

Complex Baseband Representation

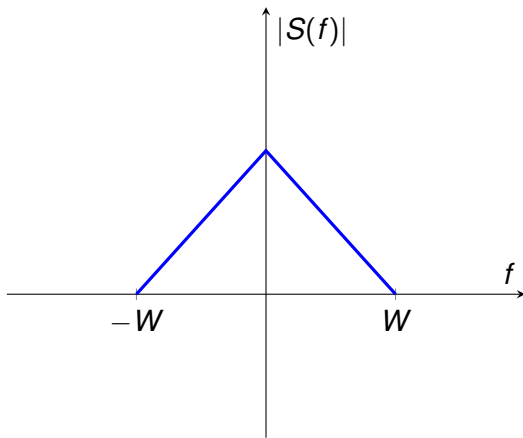
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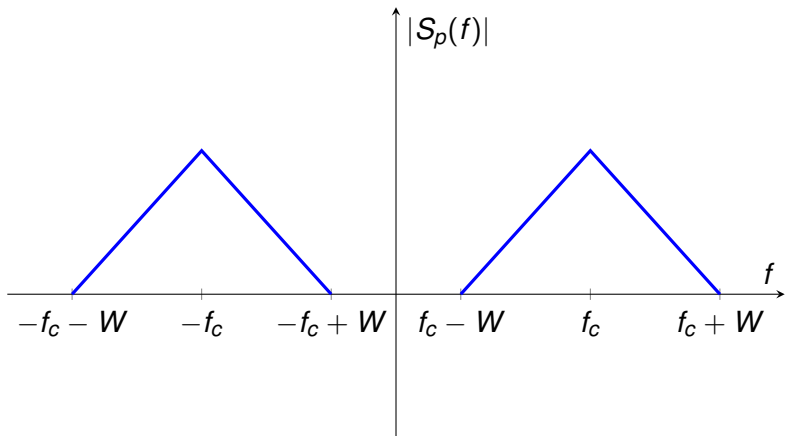
Baseband Signals

$$S(f) = 0, \quad |f| > W$$



Passband Signals

$$S(f) \neq 0, \quad |f \pm f_c| \leq W, \quad f_c > W > 0.$$



Sampling Theorem

Theorem

If a signal $s(t)$ is bandlimited to B ,

$$S(f) = 0, \quad |f| > B$$

then a sufficient condition for exact reconstructability is a uniform sampling rate f_s where

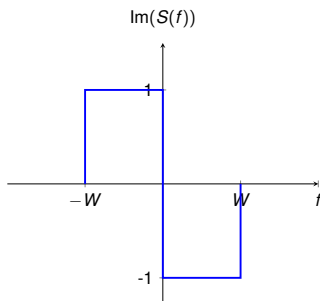
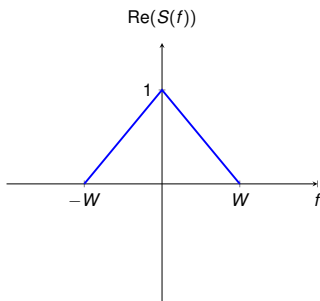
$$f_s > 2B.$$

Baseband Signals $B = W$

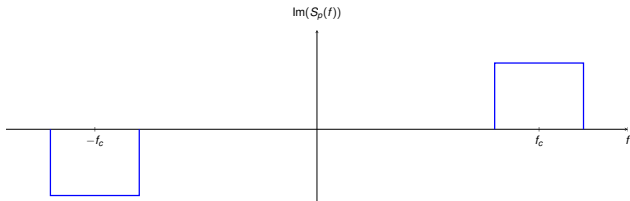
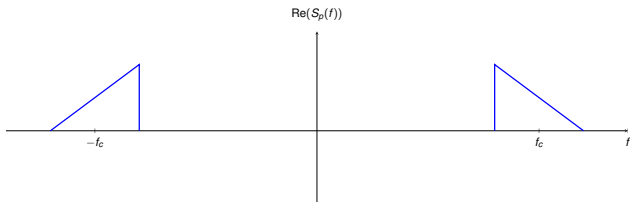
Passband Signals $B = f_c + W$ (Can we do better?)

