EE 703: Digital Message Transmission (Autumn 2016)
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Problem Set 3 — Date: September 4, 2016

1. Determine the power spectral density of the following line coding scheme:

$$u(t) = \sum_{n=-\infty}^{\infty} b_n p(t - nT)$$

where $p(t) = I_{[0,T)}(t)$ and the symbol b_n is the obtained by mapping a zero bit to amplitude $-A$ and mapping a one bit to amplitude $2A$. Assume that the bits used to generate b_n are independent and equally likely to be zero or one. Simplify your answer such that it does not contain any infinite summations.

2. Let $X \sim \mathcal{N}(0, 1)$ and let W be a discrete random variable which is equally likely to be ± 1. Assume that W is independent of X. Let $Y = WX$.
   (a) Show that $Y \sim \mathcal{N}(0, 1)$.
   (b) Show that X and Y are uncorrelated.
   (c) Show that X and Y are not independent.

3. If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ are independent Gaussian random variables, show that $X_1 + X_2 \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. *Hints: What is the pdf of the sum of independent random variables? It is enough to show that* $X_1 + X_2 - \mu_1 - \mu_2 \sim \mathcal{N}(0, \sigma_1^2 + \sigma_2^2)$.

4. If $\mathbf{X} \sim \mathcal{N}(\mathbf{m}, \mathbf{C})$ is an $n \times 1$ Gaussian random vector, find the distribution of $\mathbf{AX} + \mathbf{b}$ where $\mathbf{A}$ is a $n \times n$ matrix and $\mathbf{b}$ is an $n \times 1$ vector.

5. Let X be a Gaussian random variable with mean $\mu = -3$ and variance $\sigma^2 = 4$. Express the folllowing probabilities in terms of the Q function with positive arguments.
   (a) $P[X > 5]$
   (b) $P[X < -1]$
   (c) $P[1 < X < 4]$
   (d) $P[X^2 + X > 2]$