Viterbi Algorithm

Saravanan Vijayakumaran
sarva@ee.iitb.ac.in

Department of Electrical Engineering
Indian Institute of Technology Bombay

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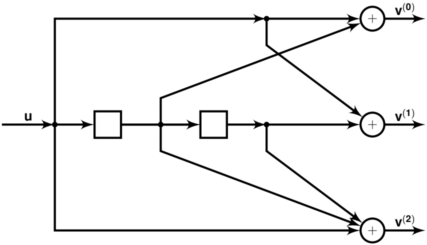
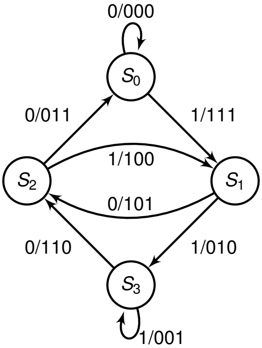
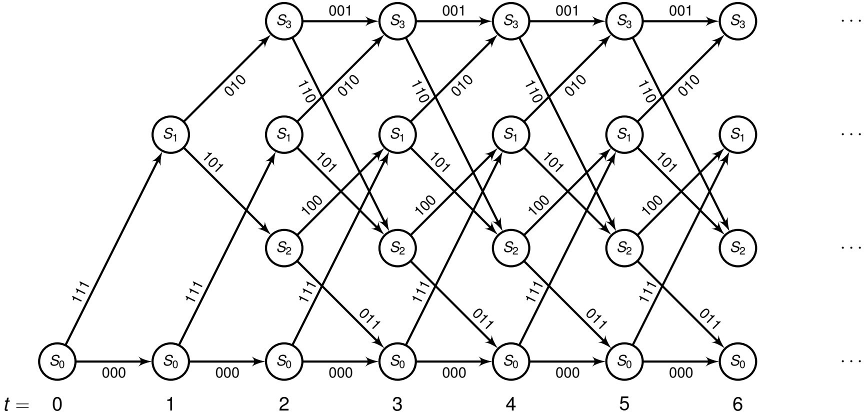
Encoder State Diagram

$$\mathbf{G}(D) = \begin{bmatrix} 1 + D & 1 + D^2 & 1 + D + D^2 \end{bmatrix}$$



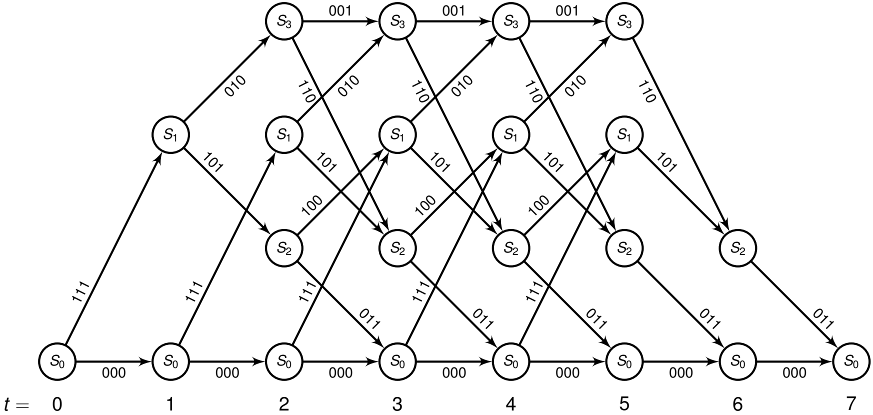


Encoder Trellis Diagram



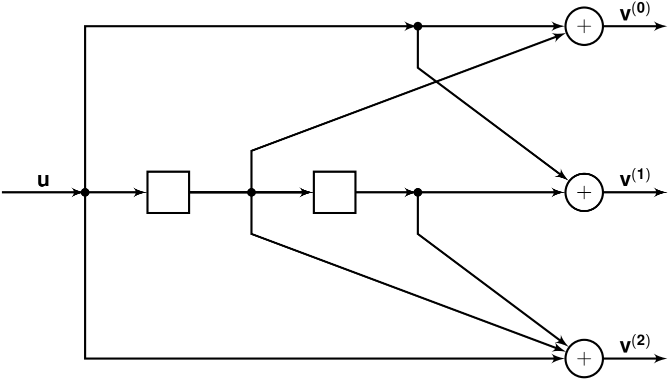
- The initial state of the encoder is the all-zeros state
- Every path in the trellis starting from the initial state gives a codeword

