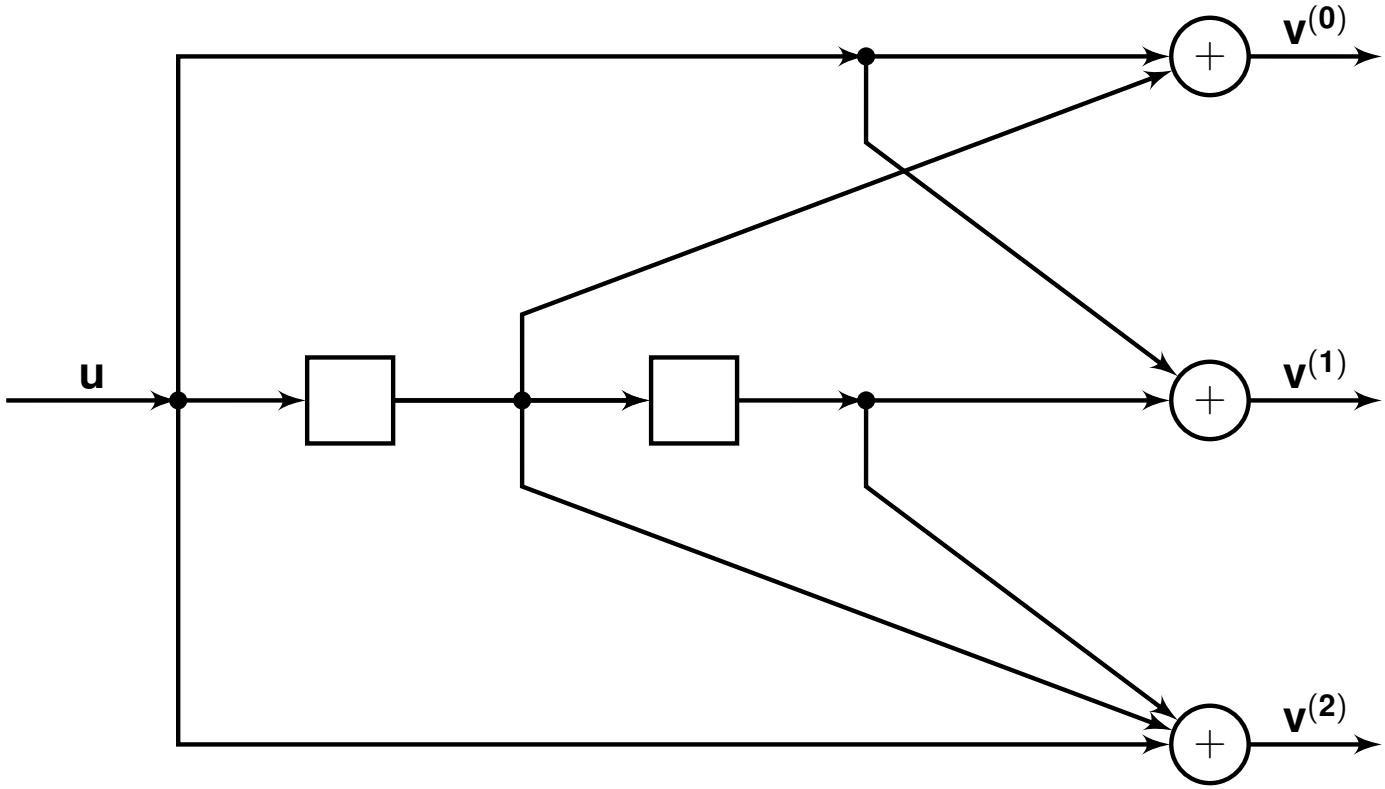
- The initial state of the encoder is the all-zeros state
- Every path in the trellis starting from the initial state corresponds to a codeword

