EE 703: Digital Message Transmission (Autumn 2016)
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Problem Set 1 — Date: July 19, 2016

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1. Prove that
$$x(at) \rightleftharpoons \frac{1}{|a|} X\left(\frac{f}{a}\right)$$
2. Prove that
$$x(t) \star y(t) \rightleftharpoons X(f)Y(f).$$
3. Prove that
$$x(t)y(t) \rightleftharpoons X(f) \star Y(f).$$
4. Prove that
$$\int_{-\infty}^{\infty} x(t)y^*(t)\ dt = \int_{-\infty}^{\infty} X(f)Y^*(f)\ df.$$
5. Let $x(t)$ be a purely imaginary-valued signal, i.e Im $[x(t)] = 0$. Prove that the Fourier transform of $x(t)$ is anti-symmetric, i.e. it satisfied the following equation:
$$X(f) = -X^*(-f).$$
6. The *rectangular pulse* of unit amplitude and unit duration centered at $t = 0$ is given by
$$\Pi(t) = \begin{cases} 1, & |t| \leq \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$
Prove that
$$\Pi\left(\frac{t}{T}\right) \rightleftharpoons T\text{sinc}(fT)$$
where $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.
7. Prove that
$$\int_{-\infty}^{t} x(\tau)\ d\tau \rightleftharpoons \frac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$$