EE 605: Error Correcting Codes
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Quiz 3 : 25 points — November 6, 2015

1. (10 points) Determine the generator polynomials of the single error correcting and double error correcting binary primitive BCH codes of length 31. Use the primitive element of the field $\mathbb{F}_{32}$ which has minimal polynomial $p(X) = 1 + X^2 + X^5$. *Hint: To find the minimal polynomial of α^3, find the polynomial representations of the field elements $\alpha^{15}, \alpha^{12}, \alpha^9, \alpha^6$ using the fact that $p(\alpha) = 0$.*

2. (5 points) Consider a rate $\frac{2}{3}$ convolutional encoder with transform domain generator matrix given by
$$\mathbf{G}(D) = \begin{bmatrix} 1+D & 1+D & D \\ D & 1 & 1 \end{bmatrix}.$$
   (a) Draw the circuit corresponding to this encoder.
   (b) Draw the state transition diagram corresponding to this encoder. All the transitions should be labelled with the corresponding inputs and outputs.

3. (10 points) A rate $\frac{1}{2}$ convolutional encoder with transform domain generator matrix given by
$$\mathbf{G}(D) = \begin{bmatrix} 1 + D + D^2 & 1 + D^2 \end{bmatrix}.$$
   is used to transmit **three** information bits over a BSC with crossover probability $p < \frac{1}{2}$.
   (a) Draw the terminated trellis diagram for this encoder. Note that the some bits have to be appended to the three information bits to bring the trellis to the all-zeros state.
   (b) If the output of the BSC is $\mathbf{r} = \begin{bmatrix} 11 & 10 & 10 & 01 & 01 \end{bmatrix}$, draw the surviving paths at each stage of the Viterbi algorithm. You will have to draw four **different** pictures corresponding to four stages ($t = 2$ to $t = 5$). At stage t, you can omit the portion of the trellis from stage $t + 1$ onwards. Label each surviving path with its partial path metric.
   (c) What are the estimated information bits obtained by the Viterbi algorithm?