Terminated Trellis Diagram



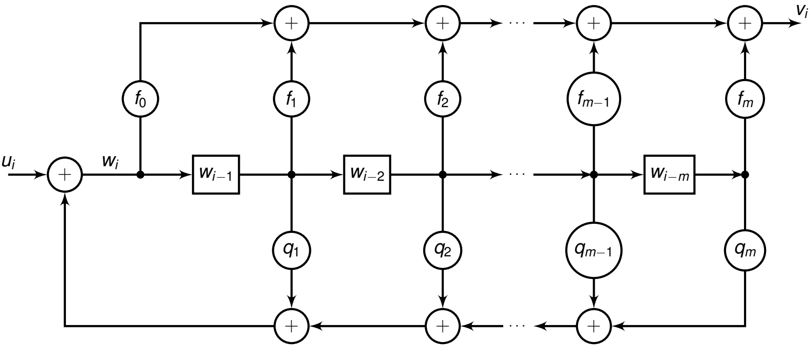
- The inputs are chosen to terminate the trellis in the all-zeros state
- Every path from the initial state to the final state is a codeword

Terminating the Trellis of Feedforward Encoders



- Two consecutive zero input bits will drive the above encoder to the all-zeros state
- In a feedforward encoder with memory order *m*, *m* consecutive zero input bits in each of the *k* inputs will terminate the trellis

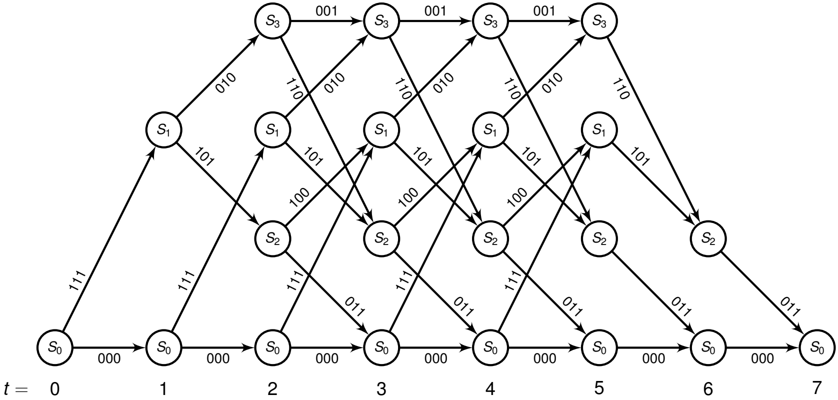
Terminating the Trellis of Feedback Encoders



- To reach the all-zeros state, the input to the shift register has to be zero for m time units

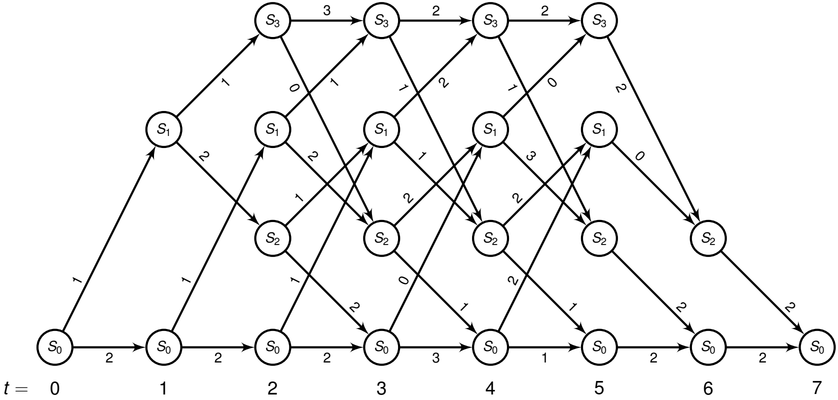
$$w_i = 0 \implies u_i = \sum_{j=1}^{m} q_j w_{i-j}$$

Maximum Likelihood Decoder for BSC



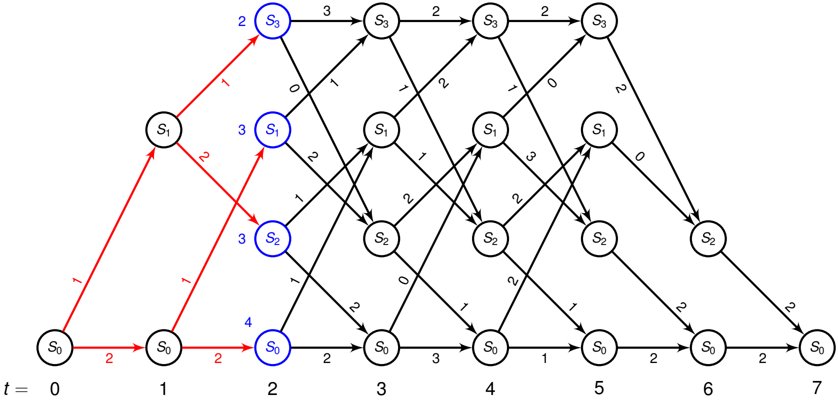
- Let $\mathbf{r} = \begin{bmatrix}110 & 110 & 110 & 111 & 010 & 101 & 101\end{bmatrix}$ be the BSC output
- The ML decoder will output a codeword $\mathbf{v}$ such that $d_H(\mathbf{r}, \mathbf{v})$ is minimum

