EE 703: Digital Message Transmission (Autumn 2016)
Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay

Assignment 1: 20 points — Date: August 4, 2016

1. [5 points] Let $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$ be an orthonormal basis for a set of signals $s_1(t), s_2(t), \dots, s_M(t)$. Let $\mathbf{s}_i \in \mathbb{C}^N$ be the signal space representation of $s_i(t)$. Show that $\langle s_i, s_j \rangle = \langle \mathbf{s}_i, \mathbf{s}_j \rangle$.

2. [5 points] Let $\hat{s}_p(t)$ be the Hilbert transform of a passband signal $s_p(t)$. Show that $\langle s_p, \hat{s}_p \rangle = 0$.

3. [5 points] Suppose we define the complex envelope of a passband signal $s_p(t)$ centered at $\pm f_c$ as

$$S(f) = 2S_p(f - f_c)u(-f + f_c)$$

where $S_p(f)$ is the Fourier transform of $s_p(t)$. Derive the following with explanations for each step.

   (a) $s_p(t)$ in terms of $s(t)$

   (b) $s_p(t)$ in terms of $s_c(t)$ and $s_s(t)$ (the in-phase and quadrature components of $s(t)$)

   (c) $s(t)$ in terms of $s_p(t)$

   (d) $S_p(f)$ in terms of $S(f)$

   (e) The relationship between $\|s\|^2$ and $\|s_p\|^2$.

4. [5 points] Consider the passband signals $s_1(t) = \sqrt{2}\cos(2\pi f_1 t)$ and $s_2(t) = \sqrt{2}\cos(2\pi f_2 t)$ where $f_1 \neq f_2$. Calculate the complex baseband representations of these signals for $f_c = f_1$.