EE 703: Digital Message Transmission (Autumn 2020)
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Quiz 1: 10 points   Date: September 27, 2020

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1. [3 points] Let $\hat{s}_p(t)$ be the Hilbert transform of a passband signal $s_p(t)$. Show that $\langle s_p, \hat{s}_p \rangle = 0$.

2. [3 points] Suppose $P = (X, Y)$ is a random point on the two-dimensional plane whose coordinates X and Y are independent Gaussian random variables. Both X and Y have mean $A > 0$ and variance $\sigma^2 > 0$.

   (a) What is the probability that the point P lies in the **first quadrant** of the two-dimensional plane? The first quadrant consists of points whose x-coordinates and y-coordinates are both positive. Express your answer in terms of the Q function.

   (b) What is the probability that the point P lies in the **third quadrant** of the two-dimensional plane? The third quadrant consists of points whose x-coordinates and y-coordinates are both negative. Express your answer in terms of the Q function.

3. [4 points] Consider the following binary hypothesis testing problem where the hypotheses are equally likely.

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\begin{aligned}
H_0 &: Y \sim U\left[-\sqrt{\frac{e^2\pi}{2}}, \sqrt{\frac{e^2\pi}{2}}\right] \\
H_1 &: Y \sim \mathcal{N}(0, 1)
\end{aligned}
$$

   U denotes the uniform distribution, $\mathcal{N}$ denotes the Gaussian distribution and e is the base of the natural logarithm. *Hint:* $\sqrt{\frac{e^2\pi}{2}}$ *is greater than* $\sqrt{2}$.

   (a) Derive the optimal decision rule.

   (b) Find the decision error probability of the optimal decision rule. Express your answer in terms of the Q function.