Maximum Likelihood Decoder for BSC



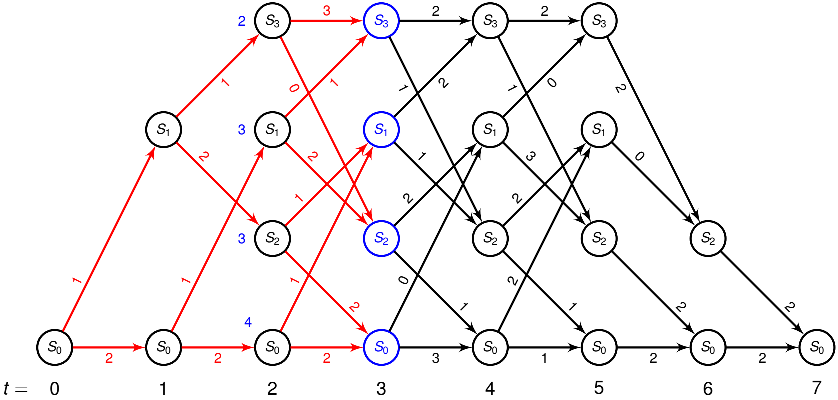
- Branch metric is the Hamming distance between the codeword bits of a state transition and the corresponding received bits
- Path metric is the sum of all the branch metrics along a path

Viterbi Algorithm (1)



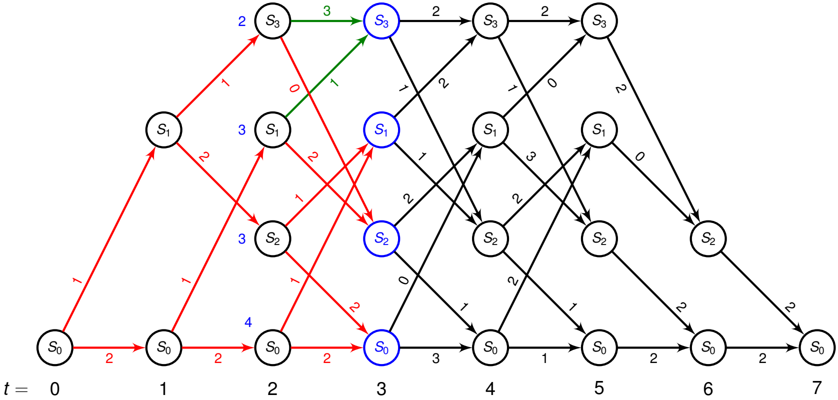
- At $t = 2$, there is only path entering each state
- The shortest path has one of these four paths as a prefix

Viterbi Algorithm (2)



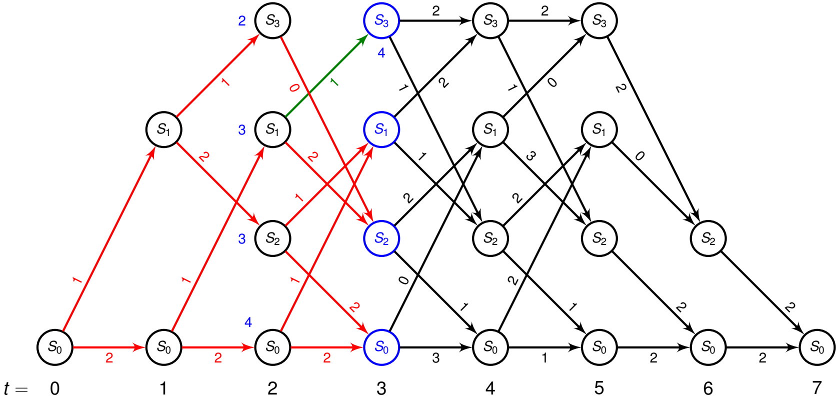
- At $t = 3$, there are two paths entering each state
- The shortest path has one of these eight paths as a prefix

Viterbi Algorithm (3)



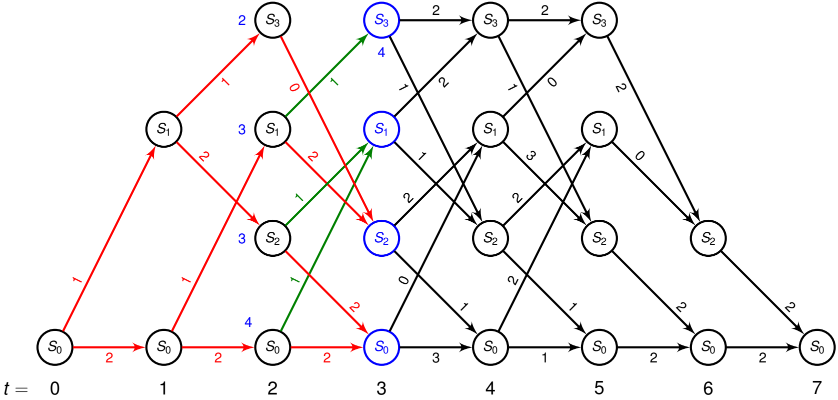
- At $t = 3$, the two paths entering state S_3 have path metrics 5 and 4
- We eliminate the path with metric 5

