Digital Modulation

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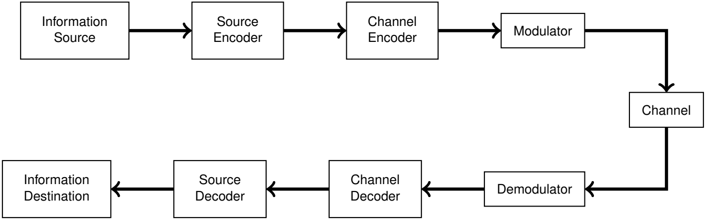
Digital Modulation

Definition

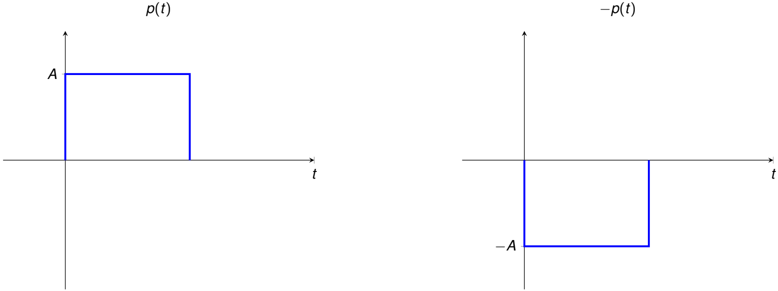
The process of mapping a bit sequence to signals for transmission over a channel.



Digital Modulation

Example (Binary Baseband PAM)

$1 \rightarrow p(t)$ and $0 \rightarrow -p(t)$



Classification of Modulation Schemes

- Memoryless
  - Divide bit sequence into k-bit blocks
  - Map each block to a signal $s_m(t), \;\; 1 \leq m \leq 2^k$
  - Mapping depends only on current k-bit block
- Having Memory
  - Mapping depends on current k-bit block and $L - 1$ previous blocks
  - L is called the constraint length
- Linear
  - Complex baseband representation of transmitted signal has the form
    $$u(t) = \sum_n b_n g(t - nT)$$
    where b_n's are the transmitted symbols and g is a fixed baseband waveform
- Nonlinear

Signal Space Representation

Signal Space Representation of Waveforms

- Given M finite energy waveforms, construct an orthonormal basis

$$s_1(t), \ldots, s_M(t) \rightarrow \underbrace{\phi_1(t), \ldots, \phi_N(t)}_{\text{Orthonormal basis}}$$

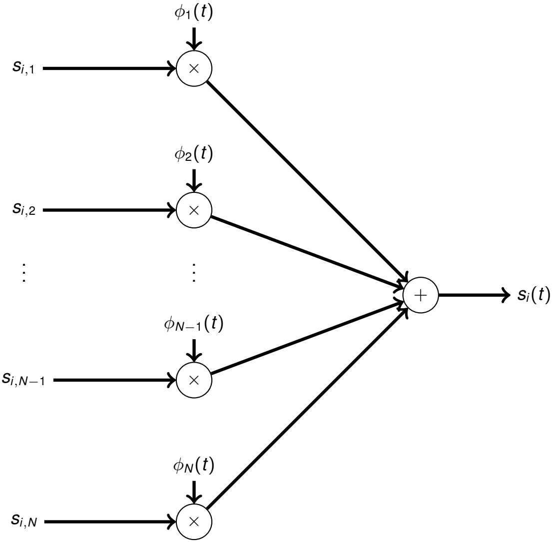
$$\langle \phi_i, \phi_j \rangle = \int_{-\infty}^{\infty} \phi_i(t)\phi_j^*(t)\, dt = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

- Each $s_i(t)$ is a linear combination of the basis vectors

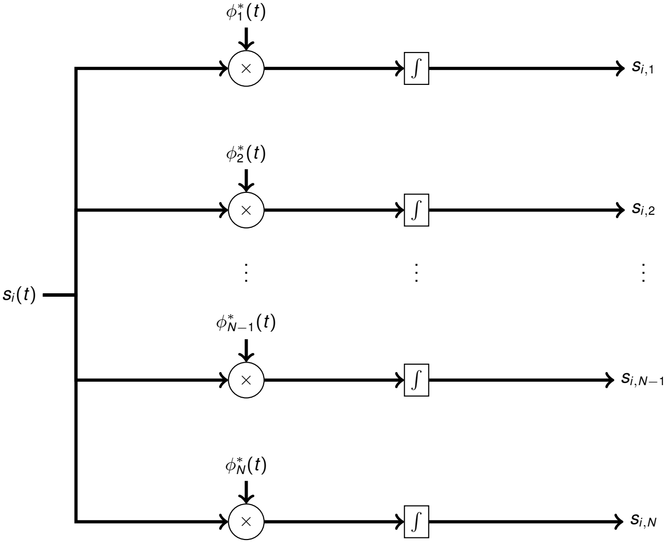
$$s_i(t) = \sum_{n=1}^{N} s_{i,n}\phi_n(t), \qquad i = 1, \ldots, M$$