Terminated Trellis Diagram



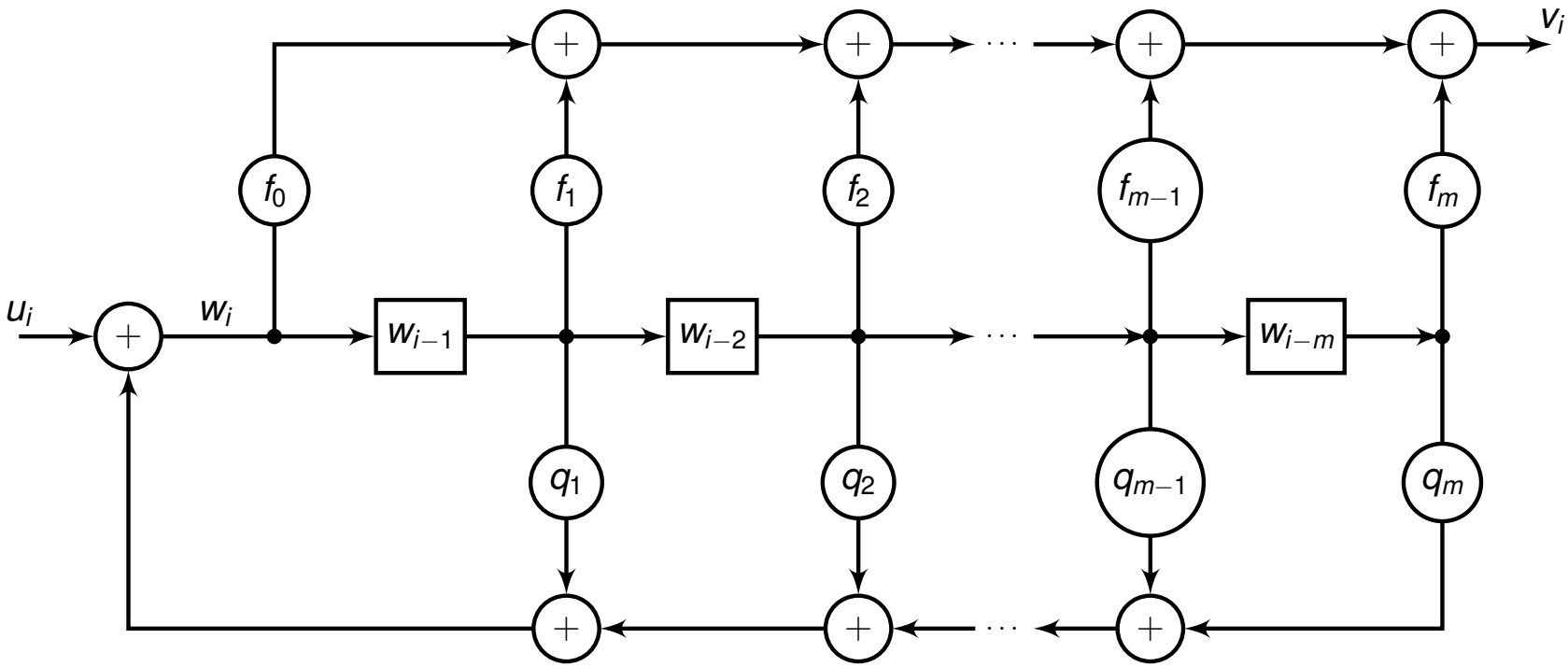
- The inputs are chosen to terminate the trellis in the all-zeros state
- Every path from the initial state to the final state is a codeword

Terminating the Trellis of Feedforward Encoders



- Two consecutive zero input bits will drive the above encoder to the all-zeros state
- In a feedforward encoder with memory order m, m consecutive zero input bits in each of the k inputs will terminate the trellis

