EE 605: Error Correcting Codes
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Endsemester Exam : 40 points — November 22, 2015

1. (5 points) Prove that $\left(C^{\perp}\right)^{\perp} = C$ when C is a linear block code.

2. (5 points) The first row of a standard array is given below where the last four entries are missing. It is known that this standard array has 8 columns.

$$000000 \quad 110001 \quad 101010 \quad 000111 \quad * \quad * \quad * \quad *$$

   (a) Complete the standard array by giving all the remaining columns and rows.

   (b) If the code corresponding to this standard array is used over a binary symmetric channel with crossover probability p, what is the probability of decoding error?

3. (5 points) Let $g(X) = 1 + X^2 + X^4 + X^6 + X^7 + X^{10}$.

   (a) Show that $g(X)$ generates a $(21, 11)$ cyclic code. Draw a syndrome computation circuit for this code.

   (b) Draw a systematic encoding circuit for this code. Compute the codeword corresponding to the input $u(X) = 1 + X + X^3 + X^9$.

4. (5 points) Prove that any finite field has a subfield isomorphic to $\mathbb{F}_p$, the field of integers $\{0, 1, 2, \ldots, p-1\}$ where p is a prime and the operations are integer addition and multiplication modulo p. Use this result to prove that any finite field has p^m elements where p is a prime number and m is a positive integer.

5. (a) ($2\frac{1}{2}$ points) Determine all the irreducible polynomials of degree 5 in $F_2[x]$.

   (b) ($2\frac{1}{2}$ points) Find all the minimal polynomials of the field of 9 elements.

6. (10 points) A rate $\frac{1}{3}$ convolutional encoder with transform domain generator matrix given by

$$\mathbf{G}(D) = \begin{bmatrix} 1 + D + D^2 & 1 + D + D^2 & 1 + D \end{bmatrix}.$$

   is used to transmit the information bits **110101** over a BSC with crossover probability $p < \frac{1}{2}$.

   (a) Draw the terminated trellis diagram for this encoder. Note that the some bits have to be appended to the information bits to bring the trellis to the all-zeros state.

   (b) Specify the output codeword corresponding to the information bits mentioned above.

   (c) If the output of the BSC is $\mathbf{r} = \begin{bmatrix} 111 & 111 & 001 & 111 & 110 & 111 & 111 & 110 \end{bmatrix}$, find the codeword which is the output of the Viterbi algorithm.

   (d) What are the estimated information bits obtained by the Viterbi algorithm?

7. (5 points) A burst of length l is defined as a vector in $\mathbb{F}_2^n$ whose nonzero components are confined to l consecutive locations, the first and last of which are nonzero. For example, for $n = 7$, the vector 0011010 corresponds to a burst of length 4. A linear code which is capable of correcting all error bursts of length l or less but not all bursts of length $l + 1$ is called an l-burst-error-correcting code. Let C be an l-burst-error-correcting binary linear block code which has length n and dimension k.

   (a) Show that no burst of length $2l$ or less can be a codeword in C.

   (b) Show that $n - k \geq 2l$. *Hint: What is the sum of two vectors in the same row of the standard array?*