EE 703: Digital Message Transmission

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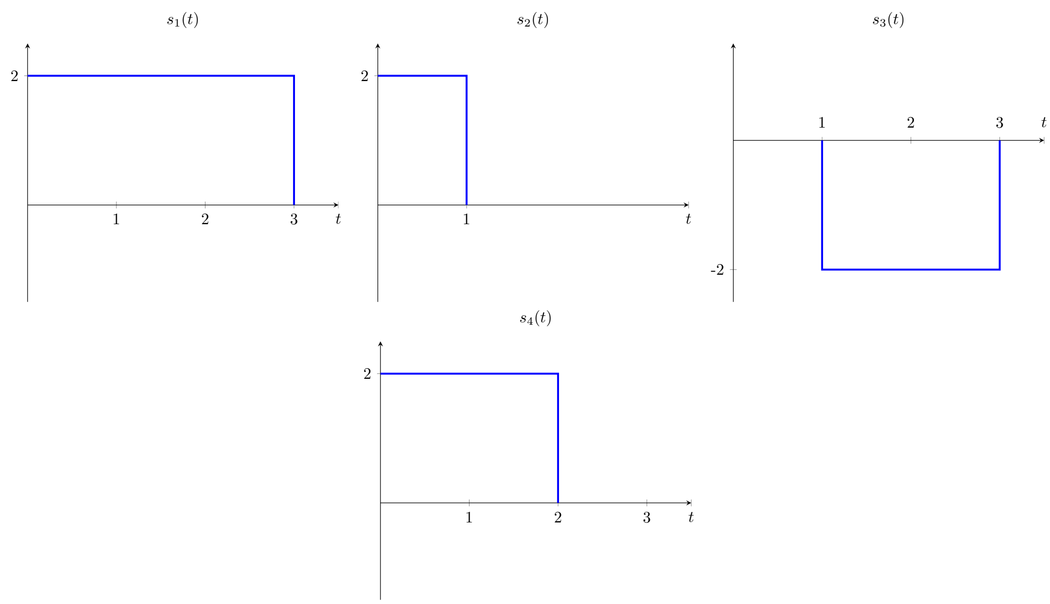
Autumn 2012

Date:
Quiz 1 : **12 points** (90 min) August 29, 2012

Each question is worth 3 points.

1. (a) Find an orthonormal basis for the following waveforms.



   (b) Use the basis functions to represent the waveforms as vectors $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4$.

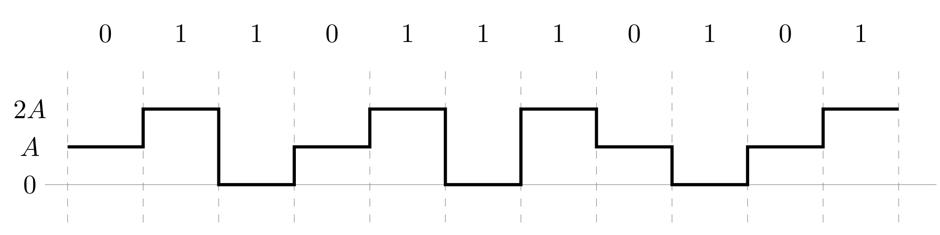
   (c) Determine the pair of waveforms which are closest to each other where distance between $s_i(t)$ and $s_j(t)$ is defined as $\int_{-\infty}^{\infty} \left[s_i(t) - s_j(t)\right]^2 \, dt$.

2. Determine the power spectral density of the following line coding scheme:

$$u(t) = \sum_{n=-\infty}^{\infty} b_n p(t - nT) \tag{1}$$

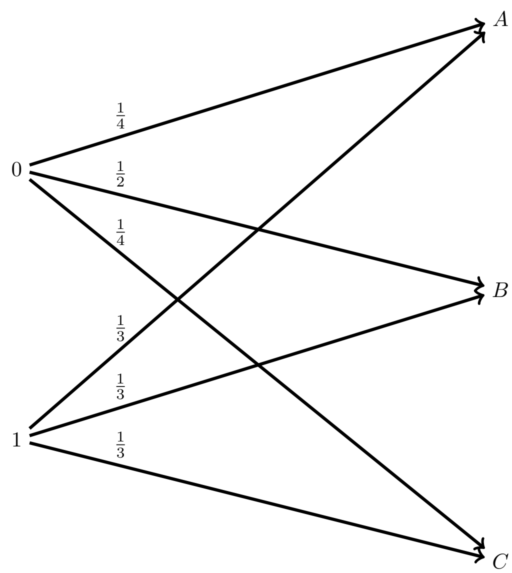
where $p(t) = I_{[0,T)}(t)$ and the symbol b_n is the obtained by mapping a zero bit to amplitude A and mapping a one bit to amplitude 0 or amplitude $2A$. Successive ones are alternately mapped to 0 and $2A$. Assume that the bits used to generate b_n are independent and equally likely to be zero or one. The formula for the PSD is as follows.

$$S_u(f) = \frac{|P(f)|^2}{T} \sum_{k=-\infty}^{\infty} R_b[k] e^{-j2\pi k f T} \tag{2}$$



3. Consider the binary input ternary output channel below. The prior probability of the input being 0 is 0.4.

   - Find the decision rule δ_{MPE} which minimizes decision error probability.
   - Find the conditional decision error probability when 0 is transmitted and δ_{MPE} is used.
   - Find the decision error probability when δ_{MPE} is used.



4. Find the maximum likelihood decision rule for the following 3-ary hypothesis testing problem where $\mu = \sqrt{2}\sigma$.

$$\begin{aligned} H_1 &: Y \sim N(-\mu, \sigma^2) \\ H_2 &: Y \sim N(0, e^2\sigma^2) \\ H_3 &: Y \sim N(\mu, \sigma^2) \end{aligned}$$