- $s_i(t)$ is represented by the vector $\mathbf{s}_i = \begin{bmatrix} s_{i,1} & \cdots & s_{i,N} \end{bmatrix}^T$
- The set $\{\mathbf{s}_i : 1 \leq i \leq M\}$ is called the signal space representation or constellation

Constellation Point to Waveform



Waveform to Constellation Point



Gram-Schmidt Orthogonalization Procedure

- Algorithm for calculating orthonormal basis for $s_1(t), \dots, s_M(t)$
- Consider $M = 1$

$$\phi_1(t) = \frac{s_1(t)}{\|s_1\|}$$

where $\|s_1\|^2 = \langle s_1, s_1 \rangle$

- Consider $M = 2$

$$\phi_1(t) = \frac{s_1(t)}{\|s_1\|}, \quad \phi_2(t) = \frac{\gamma(t)}{\|\gamma\|}$$

where $\gamma(t) = s_2(t) - \langle s_2, \phi_1 \rangle \phi_1(t)$

- Consider $M = 3$

$$\phi_1(t) = \frac{s_1(t)}{\|s_1\|}, \quad \phi_2(t) = \frac{\gamma_1(t)}{\|\gamma_1\|}, \quad \phi_3(t) = \frac{\gamma_2(t)}{\|\gamma_2\|}$$

where

$$\begin{aligned} \gamma_1(t) &= s_2(t) - \langle s_2, \phi_1 \rangle \phi_1(t) \\ \gamma_2(t) &= s_3(t) - \langle s_3, \phi_1 \rangle \phi_1(t) - \langle s_3, \phi_2 \rangle \phi_2(t) \end{aligned}$$

Gram-Schmidt Orthogonalization Procedure

- In general, given $s_1(t), \ldots, s_M(t)$ the kth basis function is

$$\phi_k(t) = \frac{\gamma_k(t)}{\|\gamma_k\|}$$

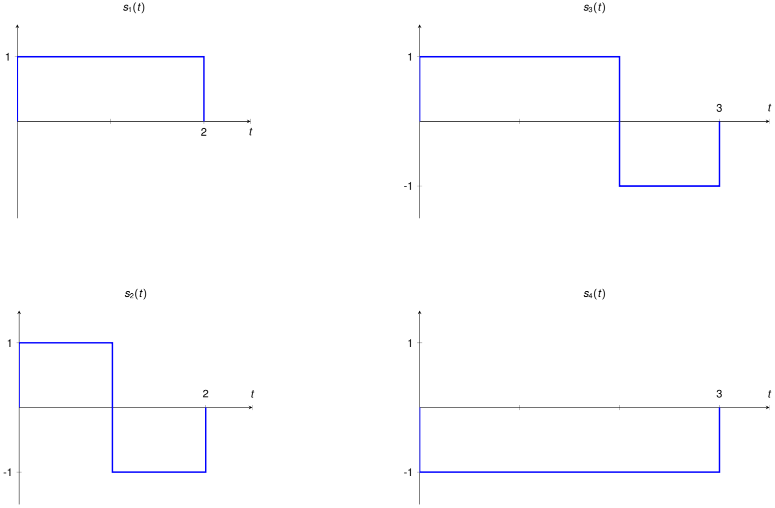
where

$$\gamma_k(t) = s_k(t) - \sum_{i=1}^{k-1} \langle s_k, \phi_i \rangle \phi_i(t)$$

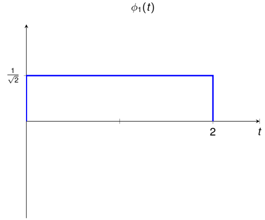
is not the zero function

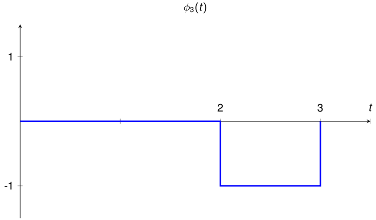
- If $\gamma_k(t)$ is zero, $s_k(t)$ is a linear combination of $\phi_1(t), \ldots, \phi_{k-1}(t)$. It does not contribute to the basis.

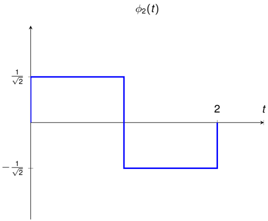
Gram-Schmidt Procedure Example



Gram-Schmidt Procedure Example







$$\mathbf{s}_1 = \begin{bmatrix}\sqrt{2} & 0 & 0\end{bmatrix}^T$$
$$\mathbf{s}_2 = \begin{bmatrix}0 & \sqrt{2} & 0\end{bmatrix}^T$$
$$\mathbf{s}_3 = \begin{bmatrix}\sqrt{2} & 0 & 1\end{bmatrix}^T$$
$$\mathbf{s}_4 = \begin{bmatrix}-\sqrt{2} & 0 & 1\end{bmatrix}^T$$

