# EE 703: Digital Message Transmission

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1. A complex random vector $\mathbf{Z} = \mathbf{X} + j\mathbf{Y}$ is said to a complex Gaussian vector if $\mathbf{X}$ and $\mathbf{Y}$ are jointly Gaussian vectors. In other words, $\mathbf{Z}$ is a complex Gaussian vector if the components of $\tilde{\mathbf{Z}}$ are jointly Gaussian where
$$\tilde{\mathbf{Z}} = \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \end{bmatrix}.$$
Prove that $\mathbf{U} = e^{j\phi}\mathbf{Z}$ is a complex Gaussian vector when $\mathbf{Z}$ is a complex Gaussian vector by showing that the components of the following vector are jointly Gaussian.
$$\tilde{\mathbf{U}} = \begin{bmatrix} \text{Re}(e^{j\phi}\mathbf{Z}) \\ \text{Im}(e^{j\phi}\mathbf{Z}) \end{bmatrix} = \begin{bmatrix} \mathbf{X}\cos\phi - \mathbf{Y}\sin\phi \\ \mathbf{X}\sin\phi + \mathbf{Y}\cos\phi \end{bmatrix}$$
*Hint:* To show that the components of $\tilde{\mathbf{U}}$ are jointly Gaussian, show that for any $a_i$'s and $b_i$'s not all of which are zero
$$\sum_{i=1}^{n} a_i(X_i\cos\phi - Y_i\sin\phi) + \sum_{i=1}^{n} b_i(X_i\sin\phi + Y_i\cos\phi)$$
is a Gaussian random variable. Think about when it can fail to be a Gaussian random variable and arrive at a contradiction.

2. Let $\psi_1(t), \psi_2(t), \dots, \psi_K(t)$ be a complex orthonormal basis. Let $n(t) = n_c(t) + jn_s(t)$ be complex white Gaussian noise with PSD $2\sigma^2$. Then $n_c(t)$ and $n_s(t)$ are independent real WGN processes with PSD $\sigma^2$. Consider the projection of $n(t)$ onto the orthonormal basis
$$\mathbf{N} = \begin{bmatrix} \langle n, \psi_1 \rangle \\ \vdots \\ \langle n, \psi_K \rangle \end{bmatrix}.$$
Show that $\mathbf{N}$ is a complex Gaussian vector i.e. the components of the following vector are jointly Gaussian.
$$\tilde{\mathbf{N}} = \begin{bmatrix} \text{Re}(\langle n, \psi_1 \rangle) \\ \vdots \\ \text{Re}(\langle n, \psi_K \rangle) \\ \text{Im}(\langle n, \psi_1 \rangle) \\ \vdots \\ \text{Im}(\langle n, \psi_K \rangle) \end{bmatrix}.$$
*Hint:* Let $\psi_i(t) = \alpha_i(t) + j\beta_i(t)$. Let
$$\begin{aligned} X_i &= \text{Re}(\langle n, \psi_i \rangle) = \langle n_c, \alpha_i \rangle + \langle n_s, \beta_i \rangle \\ Y_i &= \text{Im}(\langle n, \psi_i \rangle) = \langle n_s, \alpha_i \rangle - \langle n_c, \beta_i \rangle. \end{aligned}$$
To show that the components of $\tilde{\mathbf{N}}$ are jointly Gaussian, show that for any $a_i$'s and $b_i$'s not all of which are zero
$$\sum_{i=1}^{n} a_i X_i + \sum_{i=1}^{n} b_i Y_i$$
is a Gaussian random variable. Think about when it can fail to be a Gaussian random variable and arrive at a contradiction.