EE 703: Digital Message Transmission (Autumn 2016)
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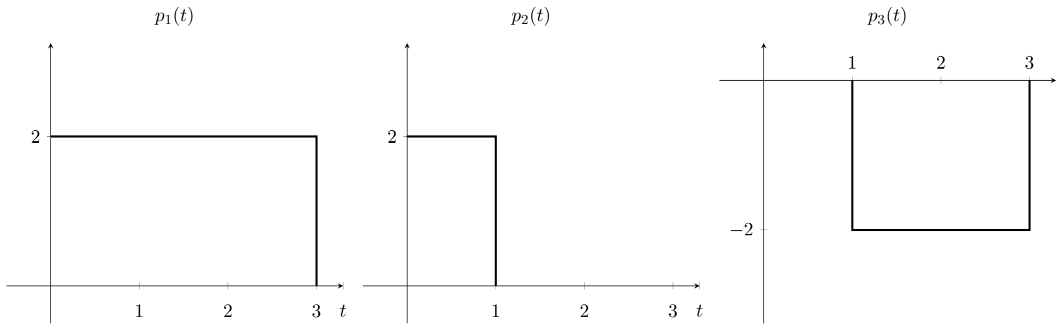
Midsemester Exam : 25 points — September 6, 2016

1. (5 points) Determine an orthonormal basis for the signals $s_1(t) = p_1(t) + jp_2(t)$, $s_2(t) = p_2(t) + jp_3(t)$, $s_3(t) = p_3(t) + jp_1(t)$ where $p_1(t)$, $p_2(t)$, and $p_3(t)$ are defined as follows.




2. (5 points) Determine the power spectral density of the following line coding scheme:

$$u(t) = \sum_{n=-\infty}^{\infty} b_n p(t - nT)$$

where $p(t) = I_{[0,T)}(t)$ and the symbol $b_n$ is the obtained by mapping 00 to amplitude $-3A$, 01 to amplitude $-A$, 11 to amplitude $A$, and 10 to amplitude $3A$. Assume that the message bits used to generate $b_n$ are independent and equally likely to be zero or one. Simplify your answer such that it does not contain any infinite summations. The formula for the PSD is as follows.

$$S_u(f) = \frac{|P(f)|^2}{T} \sum_{k=-\infty}^{\infty} R_b[k] e^{-j2\pi kfT}$$

3. (5 points) Consider the random process $X(t)$ resulting from a sinusoid with random phase.

$$X(t) = A\cos(2\pi f_c t + \Theta)$$

where $A$ and $f_c$ are constants and $\Theta$ is uniformly distributed on $[-\pi, \pi]$.

   (a) Show that $X(t)$ is wide-sense stationary.

   (b) Find the power spectral density of $X(t)$.

4. Consider the random process $X(t)$ resulting from an amplitude modulated pulse train given by

$$X(t) = \sum_{i=-\infty}^{\infty} A_i p(t - iT)$$

where the $A_i$'s are independent and identically distributed discrete random variables which are equally likely to be $\pm 1$ and $p(t)$ is a unit pulse of duration $T$ i.e. $p(t) = 1$ for $t \in [0, T)$ and 0 otherwise.

   (a) (2 points) Prove that $X(t)$ is not wide-sense stationary.

   (b) (3 points) Prove that $X(t)$ is wide-sense cyclostationary with respect to time interval $T$.

5. (5 points) Consider jointly Gaussian random variables $X_1$ and $X_2$ with $E[X_1] = 1$, $E[X_2] = -2$, $\mathrm{var}(X_1) = 4$, $\mathrm{var}(X_2) = 1$, and $\mathrm{cov}(X_1, X_2) = -2$.

   (a) Find the probability $P[2X_1 - 3X_2 < 6]$ in terms of the $Q$ function.

   (b) Suppose that $Z = X_1 - aX_2$ where $a \neq 0$. Find the constant $a$ such that $Z$ is independent of $X_1$.