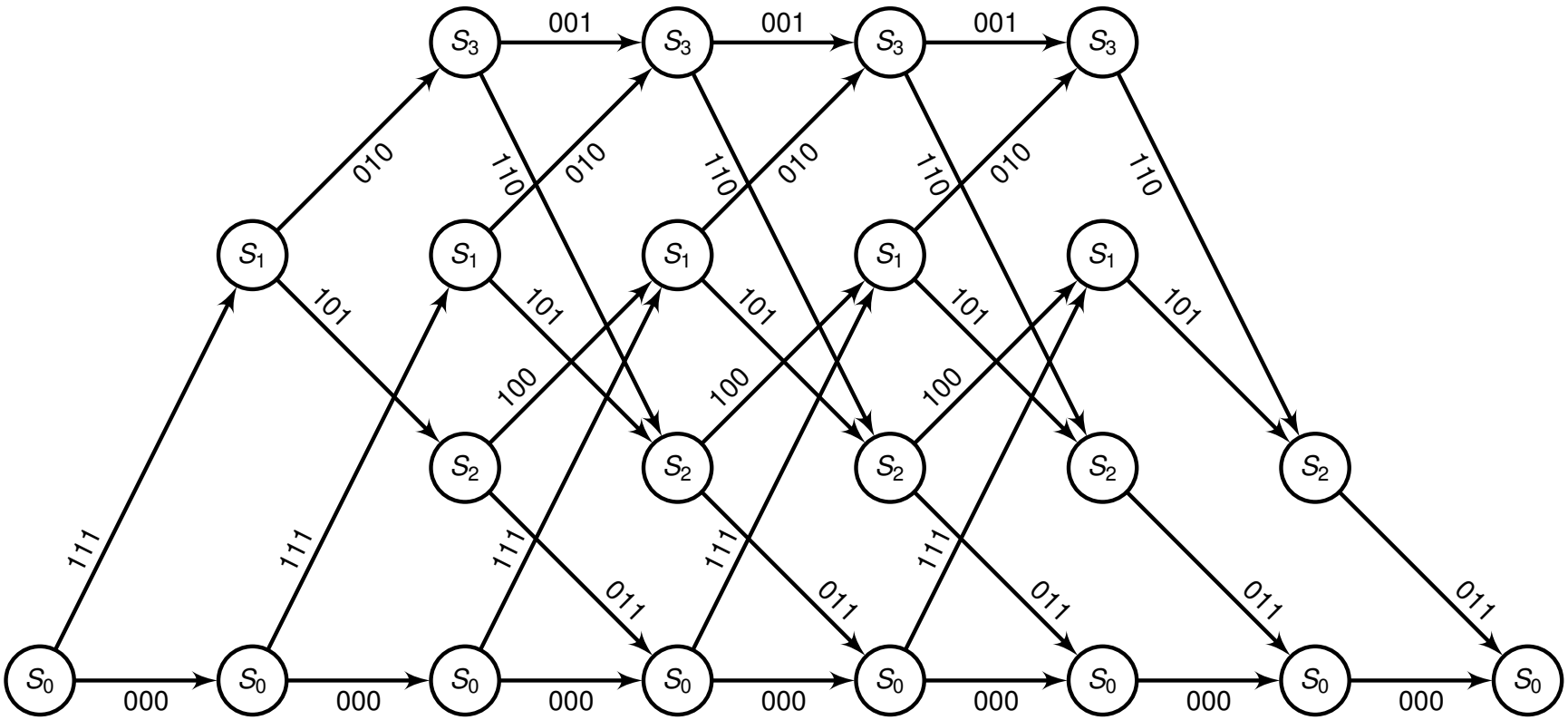
Terminating the Trellis of Feedback Encoders



- To reach the all-zeros state, the input to the shift register has to be zero for m time units

