EE 605: Error Correcting Codes (Autumn 2015)
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Problem Set 5 — Date: October 31, 2015

1. Find generator polynomials for t-error correcting binary primitive BCH codes with length 15 for $t = 1, 2, 3$. Note that the generator polynomials may not be unique. Specify one generator polynomial for each value of t. *Hint:* $x^{16} + x = x(x+1)(x^2 + x + 1)(x^4 + x + 1)(x^4 + x^3 + 1)(x^4 + x^3 + x^2 + x + 1)$

2. Consider a t-error-correcting binary BCH code of length $n = 2^m - 1$. If $2t + 1$ is a factor of n, prove that the minimum distance of the code is exactly $2t + 1$. You can assume the BCH bound in your solution ($d_{min} \geq 2t + 1$).

3. Consider a $(2, 1)$ convolutional code with encoder matrix $G(D) = \begin{bmatrix} 1 + D^2 & 1 + D + D^2 + D^3 \end{bmatrix}$.
   (a) Draw the encoder circuit.
   (b) Draw the encoder state diagram.
   (c) Is this encoder catastrophic? If yes, find an infinite weight information sequence which generates a codeword of finite weight.

4. Consider a rate $\frac{2}{3}$ convolutional encoder with transform domain generator matrix given by
$$\mathbf{G}(D) = \begin{bmatrix} 1 + D & 1 + D & D \\ D & 1 & 1 \end{bmatrix}.$$
   (a) Draw the circuit corresponding to this encoder.
   (b) Draw the state transition diagram corresponding to this encoder. All the transitions should be labelled with the corresponding inputs and outputs.

5. Consider a rate $\frac{1}{3}$ convolutional encoder with transform domain generator matrix given by
$$\mathbf{G}(D) = \begin{bmatrix} 1 & \frac{1+D^2}{1+D+D^2} & \frac{1+D}{1+D+D^2} \end{bmatrix}.$$
   (a) Draw the circuit corresponding to this encoder.
   (b) Suppose the encoder is in the all-ones state i.e. all the delay elements contain a 1. Specify the shortest input sequence which will force the encoder to the all-zeros state.

6. A rate $\frac{1}{2}$ convolutional encoder with transform domain generator matrix given by
$$\mathbf{G}(D) = \begin{bmatrix} 1 + D + D^2 & 1 + D^2 \end{bmatrix}.$$
is used to transmit four information bits over a BSC with crossover probability $p < \frac{1}{2}$.
   (a) Draw the terminated trellis diagram for this encoder. Note that the some bits have to be appended to the four information bits to bring the trellis to the all-zeros state.
   (b) If the output of the BSC is $\mathbf{r} = \begin{bmatrix} 10 & 01 & 10 & 01 & 01 & 00 \end{bmatrix}$, draw the surviving paths at each stage of the Viterbi algorithm. You will have to draw five **different** pictures corresponding to five stages ($t = 2$ to $t = 6$). At stage t, you can omit the portion of the trellis from stage $t + 1$ onwards. Label each surviving path with its partial path metric.
   (c) What are the estimated information bits obtained by the Viterbi algorithm?