Viterbi Algorithm (4)



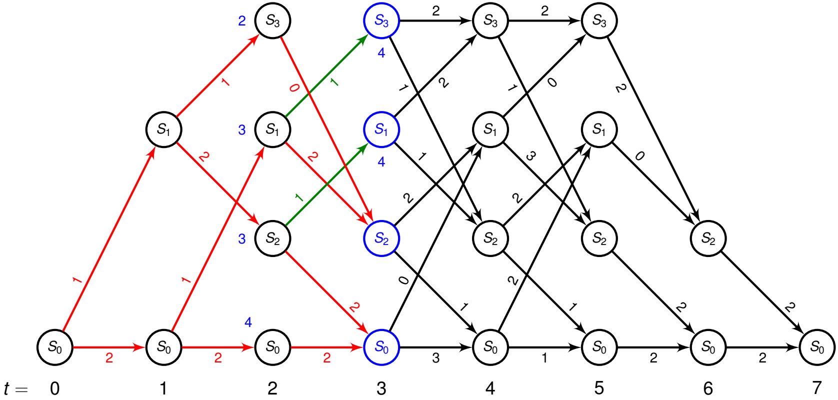
- The state S_3 at time $t = 3$ is now labelled with its partial path metric

Viterbi Algorithm (5)



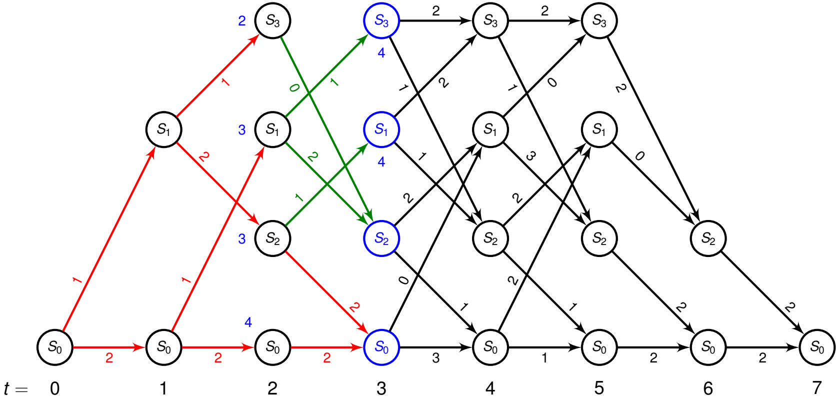
- At $t = 3$, the two paths entering state S_1 have path metrics 4 and 5
- We eliminate the path with metric 5

Viterbi Algorithm (6)



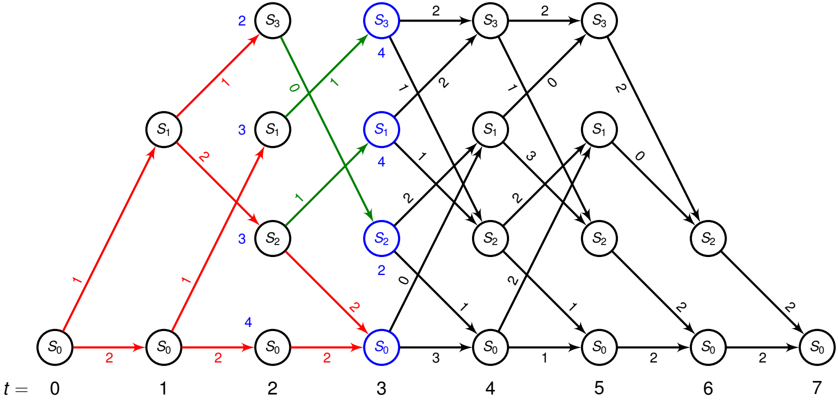
- The state S_1 at time $t = 3$ is now labelled with its partial path metric

Viterbi Algorithm (7)



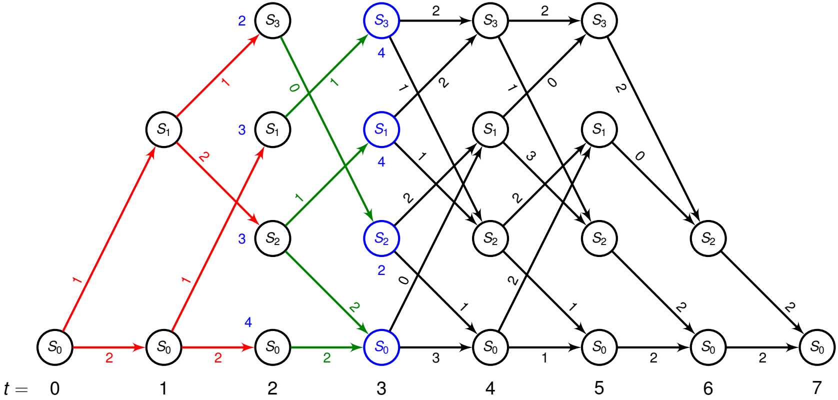
- At $t = 3$, the two paths entering state S_2 have path metrics 2 and 5
- We eliminate the path with metric 5