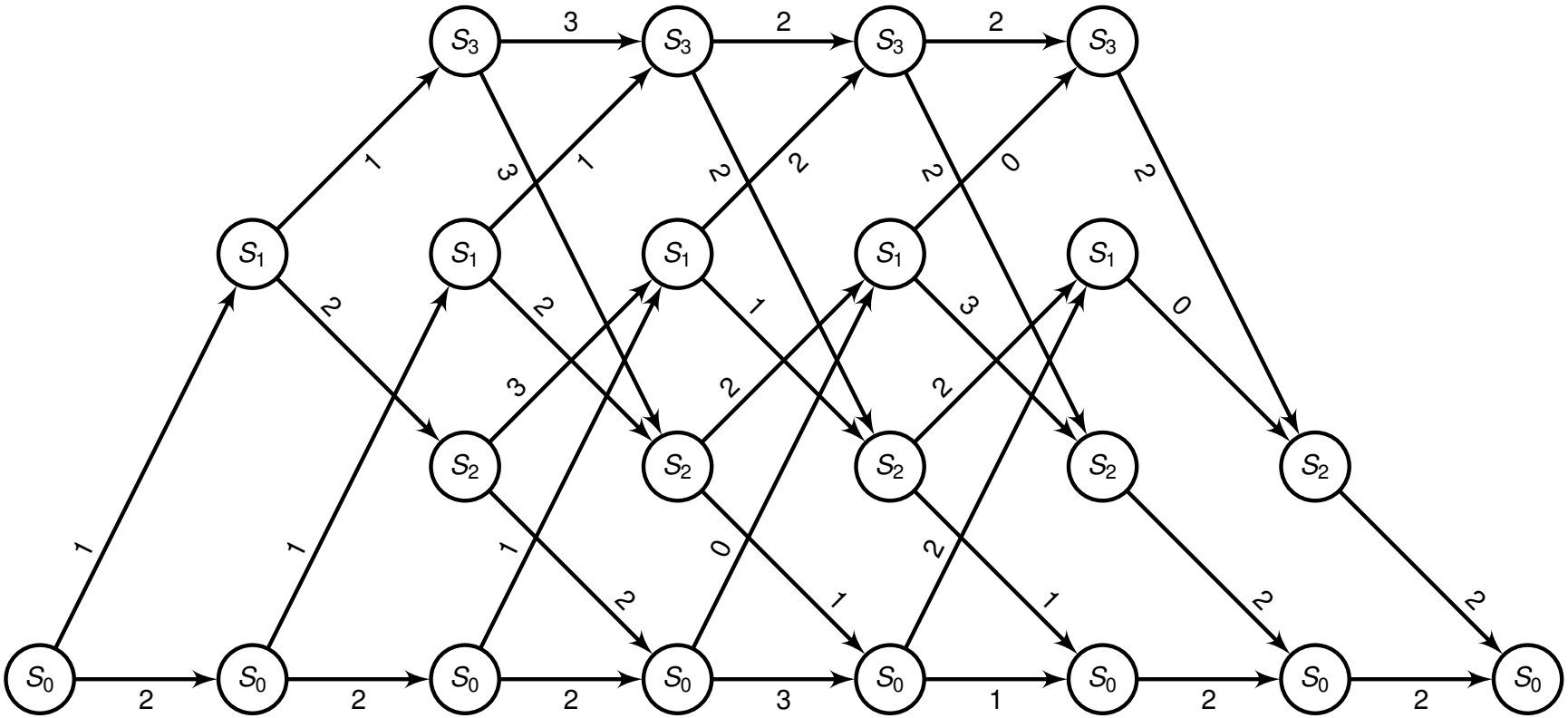
$$w_i = 0 \implies u_i = \sum_{j=1}^{m} q_j w_{i-j}$$

Maximum Likelihood Decoder for BSC



- Let $\mathbf{r} = \begin{bmatrix} 110 & 110 & 110 & 111 & 010 & 101 & 101 \end{bmatrix}$ be the BSC output
- The ML decoder will output a codeword $\mathbf{v}$ such that $d_H(\mathbf{r}, \mathbf{v})$ is minimum
- The Viterbi algorithm is an efficient way to find the $\mathbf{v}$ closest to $\mathbf{r}$

Maximum Likelihood Decoder for BSC



- Branch metric is the Hamming distance between the codeword bits of a state transition and the corresponding received bits
- Path metric is the sum of all the branch metrics along a path

Questions? Takeaways?