Fourier Transform for Real Signals

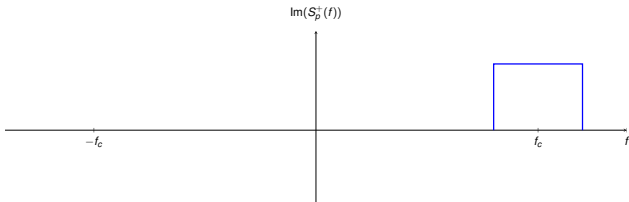
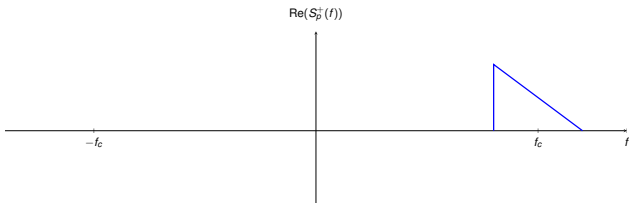
$$\begin{aligned}\text{Im}[s(t)] = 0 &\Rightarrow S(f) = S^*(-f) \quad (\text{Conjugate Symmetry}) \\ &\Rightarrow |S(f)| = |S(-f)|, \arg(S(f)) = -\arg(S(-f))\end{aligned}$$



Fourier Transform of a Real Passband Signal

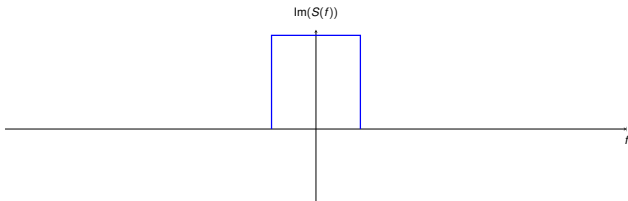
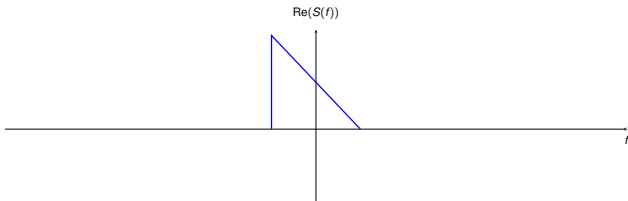


Positive Spectrum of a Real Passband Signal



$$S_p^+(f) = S_p(f)u(f)$$

Complex Envelope of a Real Passband Signal



$$S(f) = \sqrt{2}S_p^+(f + f_c) = \sqrt{2}S_p(f + f_c)u(f + f_c)$$

Complex Envelope in Time Domain

Frequency Domain Representation

$$S(f) = \sqrt{2}S_p^+(f + f_c) = \sqrt{2}S_p(f + f_c)u(f + f_c)$$

Time Domain Representation of Positive Spectrum

$$\begin{aligned}S_p^+(f) &= S_p(f)u(f) \\s_p^+(t) &= s_p(t) \star \mathcal{F}^{-1}[u(f)]\end{aligned}$$

Time Domain Representation of Frequency Domain Unit Step

$$\begin{aligned}u(t) &\Leftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f) \\u(f) &\Leftrightarrow \frac{1}{-j2\pi t} + \frac{1}{2}\delta(-t) \\&= \frac{j}{2\pi t} + \frac{1}{2}\delta(t)\end{aligned}$$

Complex Envelope in Time Domain

Time Domain Representation of Positive Spectrum

$$\begin{aligned} s_p^+(t) &= s_p(t) \star \left[\frac{1}{2} \delta(t) + \frac{j}{2\pi t} \right] \\ &= \frac{1}{2} [s_p(t) + j\hat{s}_p(t)] \end{aligned}$$

Time Domain Representation of Complex Envelope

$$\begin{aligned} \sqrt{2}S_p(f)u(f) &\Leftrightarrow \frac{1}{\sqrt{2}} [s_p(t) + j\hat{s}_p(t)] \\ \sqrt{2}S_p(f + f_c)u(f + f_c) &\Leftrightarrow \frac{1}{\sqrt{2}} [s_p(t) + j\hat{s}_p(t)] e^{-j2\pi f_c t} \\ S(f) &\Leftrightarrow \frac{1}{\sqrt{2}} [s_p(t) + j\hat{s}_p(t)] e^{-j2\pi f_c t} \\ s(t) &= \frac{1}{\sqrt{2}} [s_p(t) + j\hat{s}_p(t)] e^{-j2\pi f_c t} \end{aligned}$$