Viterbi Algorithm (8)



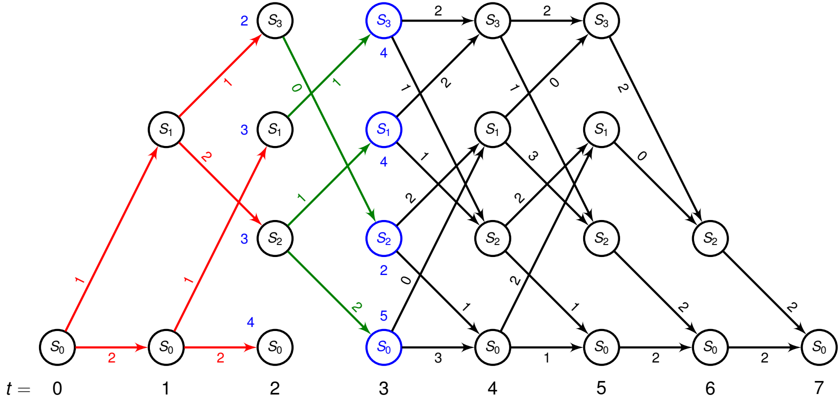
- The state S_2 at time $t = 3$ is now labelled with its partial path metric

Viterbi Algorithm (9)



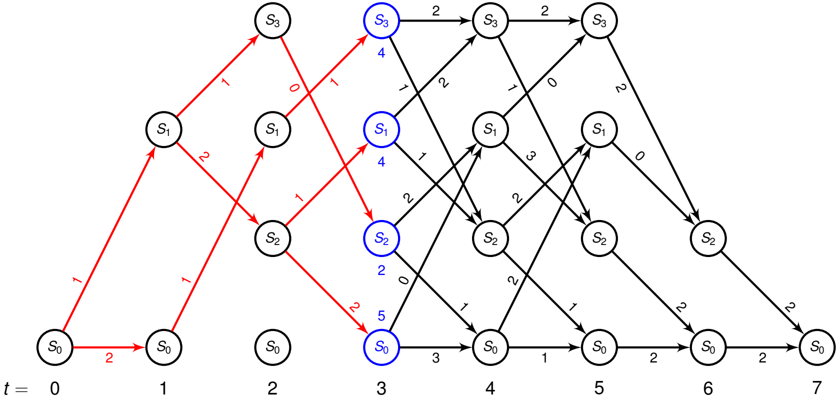
- At $t = 3$, the two paths entering state S_0 have path metrics 5 and 6
- We eliminate the path with metric 6

Viterbi Algorithm (10)



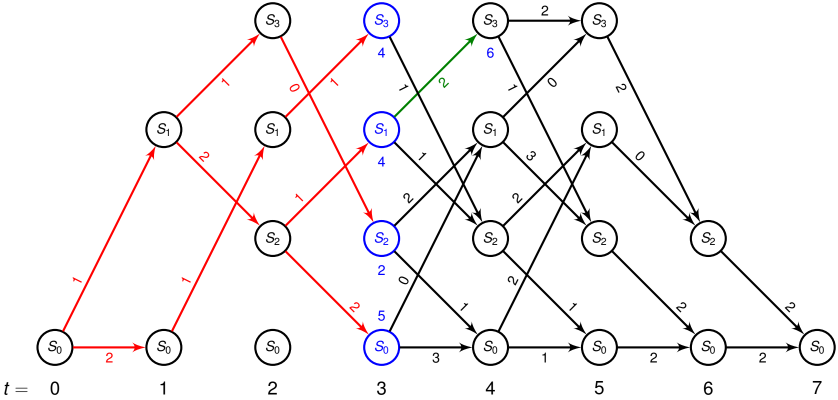
- The state S_0 at time $t = 3$ is now labelled with its partial path metric

Viterbi Algorithm (11)