Properties of Signal Space Representation

- Energy

$$E_m = \int_{-\infty}^{\infty} |s_m(t)|^2 \, dt = \sum_{n=1}^{N} |s_{m,n}|^2 = \|\mathbf{s}_m\|^2$$

- Inner product

$$\langle s_i(t), s_j(t) \rangle = \langle \mathbf{s}_i, \mathbf{s}_j \rangle$$

Modulation Schemes

Pulse Amplitude Modulation

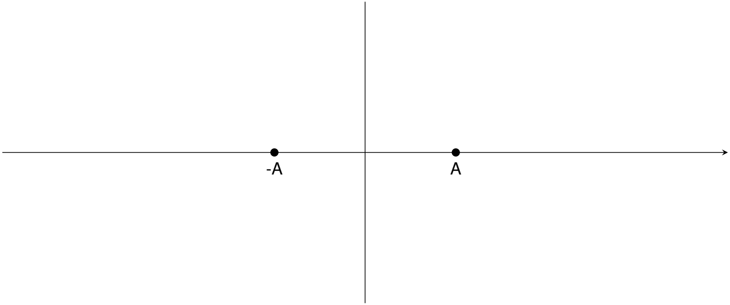
- Signal Waveforms

$$s_m(t) = A_m p(t), \quad 1 \leq m \leq M$$

where $p(t)$ is a pulse of duration T and A_m's denote the M possible amplitudes.

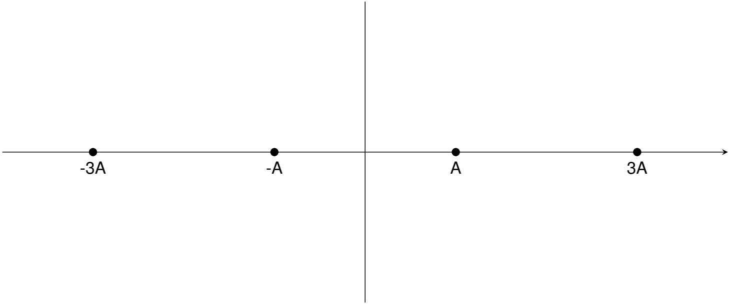
- Example $M = 2$, $p(t)$ is a real pulse

$A_1 = -A, A_2 = A$ for a real number A



Pulse Amplitude Modulation

- Example $M = 4$, $p(t)$ is a real pulse
  $A_1 = -3A, A_2 = -A, A_3 = A, A_4 = 3A$



Phase Modulation

- Complex envelope of phase modulated signals

$$s_m(t) = p(t)e^{j\frac{\pi(2m-1)}{M}}, \quad 1 \leq m \leq M$$

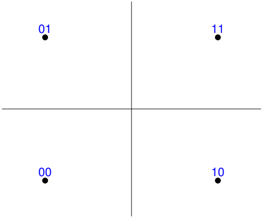
  where $p(t)$ is a real baseband pulse of duration T

- Corresponding passband signals

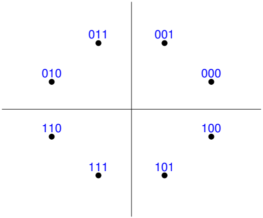
$$\begin{aligned} s_m^p(t) &= \text{Re}\left[\sqrt{2}s_m(t)e^{j2\pi f_c t}\right] \\ &= \sqrt{2}p(t)\cos\left(\frac{\pi(2m-1)}{M}\right)\cos 2\pi f_c t \\ &\quad -\sqrt{2}p(t)\sin\left(\frac{\pi(2m-1)}{M}\right)\sin 2\pi f_c t \end{aligned}$$

Constellation for PSK

QPSK, $M = 4$



Octal PSK, $M = 8$



Quadrature Amplitude Modulation

- Complex envelope of QAM signals

$$s_m(t) = (A_{m,i} + jA_{m,q})p(t), \quad 1 \leq m \leq M$$

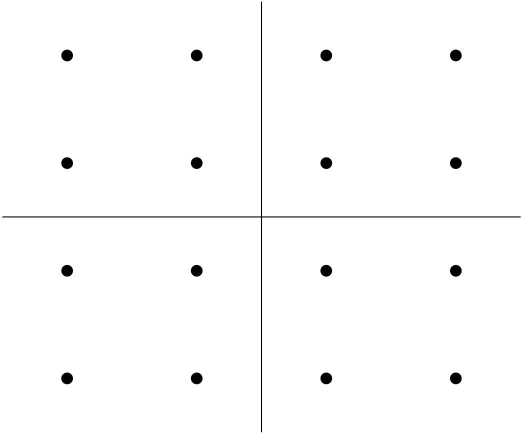
where $p(t)$ is a real baseband pulse of duration T

- Corresponding passband signals

$$\begin{aligned} s_m^p(t) &= \text{Re}\left[\sqrt{2}s_m(t)e^{j2\pi f_c t}\right] \\ &= \sqrt{2}A_{m,i}p(t)\cos 2\pi f_c t - \sqrt{2}A_{m,q}p(t)\sin 2\pi f_c t \end{aligned}$$

Constellation for QAM

16-QAM



Thanks for your attention