EE 605: Error Correcting Codes
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Quiz 1 : 25 points August 21, 2015

1. (5 points) Prove that for a binary block code with minimum distance d_{min}, the minimum distance decoder can correct upto $\lfloor \frac{d_{min}-1}{2} \rfloor$ errors.

2. (5 points) Construct a standard array for a binary linear block code with the following generator matrix if it is to be used over a binary symmetric channel with crossover probability $p < \frac{1}{2}$.

$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Construct the syndrome-error pattern lookup table for this code, i.e. a one-to-one mapping between the set of syndromes and set of correctable error patterns.

3. (5 points) Let C be an (n, k) binary linear block code with $k \geq 1$. Let $\mathbf{v} \in \mathbb{F}_2^n$ be a vector not in the dual code of C, i.e. $\mathbf{v} \notin C^\perp$. Show that $\mathbf{v}$ is orthogonal to exactly half of the codewords in C.

4. (5 points) Consider a binary linear code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

   (a) Find the weight distribution of this code.
   (b) Find a parity check matrix for this code.

5. (5 points) Let $f : \mathbb{F}_2^n \mapsto \mathbb{R}$ be a function. The Hadamard transform of f is given by

$$\hat{f}(\mathbf{u}) = \sum_{\mathbf{v} \in \mathbb{F}_2^n} (-1)^{\mathbf{u} \cdot \mathbf{v}} f(\mathbf{v})$$

where $\mathbf{u} \in \mathbb{F}_2^n$ and $\mathbf{u} \cdot \mathbf{v} = \mathbf{u}\mathbf{v}^T$. Let C be an (n, k) binary linear block code. Prove that

$$\sum_{\mathbf{u} \in C^\perp} f(\mathbf{u}) = \frac{1}{2^k} \sum_{\mathbf{u} \in C} \hat{f}(\mathbf{u}).$$