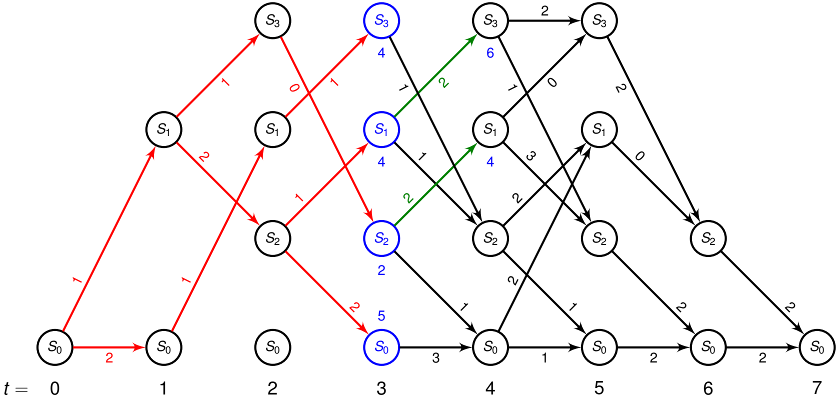
- At $t = 3$, there is now only one surviving path entering each state
- The shortest path has one of these four paths as a prefix

Viterbi Algorithm (12)



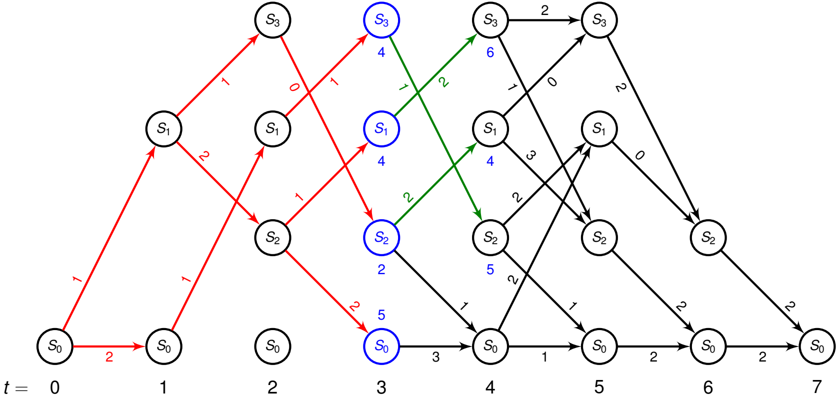
- We eliminate one of the paths with metric 6 entering S_3 at $t = 4$

Viterbi Algorithm (13)



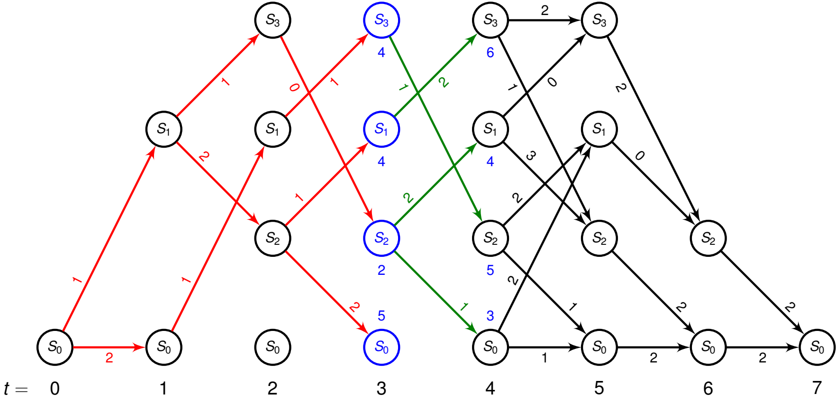
- We eliminate the path with metric 5 entering S_1 at $t = 4$

Viterbi Algorithm (14)



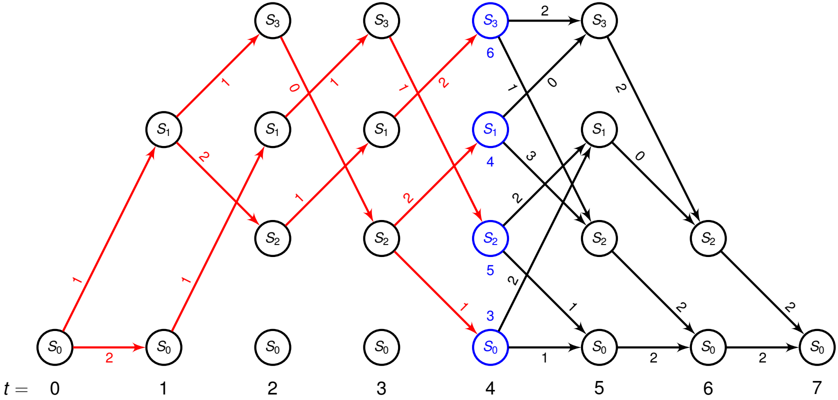
- We eliminate one of the paths with metric 5 entering S_2 at $t = 4$

Viterbi Algorithm (15)