Passband Signal in terms of Complex Envelope

Complex Envelope

$$s(t) = s_c(t) + js_s(t)$$

$s_c(t)$ In-phase component

$s_s(t)$ Quadrature component

Time Domain Relationship

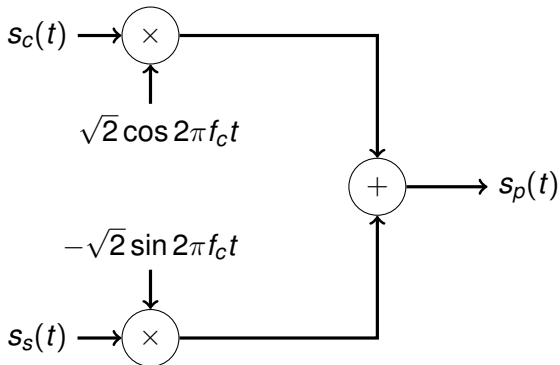
$$\begin{aligned} s_p(t) &= \operatorname{Re} \left[\sqrt{2}s(t)e^{j2\pi f_c t} \right] \\ &= \operatorname{Re} \left[\sqrt{2}\{s_c(t) + js_s(t)\}e^{j2\pi f_c t} \right] \\ &= \sqrt{2}s_c(t) \cos 2\pi f_c t - \sqrt{2}s_s(t) \sin 2\pi f_c t \end{aligned}$$

Frequency Domain Relationship

$$S_p(f) = \frac{S(f - f_c) + S^*(-f - f_c)}{\sqrt{2}}$$

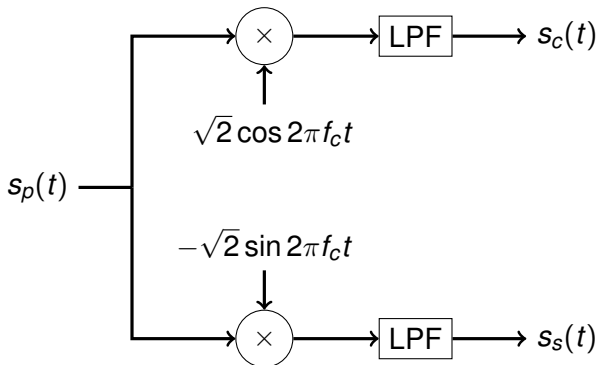
Upconversion

$$s_p(t) = \sqrt{2}s_c(t) \cos 2\pi f_c t - \sqrt{2}s_s(t) \sin 2\pi f_c t$$



Downconversion

$$\begin{aligned}\sqrt{2}s_p(t) \cos 2\pi f_c t &= 2s_c(t) \cos^2 2\pi f_c t - 2s_s(t) \sin 2\pi f_c t \cos 2\pi f_c t \\ &= s_c(t) + s_c(t) \cos 4\pi f_c t - s_s(t) \sin 4\pi f_c t\end{aligned}$$



Inner Product and Energy

Let $s(t)$ and $r(t)$ be signals.

Definition (Inner Product)

$$\langle \mathbf{s}, \mathbf{r} \rangle = \int_{-\infty}^{\infty} s(t)r^*(t) dt$$

Definition (Energy)

$$E_s = \|\mathbf{s}\|^2 = \langle \mathbf{s}, \mathbf{s} \rangle = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

I and Q Channels of a Passband Signal

$$s_p(t) = \underbrace{\sqrt{2}s_c(t) \cos 2\pi f_c t}_{I \text{ Component}} - \underbrace{\sqrt{2}s_s(t) \sin 2\pi f_c t}_{Q \text{ Component}}$$

$$x_c(t) = \sqrt{2}s_c(t) \cos 2\pi f_c t$$

$$x_s(t) = \sqrt{2}s_s(t) \sin 2\pi f_c t$$

I and Q Channels of a Passband Signal are Orthogonal

$$\langle x_c, x_s \rangle = 0$$

Passband and Baseband Inner Products

$$\langle u_p, v_p \rangle = \langle u_c, v_c \rangle + \langle u_s, v_s \rangle = \text{Re}(\langle u, v \rangle)$$

Energy of Complex Envelope = Energy of Passband Signal

$$\|s\|^2 = \|s_p\|^2$$

Complex Baseband Equivalent of Passband Filtering

$s_p(t)$ Passband signal

$h_p(t)$ Impulse response of passband filter

$y_p(t)$ Filter output

$$y_p(t) = s_p(t) \star h_p(t)$$

$$Y_p(f) = S_p(f)H_p(f)$$

$$S_+(f) = S_p(f)u(f)$$

$$H_+(f) = H_p(f)u(f)$$

$$Y_+(f) = Y_p(f)u(f)$$

$$Y_+(f) = S_+(f)H_+(f)$$

$$Y(f) = \sqrt{2}Y_+(f + f_c) = \sqrt{2}S_+(f + f_c)H_+(f + f_c) = \frac{1}{\sqrt{2}}S(f)H(f)$$

Complex Baseband Equivalent of Passband Filtering

$$y(t) = \frac{1}{\sqrt{2}} s(t) \star h(t)$$

$$y_c = \frac{1}{\sqrt{2}} (s_c \star h_c - s_s \star h_s)$$

$$y_s = \frac{1}{\sqrt{2}} (s_s \star h_c + s_c \star h_s)$$

Thanks for your attention