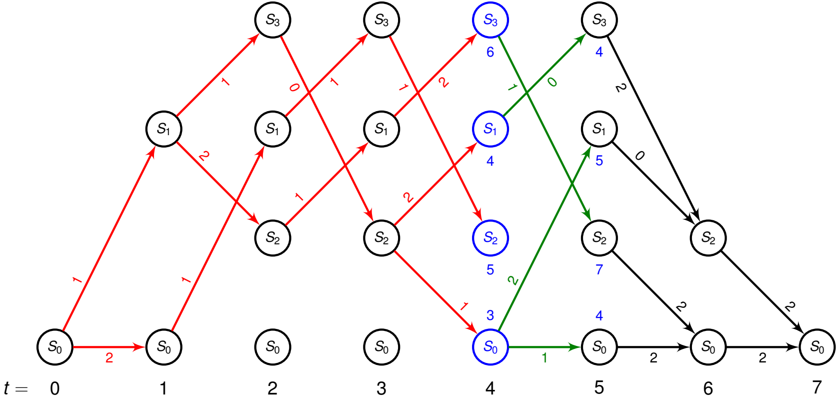
- We eliminate the path with metric 8 entering S_0 at $t = 4$

Viterbi Algorithm (16)



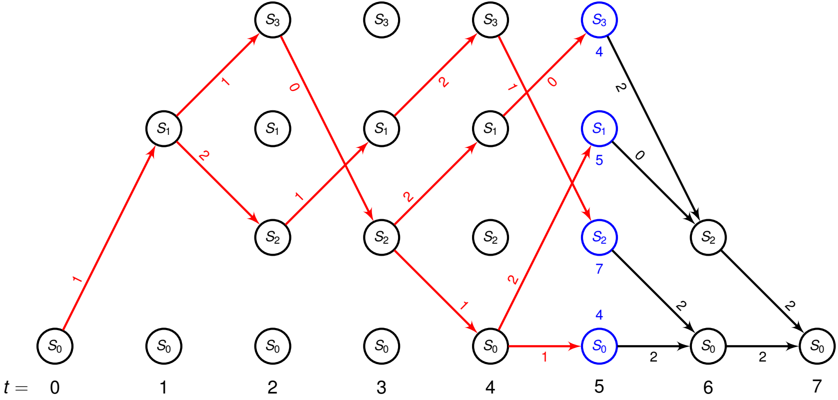
- At $t = 4$, there is now only one surviving path entering each state
- The shortest path has one of these four paths as a prefix

Viterbi Algorithm (17)



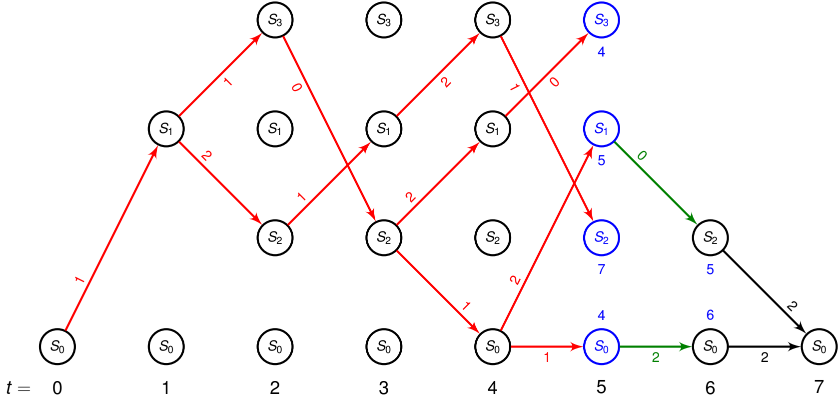
- Using the same procedure, we eliminate one path per state at $t = 5$

Viterbi Algorithm (18)



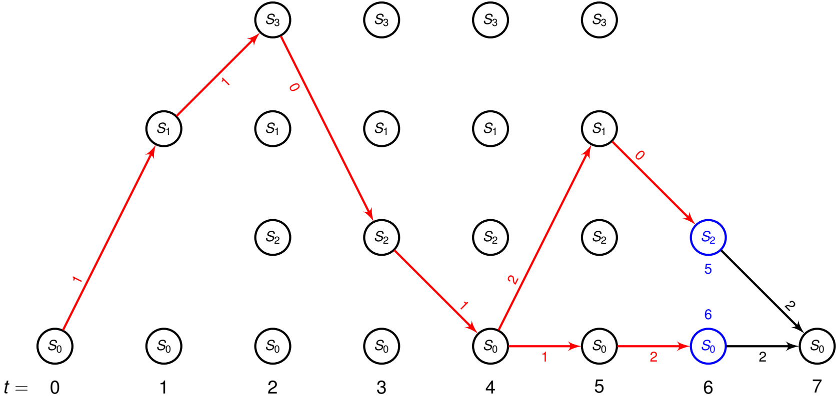
- At $t = 5$, there is now only one surviving path entering each state
- The shortest path has one of these four paths as a prefix

Viterbi Algorithm (19)



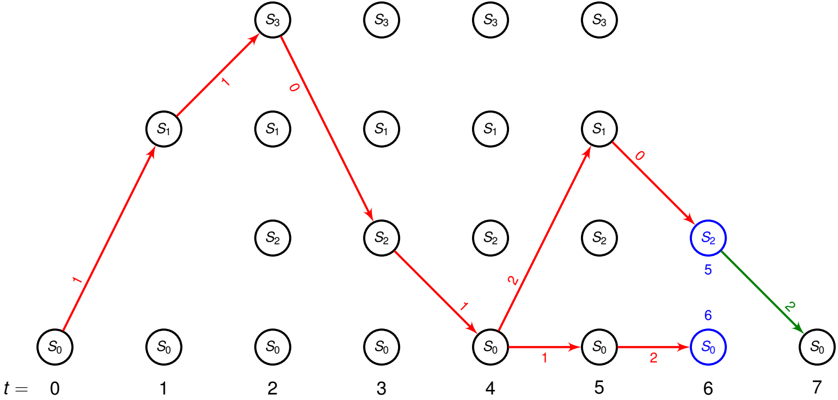
- Using the same procedure, we eliminate one path per state at $t = 6$

Viterbi Algorithm (20)



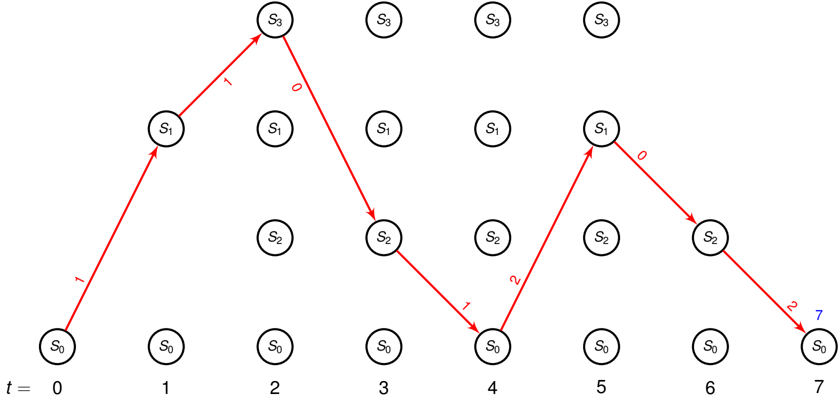
- At $t = 6$, there is now only one surviving path entering each state
- The shortest path has one of these two paths as a prefix

Viterbi Algorithm (21)



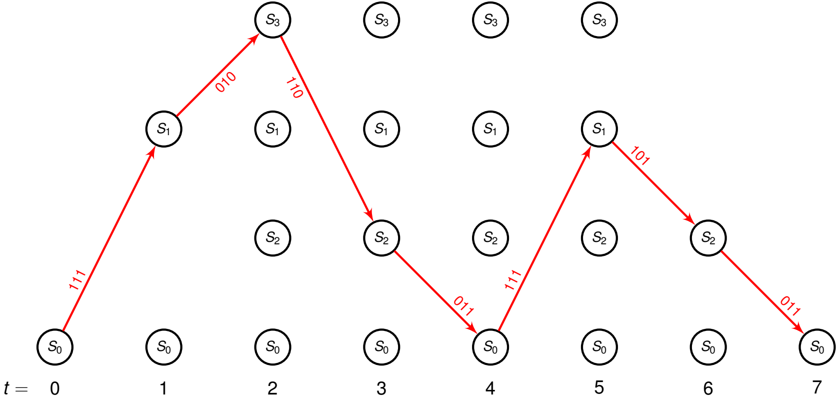
- We eliminate the path with metric 8 entering S_0 at $t = 7$

Viterbi Algorithm (22)



- A single path from state S_0 at $t = 0$ to state S_0 at $t = 7$ has survived

Viterbi Algorithm (23)



- $\hat{\mathbf{v}} = \begin{bmatrix}111 & 010 & 110 & 011 & 111 & 101 & 011\end{bmatrix}$ is the output of the Viterbi algorithm

Viterbi Algorithm in General

- Consider a convolutional code with k inputs, n outputs, memory order m and constraint length ν
- The trellis has at most 2^ν states at each time instant
- At $t = m$, there is one path entering each state
- At $t = m + 1$, there are 2^k paths entering each state, out of which $2^k - 1$ have to be eliminated
- At each time instant t, at most 2^ν surviving paths exist
- For a discrete memoryless channel with channel transition probability function $p(\cdot|\cdot)$, we maximize

$$\log p(\mathbf{r}|\mathbf{v}) = \sum_{i=0}^{h+m-1} \log p(\mathbf{r}_i|\mathbf{v}_i)$$

where h is the number of k-bit input blocks, $\mathbf{r}_i$ and $\mathbf{v}_i$ are n symbols long

Questions? Takeaways?